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FREQUENCY METHODS FOR ROBUST ANALYSIS

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Abstract. *The following paper aims at giving analytical expression and confirming unequivocally the results achieved through frequency methods for robust analysis of stability and industrial control systems' performance in conditions under uncertainty. A numerical example of robust analysis of a system with internal model is presented. Methods of Nyquist- and Black-Nichols- analysis and the analisis through sensitivity functions for evaluation of robust properties of control systems are examined.*

Key words: *robust stability, robust performance, frequency methods for robust analysis*

1. INTRODUCTION

Industrial plants of control as dynamical systems are characterized by a priori uncertainty. Various synthesis approaches are used to achieve the desired performance.

One of them, where the designed system possesses robust properties (stability and performance), imposes that the controller must be designed as a robust one. The analysis of the robust properties of the designed system uses robust analysis tools. For this purpose various methods are used including frequency robust analysis methods.

This paper addresses the ***Nyquist-*** and ***Black-Nichols*** analysis methods as well as robust analysis methods using sensitivity functions. The results from their applications on the designed control systems under a priori uncertainty ensure complete understanding of the systems bahaviour and provide ability for robust analysis of their properties.

2. AIMS AND PROBLEM SETUP

The aim of the present work is to systematize the nature of the frequency methods for robust analysis of control systems and to explore comparatively the results from their application. Set forth and solved are the following tasks:

- to express analytically and visualize the influence of the a priori uncertainty upon the control systems stability;

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- to express analytically and visualize the requirements of the used frequency methods for robust analysis of the stability and the performance of industrial plant control systems under a priori uncertainty;
- to design robust internal model systems (with static and astatic plant) using the free parameter method using criteria as robust stability, robust performance and transient response with minimum integral of square error (ISE);
- to develop robust analysis of the designed systems and to confirm equivalency of the results obtained using frequency methods for robust analysis.

3. METHODS USED

The following approach is used to model the uncertainty and to express analytically and visualize the requirements of the frequency methods for analysis of the robust properties of the control systems.

Consider the structure schematics of a ***classical linear feedback control system*** (Fig.1). R denotes controller, G - plant, W_3 - influence of the disturbances ζ upon the controlled variable y, and y^0, u, ε, ϑ, n - setpoint, control signal, error, load and measurement noise. With reliable ($W_4(p) = 1$, $n(p) = 0$) measurements a simplified structure is used (Fig.2) of *a* ***classical linear feedback control system***.

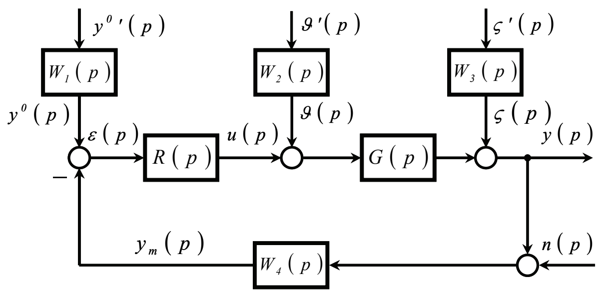


Fig.1

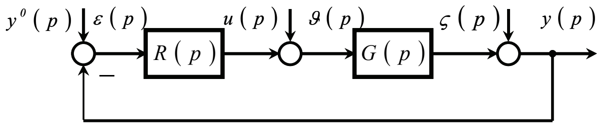


Fig.2

One possibility to model the manifestation of uncertainty in the real plant with frequency characteristics is ***the functional set*** $\Pi(j\omega)$ (1),

$$\Pi(j\omega) = \left\{ \begin{array}{l} \Delta\, G(j\omega) : \left| G(j\omega) - G^*(j\omega) \right| \le \bar{\ell}_a\ (\omega), \\ \Delta\, G(j\omega) : \dfrac{\left| G(j\omega) - G^*(j\omega) \right|}{\left| G^*(j\omega) \right|} \le \bar{\ell}_m\ (\omega), \\ \qquad (\omega \in [0;\infty)) \end{array} \right\}, \tag{1}$$

where $\Pi(j\omega) \in G(j\omega)$.

It is determined by the variations $\Delta G(j\omega)$ of the real plant characteristics $G(j\omega)$ around its ***nominal model*** $G^*(j\omega)$. The variations of $G(j\omega)$ are caused by the additive $l_a(j\omega)$ or multiplicative $l_m(j\omega)$ internal disturbance in the plant. The maximal value of this reparametrization and/or of the restructuring $\overline{\ell}_a(\omega)$ ($\overline{\ell}_m(\omega)$ respectively) determines the so called ***"disturbed at upper limit"*** plant model $G^{\blacksquare}(j\omega)$. The variations of $G(j\omega)$ are the cause for changes in the open-loop system characteristics, modelled using the functional set $\pi(j\omega)$ (2)

$$\pi(j\omega) \in \mathrm{W}(j\omega)\ , (\omega \in [0;\infty))\ . \tag{2}$$

The Nyquist-visualization methods of uncertainty and of the ***robust Nyquist-analysis***, present graphically the form of the $\pi(j\omega)$ set (2) with a family of circles $\pi(j\omega_i)$ (Fig.3). Fig.3. shows an astatic system characteristics and Fig.3.b - a static system. The centers of the circles $\pi(j\omega_i)$ are the representing points ω_i of the ***nominal open-loop system*** hodograph, presented by

$$W^*(j\omega_i) = R(j\omega_i)\, G^*(j\omega_i)\ .$$

For every sample ω_i of the frequency ω, the corresponding circle $\pi(j\omega_i)$ is the geometrical set of points taken by the depicting point $\omega = \omega_i$ resulting from the variations of the real system $W(j\omega_i) = R(j\omega_i)\, G(j\omega_i)$, from $W^*(j\omega_i)$ up to the **"*disturbed at upper limit*"** system, defined by

$$W^{\blacksquare}(j\omega_i) = R(j\omega_i)\, G^{\blacksquare}(j\omega_i)\ .$$

The radius $r^0(\omega_i)$ of the circle $\pi(j\omega_i)$ corresponding to ω_i, (Fig.3) is determined by (3) and the parametrical equation of the circle $\pi^0(j\omega_i)$, depicting the circle $\pi(j\omega_i)$ (Fig.3), is shown in (4). Fig.3.c and Fig.3.d present the corresponding analogs of the ***Black-Nichols-visualization*** of uncertainty and of the ***robust Black-Nichols-analysis***. Due to the specific axes scales the circles $\pi^0(j\omega_i)$ are transformed into elypses and when using logarithmic scale for ordinate axis - with waterdrop-shaped ovals.

$$r^0(\omega_i) = \left| l_a(\omega_i)\, R(\omega_i) \right| = \left| l_m(\omega_i)\, R(\omega_i)\, G^*(\omega_i) \right| \tag{3}$$

$$\pi^0(j\omega_i) = \begin{cases} \mathrm{Re}^0(\omega_i) = \mathrm{Re}^*(\omega_i) + r(\omega_i)\, cos\,\Omega\ , (\Omega \in [0, \infty)) \\ \mathrm{Im}^0(\omega_i) = \mathrm{Im}^*(\omega_i) + r(\omega_i)\, sin\,\Omega\ , (\Omega \in [0, \infty)) \end{cases} . \tag{4}$$

The controller synthesis requires knowledge of the uncertainty $\Pi(j\omega)$ (1). With its existence the closed-looped system should be stable. Otherwise the designed controller would be inefficient in practice.

Hence, the aim of the controller design should be a system with robust stability and robust performance.

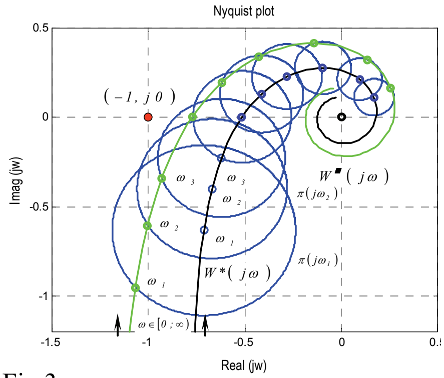


Fig.3.a

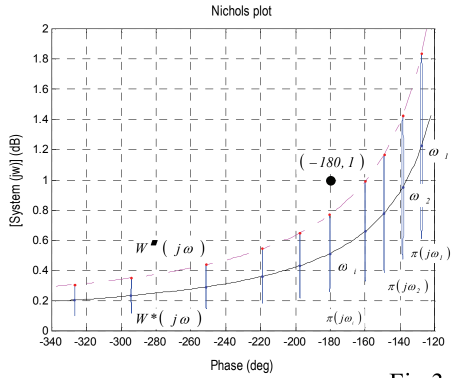


Fig.3.c

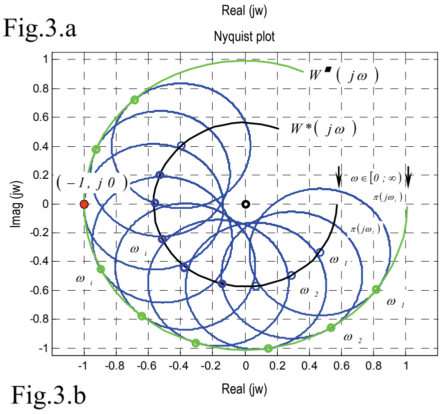


Fig.3.b

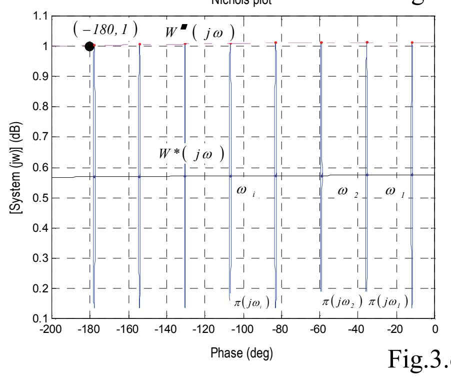


Fig.3.d

In reality the input (externel, signal) influence to the system is summerized and may be presented by the vector v of the exogenious signal disturbances (5):

$$v\ (p) = [\,y^0\ (p)\quad \vartheta\,(p)\quad \varsigma\,(p)\,]^T = W_v\ (p)\ v'\ (p)\,. \qquad (5)$$

In theory the disturbances are often normalized during control systems analysis in order to obtain possibilities for setting input influences of rich spectrum (step, pulse, sine, etc.). Appropriate matrix vector norms are used then.

Upon normalization (6) of $y^{0'}$ into y^0, and of ϑ' into ϑ and of ς' into ς for example, W_1, W_2 and W_3 are the corresponding normalization functions.

$$y^0(p) = W_1(p)\ y^{0'}(p)\,;\ \vartheta(p) = W_2(p)\ \vartheta'(p)\,;\ \varsigma(p) = W_3(p)\ \varsigma'(p)\,. \qquad (6)$$

In the following case for the summerized influence v (5) the normalization (weight) function $W_v(p)$ is used. With this generalized setup, the expressions, connecting the quantities that characterize the control system - normalized input signals (y^0, ϑ) and fed to the plant output disturbances ζ (Fig.2), are presented by the matrix form as in (7). From a generalized for real conditions point of view the neccessary and sufficient condition for system stability requires the all locii of the characteristic polynom ($1 + R(p)\ G(p) = 0$) are within the leftside complex semiplane. For estimation of systems properties in control systems theory are defined:

$$\begin{bmatrix} y \\ u \\ \varepsilon \end{bmatrix} = \begin{bmatrix} \dfrac{R\ G}{1+R\ G} & \dfrac{G}{1+R\ G} & \dfrac{-R\ G}{1+R\ G} \\ \dfrac{R}{1+R\ G} & \dfrac{-R\,G}{1+R\ G} & \dfrac{-R}{1+R\ G} \\ \dfrac{1}{1+R\ G} & \dfrac{-G}{1+R\ G} & \dfrac{-1}{1+R\ G} \end{bmatrix} \cdot \begin{bmatrix} y^{0} \\ \vartheta \\ \varsigma \end{bmatrix}. \tag{7}$$

- ***sensitivity function*** *e* (8), which represents the influence of the disturbance ζ upon the controlled variable *y* and is of basic importance when estimating controller efficiency in a feedback control system.

$$e(p) = \frac{1}{1+R(p)\ G(p)} = \frac{y\ (p)}{\varsigma\ (p)} = \frac{\varepsilon(p)}{y^{0}(p)}; \tag{8}$$

- ***complementary sensitivity function*** η (9), which represents the influence of the measurable noise *n* upon *y*

$$\eta(p) = \frac{R(p)\ G(p)}{1+R(p)\ G(p)} = \frac{y(p)}{y^{0}(p)-n(p)}\ , \tag{9}$$

which functions (*e* and η) always satisfy equation (10)

$$e(p)+\eta(p)=1. \tag{10}$$

When designing control systems, requirements may be set forth for: • reducing the measurable noise *n* ($e(p) \approx 1$, $\eta(p) \approx 0$); • exact tracking of the setpoint y^0 and effective actions against the disturbances, i.e. ($e(p) \approx 0$, $\eta(p) \approx 1$). These two requirements are contardictory. If it is assumed that the level of measurable noise *n* is low ($n(p) \to 0$) and rendering the effective action against internal disturbances (reparameterization and/or restructuring of the control plant) as basic requirement for robust control systems when designing robust systems the second requirement is accepted - for an exact tracking of the controlled varibable setpoint and counteraction against disturbances (11)

$$\begin{cases} e(p) = \dfrac{1}{1+G(p)R(p)} = \dfrac{y(p)}{\varsigma(p)} = \dfrac{\varepsilon(p)}{y^{0}(p)} \approx 0 \\ \eta(p) = \dfrac{G(p)R(p)}{1+G(p)R(p)} = \dfrac{y(p)}{y^{0}(p)-n(p)} \approx 1 \end{cases}. \tag{11}$$

Requirement (11) determines the basic goal of the control system to be: • minimization of error (12) (when the system is influenced by external distubance at y^0 and at ζ); • the modulus $|\,e\,|$ may be minimized (13); • the modulus $|\,\eta\,|$ to be of value as close as possible to unity (14) at (15)

$$\varepsilon\,(p) = y^0(p) - y\,(p)\,, \tag{12}$$

$$\lim_{\omega\to\infty} \left| e\,(\omega) \right| = \lim_{\omega\to\infty} \left| \frac{1}{1+G(\omega)\,R\,(\omega)} \right| = 0\ , \tag{13}$$

$$\lim_{\omega\to\infty} \left| \eta(\omega) \right| = \lim_{\omega\to\infty} \left| \frac{G(\omega)\,R(\omega)}{1+G(\omega)\,R(\omega)} \right| = 1\ , \tag{14}$$

$$\left| e(\omega) \right| + \left| \eta(\omega) \right| = 1\,. \tag{15}$$

Under summerized influence v (5), the error ε (12) is analytically presented with expressions (16) and (17)

$$\varepsilon(p) = \frac{1}{1+R(p)\,G(p)}\,v(p) = e(p)\,v(p)\ ;\ \left|\varepsilon(\omega)\right| = \left| e(\omega)\,v(\omega) \right|\ , \tag{16}$$

$$\begin{cases} \varepsilon^2(p) = v^2(p)\left(\dfrac{1}{1+R(p)\,G(p)}\right)^2; \\ \left|\varepsilon(\omega)\right|^2 = \left|v(\omega)\right|^2 \cdot \left|\dfrac{1}{1+R(\omega)\,G(\omega)}\right|^2 \end{cases} . \tag{17}$$

With robust systems design under summerized influence (5) it is assumed for the performance criterion to be that for minimal integral of square error $\Im$ (18). σ (19) may be adopted as equivalent reciprocal analog of $\Im$ in the frequency characteristics domain

$$\Im = \min_R \max_{G\in\Pi} \int_0^\infty \varepsilon^2(t)\,dt\ , \tag{18}$$

$$\sigma = \min_R \max_{G\in\Pi} \frac{1}{2\pi} \int_{-\infty}^{\infty} \left| \frac{1}{1+G(\omega)R(\omega)} \right|^2 \left| v(\omega) \right|^2 d\,\omega\,. \tag{19}$$

In this case the requirements for the controller R in the system are to be defined in such a way that with summarized influence v to minimize the integral of square error (18). the optimization task (20), solved during R synthesis and taking into account ***Parseval's theorem*** is presented in the frequency domain, as (21)

$$\left\|\varepsilon(t)\right\|_2^2 = \int_0^\infty \varepsilon^2(t)\,dt\ , \tag{20}$$

$$\min_R \left\|\,\varepsilon(\omega)\right\|_2^2 = \min_R \frac{1}{2\pi}\int_{-\infty}^{\infty} \left|\,\varepsilon(\omega)\,\right|^2 d\omega\ , \tag{21}$$

where the integrating result defines the average value of $|\,\varepsilon\,|$. When using (5) as an equivalent of (21) we get (22)

$$\min_R \| e(\omega)\, v(\omega) \|_2^2 = \min_R \frac{1}{2\pi} \int_{-\infty}^{\infty} |e(\omega)\, v(\omega)|^2 d\omega \,. \tag{22}$$

The integral expression in (22) is the average value of the sensitivity function e modulus, weighed for W_v.

4. Robust Stability and Performance

The condition for robust stability of the system may be obtained using ***Nyquis-analysis*** in the following way. Fig.4. shows an astatic system characteristics and Fig.4.b - a static system characteristics. In order for the system to be stable in the whole range $\Pi(j\omega)$ of variations $\Delta G(j\omega)$ (and robustly stable in this sense), it is neccessary for the set $\pi(j\omega)$, corresponding to $\Pi(j\omega)$, not to cover (Fig.4) the point $(-1, j\,0)$ for not a single value of the frequency ω in the range $\omega \in [0,\infty)$. It is obviously possible only in the cases (Fig.4), when the distance from any point $\omega = \omega_i$ of $W^*(j\omega)$, determined by the modulus value

$$|1 + G^*(\omega_i)\, R(\omega_i)|,$$

to the point $(-1, j\,0)$ is greater than the radius (23) $r^0(\omega_i)$

$$r^0(\omega_i) = |G^*(\omega_i)\, R(\omega_i)|\; \overline{\ell}_m(\omega_i)\,. \tag{23}$$

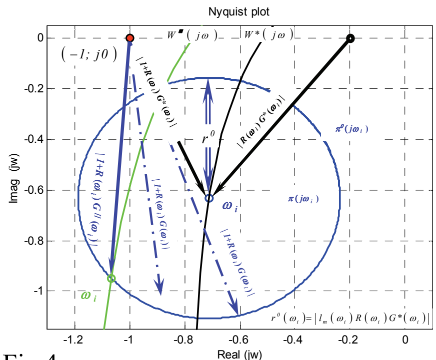


Fig.4.a

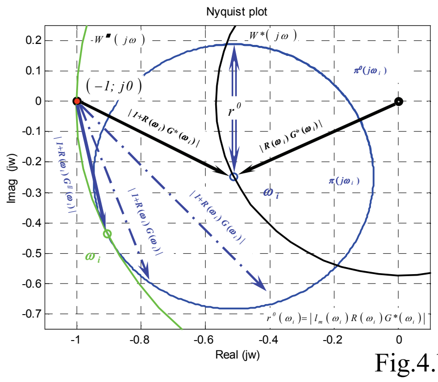


Fig.4.b

The analytical conditions for obtaining robust stability of the system for all points of the set $\pi(j\omega)$ (2) in these cases are depicted in (24), (25)

$$|1 + G^*(\omega)\, R(\omega)| > r^0(\omega), \;\; \forall\omega \,, \tag{24}$$

$$|1 + G^*(\omega)\, R(\omega)| > |G^*(\omega)\, R(\omega)|\; \overline{\ell}_m(\omega)\,, \;\; \forall\omega \,. \tag{25}$$

If it is accounted for that

$$G^*(j\omega)\, R(j\omega)\, (1 + G^*(j\omega)\, R(j\omega))^{-1} = \eta^*(j\omega)\,,$$

then (24), (25) are equivalently presented by (26), (27)

$$\left| \frac{G^*(\omega)R(\omega)}{1+G^*(\omega)R(\omega)} \right| \bar{\ell}_m(\omega) = \left|\eta^*(\omega)\right| \bar{\ell}_m(\omega) < 1, \ \forall\omega, \tag{26}$$

$$\left|\eta^*(\omega)\right|^{-1} > \bar{\ell}_m(\omega), \ \forall\omega. \tag{27}$$

If it is assumed that the controller R is optimally tuned in the context of the system local performance criterion σ to the nominal model of the plant G^*, i.e.

$$R(j\omega) \underset{\{\sigma = const\}}{\Leftrightarrow} G^*(j\omega) \ ,$$

then from (26) and (27) follows that the control system is ***robustly stable*** with controller $R(j\omega)$ (stable for the whole region $\Pi(j\omega)$ of variations) $\Delta G(j\omega)$) if and only if the complementary sensitivity function $\eta^*(j\omega)$ of the system for the nominal plant model $G^*(j\omega)$ satisfies the condition for exact limit (28)

$$\begin{cases} \left\| \eta^*(\omega)\,\bar{\ell}_m(\omega) \right\|_\infty = \\ = \sup\limits_{\omega} \left| \eta^*(\omega)\,\bar{\ell}_m(\omega) \right| < 1, \ \forall\omega \end{cases}. \tag{28}$$

(28) is a sufficient and neccessary condition for ***robust stability*** of the system. If (28) is not satisfied, this means that in the set Π (1) there exists a plant G, for which the closed-loop system with controller G is unstable or is on the limit of stability. That is why only robust stability is not enough and controller design seeks to satisfy a stronger condition defined by the conditions for robust performance of the control system. In this sense the aim is to design such a controller R, which for the plant $G \in \Pi$ with **"*disturbed at upper limit*"** model $G^{\blacksquare}$ to minimize the system error formed as a result from the influence of ζ. The plant with model $G^{\blacksquare}$ is this plant whose control is connected with the greatest increase of the system error. Thus expressed design of a goal is analytically presented by (29)

$$\min_R \max_{G\in\Pi} \int_0^\infty (\varepsilon(t))^2\,dt \,\hat{=}\, \min_R \max_{G\in\Pi} \frac{1}{2\pi} \int_{-\infty}^{\infty} \left| \frac{1}{1+G(\omega)R(\omega)} \right|^2 |v|^2\, d\omega. \tag{29}$$

Such a goal is reachable only in the cases (Fig.4), where the distance from any point $\omega = \omega_i$ of $W^*(j\omega)$ to the point $(-1, j\,0)$, decreased with the radius $r^0(\omega_i)$, is not greater than the corresponding distance for the frequency $\omega = \omega_i$ of $W(j\omega)$ to the point $(-1, j\,0)$. The analytical requirements for all points of the set $\pi(j\omega)$ in these cases are expressed by (30). After transformations, (30) is expressed by (31) and transforms the relation for the error (32) in the system into (33)

$$\left|1 + G(\omega)R(\omega)\right| \geq 1 + G^*(\omega)\,R(\omega) - r^0 \ , \ \ (\forall\ G \in \Pi;\ \forall\ \omega), \tag{30}$$

$$\left|e(\omega)\right| \leq \frac{\left|e^*(\omega)\right|}{1 - \left|\eta^*(\omega)\right| \cdot \bar{\ell}_m(\omega)} \ , \forall G \in \Pi; \forall\omega, \tag{31}$$

$$\varepsilon^2(t) \mathrel{\hat{=}} |\varepsilon(\omega)|^2 \equiv \left|\frac{1}{1+G(\omega)\,R(\omega)}\right|^2 |v(\omega)|^2 \tag{32}$$

$$\varepsilon^2(t) \mathrel{\hat{=}} |\varepsilon(\omega)|^2 \equiv \left|\frac{1}{1-|\eta^*(\omega)|\cdot\bar{\ell}_m(\omega)}\right|^2 |e^*(\omega)\cdot v(\omega)|^2, \forall G\in\Pi; \forall\omega. \tag{33}$$

From here follows that the basic requirement for robust control is expressed by (34), which is transformed to (35) and into (36) with respect to (31)

$$\|e(\omega)\,v(\omega)\|_\infty = \sup_\omega |e(\omega)\,v(\omega)| < 1 \;,\; \forall\; G\in\Pi\,, \tag{34}$$

$$\frac{|e^*(\omega)\;v(\omega)|}{1-|\eta^*(\omega)|\;\bar{\ell}_m(\omega)} < 1,\; \forall\omega\;, \tag{35}$$

$$|\eta^*(\omega)\;\bar{\ell}_m(\omega)| + |e^*(\omega)\;v(\omega)| < 1\,, \forall\omega\,. \tag{36}$$

The closed system will satisfy the requirements (34), if and only if the nominal ($G(p) \mathrel{\hat{\equiv}} G^*(p)$) closed-loop system is stable and the functions e^* and η^* satisfy (37)

$$|\eta^*(\omega)\;\bar{\ell}_m(\omega)| + |e^*(\omega)\;v(\omega)| = 1,\;\forall\omega\,. \tag{37}$$

Robust performance is defined by (36) and denotes robust stability (28) and ***nominal performance*** (38). The improvement of the indices of nominal performance (decrease the value of the modulus $|e^*(\omega)\;v(\omega)|$) worsens the robustness and increases the value of the modulus $|\eta^*(\omega)\;\ell_m(\omega)|$ while moving the closed-loop system to the point of instability for the plant $G\in\Pi$. Based on the above stated, the controller optimizing the system in the context of reaching robust performance, should satisfy the requirements (39)

$$|e^*(\omega)| < |v(\omega)|^{-1},\;\forall\;\omega\;\;, \tag{38}$$

$$\min_R\;\sup_\omega\;(|\eta^*(\omega)\;\bar{\ell}_m(\omega)| + |e^*(\omega)\;v(\omega)|)\;. \tag{39}$$

5. Frequency Methods for Robust Analysis

Robust analysis consists of verification for satisfied conditions (24), (28), (30) and (36) of the designed system, according to the criteria used at its synthesis. Its core is the exploration of the influence of uncertainty upon the stability of the designed system in a preliminary denoted (as one of the initial conditions for system synthesis) range of reparameterization and/or restructuring in the model of the controlled plant. The frequency methods used as tools in this study are:

- ***Nyquist-analysis methos*** for estimation of the robust stability (24), and robust performance (30) of the designed system (based on the characteristics of the open-loop system) through visualization of the range of reparameterization and/or restructuring in the model of the controlled plant;

• ***Black-Nichols-analysis method*** for estimation of the robust stability (24), and robust performance (30) of the designed system (based on the characteristics of the open-loop system) through visualization of the range of reparameterization and/or restructuring in the model of the controlled plant;

• ***method of sensitivity functions*** for estimation of the robust stability (24), and robust performance (30) of the designed system (based on the characteristics of the closed-loop system).

It has been proven [4] that the requirements for robust stability (24) and robust performance (30) using ***Nyquist-*** and ***Black-Nichols-analysis***, are equivalent to the conditions for robust stability (28) and robust performance (36) using the method of ***sensitivity functions***. The positive estimate obtained using these methods is an ***analytical proof*** that (in the context of the given range of reparametrization and/or restructuring of the analyzed systems):

• possess relevant robust properties;

• satisfy the conditions of the concrete criteria used at their synthesis.

The analytically formulated requirements for robust stability (24), (28) and robust performance (30) and (36) are the essence of the frequency methods for robust analysis and a basis of the criteria used at the synthesis of robust systems.

6. Numerical Examples

The structure of a known class [1] of internal model robust systems is shown in Fig.5. In [1,3] the free parameter method and the criteria for their synthesis are examined. The criteria used are: robust stability (24), (28), (40.a); robust performance (30), (36), (40.b) and a local performance criterion (40.c) - transient response with minimum ISE. The equivalent structure transformation of the setup in Fig.5 using (41) leads synonymously to the structure of a classical feedback system (Fig.6). And in reverse order, by using (42), the conformity between the structures of a classical (Fig.6) with $R_{(M)}$ and a robust with Q internal model systems (Fig.5) is complete. From this follows that the dependencies of the control action u and the controlled variable y on the setpoint y^0 and the disturbance ζ for the control systems in the structures from Fig.5 and Fig.6 are synonymously equivalent and the controllers $R_{(M)}$ and Q are connected by (41) and (42). The conformity between the two systems lets the robust system synthesis procedure be equivalently taken to the design either of Q, or $R_{(M)}$.

$$\Pi = \left\{ \begin{array}{l} \left\{ \begin{array}{l} \Delta\, G : \left| \dfrac{G\,(j\omega) - G^*(j\omega)}{G^*(j\omega)} \right| = \dfrac{\bar{\ell}_a(\omega)}{\left| G^*(j\omega) \right|} \le \bar{\ell}_m(\omega) = G^{\blacksquare} \\ a)\quad \| \eta\, \bar{\ell}_m \|_\infty = \sup\limits_\omega \left| \eta\, \bar{\ell}_m \right| < 1 \,; \qquad \bar{\ell}_m(\omega) < \left| \eta \right|^{-1},\ \forall\, \omega \\ b)\quad \| e v \|_\infty = \sup\limits_\omega \left| e v \right| < 1 \,; \qquad \left| \eta\, \bar{\ell}_m \right| + \left| e\, v \right| < 1 \,,\ \forall\, \omega \end{array} \right\} \\ c)\quad \| e \|_2^2 = \int\limits_0^\infty e^2(t)\, dt \,;\ \min\limits_{R_M} \| e \|_2^2 = \min \dfrac{1}{2\pi} \int\limits_{-\infty}^{\infty} \left| e\,(j\omega) \right|^2 d\,\omega \end{array} \right\} \tag{40}$$

$$R_{(M)}(p) = \frac{Q(p)}{1 - G^*(p)Q(p)}, \tag{41}$$

$$Q(p) = \frac{R_{(M)}(p)}{1 + G^*(p)\; R_{(M)}(p)} \quad . \tag{42}$$

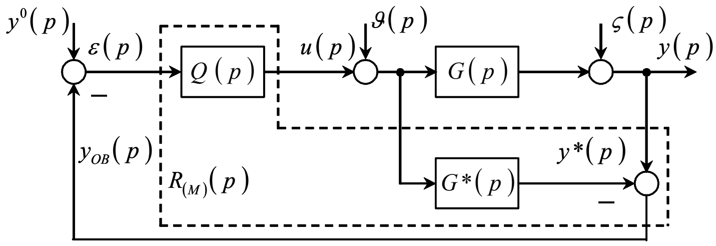


Fig. 5

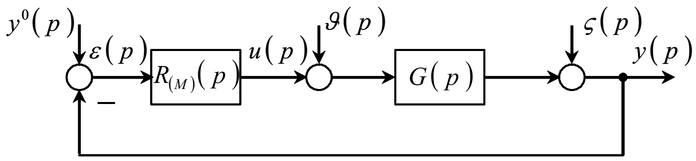


Fig. 6

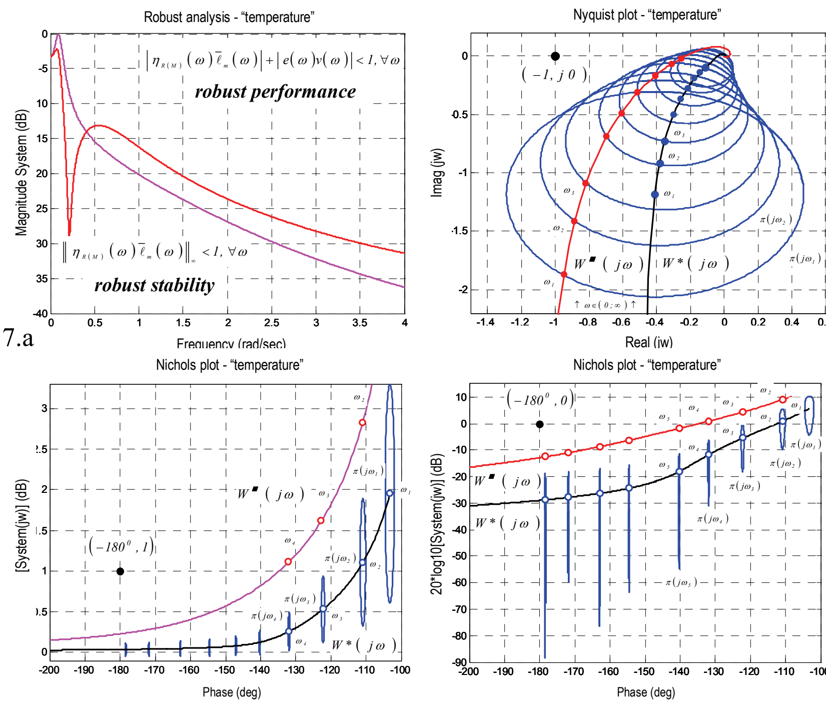


Fig. 7.a Fig. 7.b

Fig. 7.c Fig. 7.d

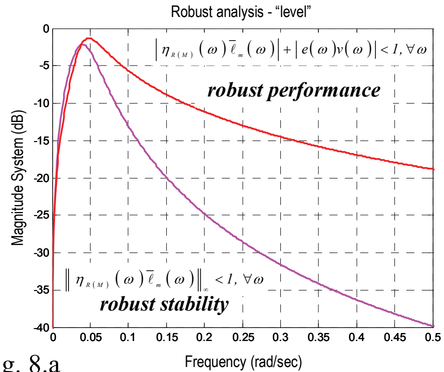


Fig. 8.a

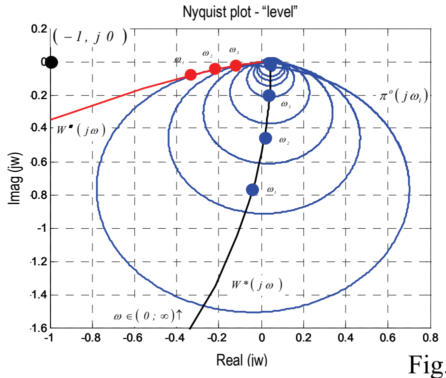


Fig. 8.b

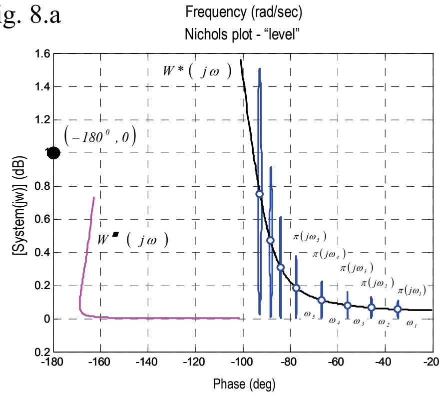


Fig. 8.c

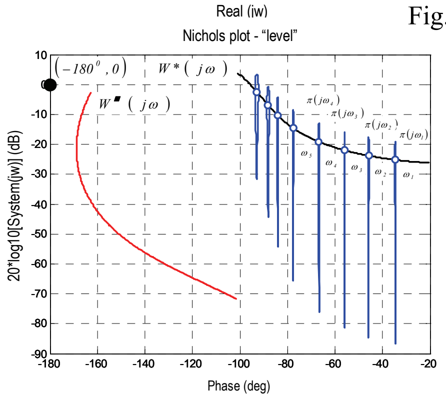


Fig. 8.d

The first numerical example is based on an industrial plant control system - the temperature of acute steam at the output of the drum boiler of an energy generating block turbine. In this case the controlled variable is the acute steam temperature and the control signal is the flow of feeding water to the injection installation. The ***nominal*** (43) and **"*disturbed at upper limit*"** (44) models of the summarized plant are given (simplified after approximation).

The second example for the implementation of frequency methods for robust analysis examines a system for regulating the feeding water level in a drum boiler. The input flow of the feeding water is the control action and the change of the water level in the drum boiler is the controlled variable of the plant. The ***nominal*** (46) and **"*disturbed at upper limit*"** (47) models of the drum boiler water level are given (simplified after approximation).

These data for the static (43), (44) and astatic (46), (47) plant models present the possibility to model the uncertainty with the functional set $\Pi(j\omega)$ (1) following the ***Nyquist-*** and ***Black-Nichols-visualization*** methods

$$G^*(p) = \frac{1{,}25\, e^{-2p}}{10\,p+1}, \tag{43}$$

$$G^{\blacksquare}(p) = \frac{2{,}1\, e^{-5p}}{10\,p+1}, \tag{44}$$

$$R_{(M)}(p) = 0{,}243335\left(\frac{1}{(0{,}967742p+1)} + \frac{0{,}909p}{(0{,}967742p+1)} + \frac{1}{(4{,}8387\,p^2+5\,p)}\right), \tag{45}$$

$$G^*(p) = \frac{(p+1)}{(0{,}05\,p+1)}\cdot\frac{1}{(0{,}01\,p+1)}\cdot\frac{1}{200p(14{,}25\,p+1)}, \tag{46}$$

$$G^{\blacksquare}(p) = \frac{(p+1)}{(63{,}01\,p+1)}\cdot\frac{1}{(0{,}01\,p+1)}\cdot\frac{1}{200p\;(14{,}25\,p+1)}, \tag{47}$$

$$R_{(M)}(p) = 7{,}043\left(\frac{1}{(0{,}05p+1)} + \frac{0{,}1\,p}{(0{,}05p+1)} + \frac{1}{(100\,p^2+150\,p)}\right) \;. \tag{48}$$

Based on the models given for the static (43), (44) and astatic (46), (47) plants and taking into account the criteria (40.a), (40.b), (40.c) and using the iterative free parameter method [1,3], the following systems have been designed: an internal model robust system (Fig.6) to control the acute steam temperature and an internal model robust system (Fig.6) to control the water level in the drum boiler of an energy generating block.

The synthesis results for the two systems are shown by: controller $R_{(M)}$ (45) -for acute steam temperature control; controller $R_{(M)}$ (48) -to control the water level in the drum boiler.

The designed robust systems with controllers (45) and (48) are modeled in order to demonstrate the application of the frequency methods for robust analysis for the concrete numerical examples. The models simulation gives an opportunity to estimate the systems properties from their characteristics. For this purpose frequency methods for robust analysis are used - the ***Nyquist-analysis method,*** the ***Black-Nichols-analysis method***, the ***sensitivity functions method***.

With the ***sensitivity functions method*** the fulfillment of the rquirements (28) and (36) has been estimated. The simulation results which prove robust stability and robust performance of the designed system for control of a static plant (the temperature) are shown in Fig.7.a, and for the control of the astatic plant (the level) - in Fig.8.a.

Nyquist- and ***Black-Nichols-robust analysis*** of the designed systems use the requirements (24) and (30). The simulation results which prove robust stability and robust performance of the designed system for control of a static plant (the temperature) are shown in Fig.7.b, Fig.7.c, Fig.7.d, and for the control of the astatic plant (the level) - in Fig.8.b, Fig.8.c, Fig.8.d. These figures show also the family of circles $\pi(j\omega_i)$ which visualize the influence of restructuring and/or repameterization upon the corresponding systems stability.

The third numerical example. Repetitive control systems (Fig.9) use a rejection filter ***ML*** (***Memory Loop***) (Fig.10).

A modified $\mathcal{ML}$-loop shown in Fig.11 is proposed as repetitive control systems (Fig.12). On the basis of $\mathcal{ML}$, proposed are five $\mathcal{ML}_i$ - structures (Fig.13 ÷ Fig.17). Their characteristics are presented in Fig.18, Fig.19.

For an industrial plant presented by ***nominal*** G^* model (49) and **"*disturbed at upper limit*"** G' (50) model, and the periodic signal disturbances with time period $T_p = 400\ s$ are designed robust repetitive control systems (Fig.12) with $\mathcal{PID}$-controller (51) and $\mathcal{ML}_i$-structures proposed (52) ÷ (57) (Fig.13 ÷ Fig.17), and a classic control system with $\mathcal{PID}$-controller (51).

The models simulation gives an opportunity to estimate the systems properties from their characteristics.

For these purpose frequency methods for robust analyses are used - the ***Nyquist-analysis method*** (Fig.20), the ***sensitivity functions method*** (Fig.21)

Fig.9

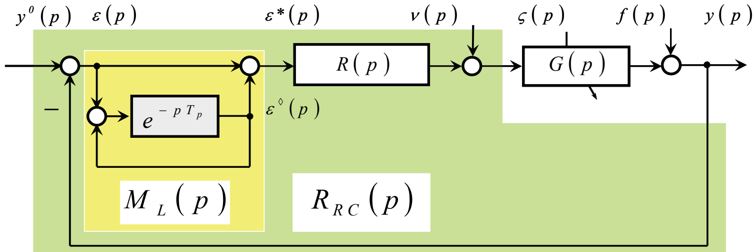
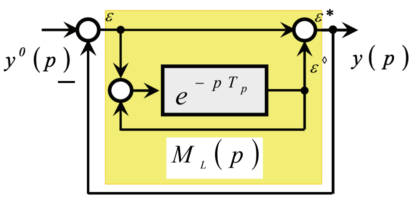


Fig.10



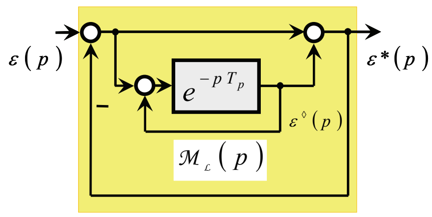


Fig.11

Fig.12

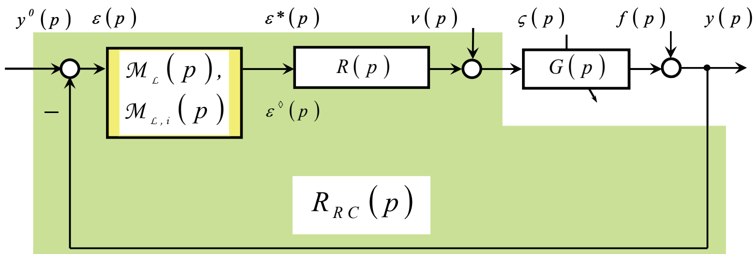


Fig.13

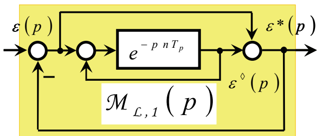


Fig.16

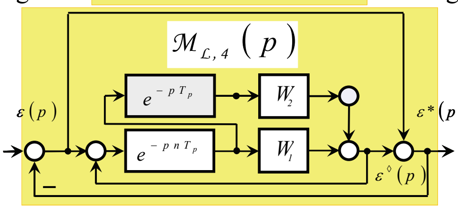
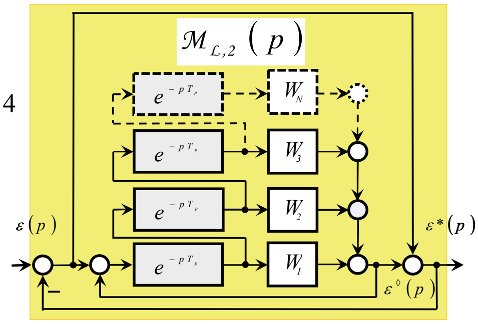


Fig.14



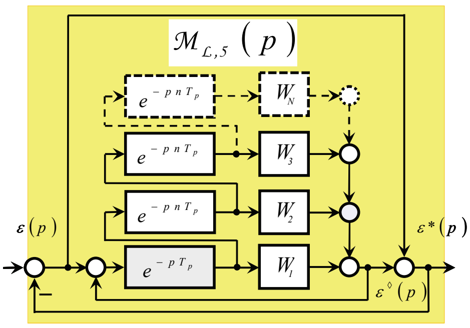


Fig.15

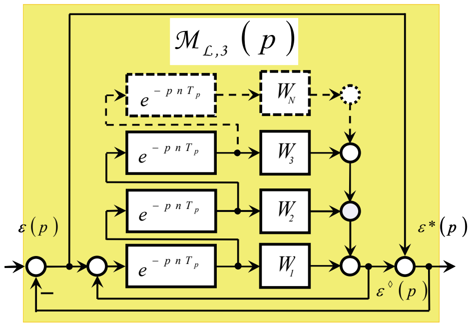


Fig.17

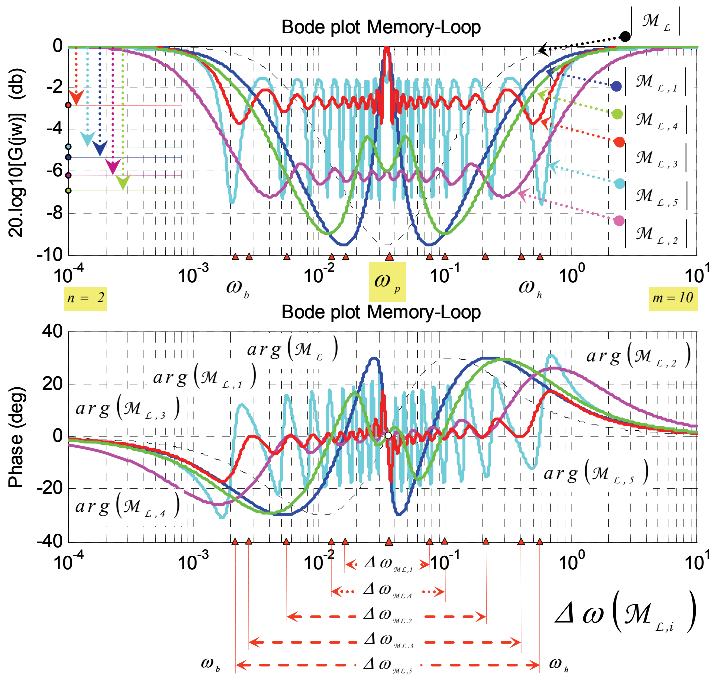


Fig. 18

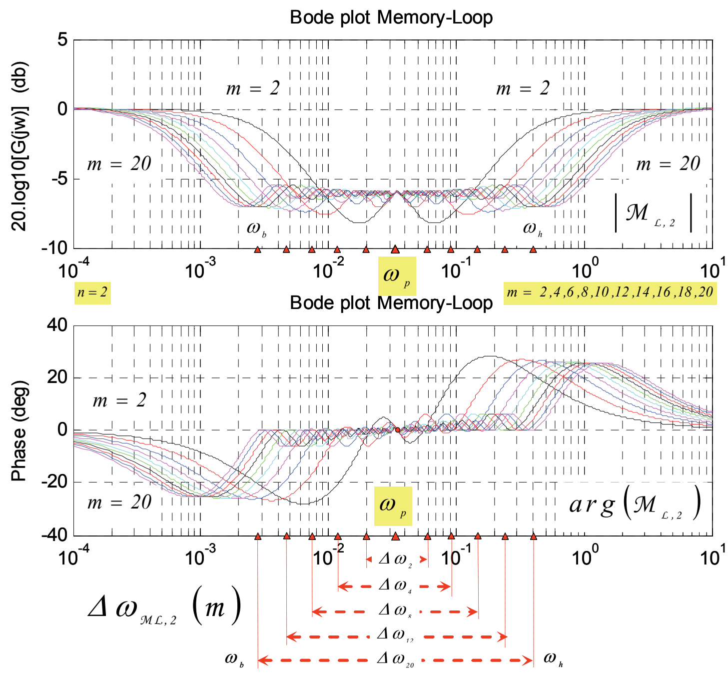


Fig. 19

$$G^*(p) = 0{,}15\ (1+4p)^{-1} e^{-10p} \quad (49)$$

$$G^{\blacksquare}(p) = 0{,}24(1+3p)^{-1} e^{-10p} \quad (50)$$

$$R(p) = 2{,}35(1+8p)(2p+1)(8p\,(0.4\,p+1))^{-1} \quad (51)$$

$$\mathrm{M}_{\mathrm{L}}(p) = (2 - e^{-pT_p})^{-1} = (2 - R_{2,2}^{(400)}(p))^{-1} \quad (52)$$

$$\mathrm{M}_{\mathrm{L},1}(p) = (2 - e^{-p\,i\,T_p})^{-1} = (2 - (R_{2,2}^{(400)}(p))^{i})^{-1} \quad (53)$$

$$\mathrm{M}_{\mathrm{L},2}(p) = \left(2 - \sum_{k=1}^{10} W_k(p)\, R_{2,2}^{(400)}(p)\right)^{-1} \quad (54)$$

$$\mathrm{M}_{\mathrm{L},3}(p) = \left(2 - \sum_{k=1}^{10} W_k(p)\, (R_{2,2}^{(400)}(p))^{10}\right)^{-1} \quad (55)$$

$$\mathrm{M}_{\mathrm{L},4}(p) = (2 - (W_0(p)\, R_{2,2}^{(400)}(p) + W_1(p)\, (R_{2,2}^{(400)}(p))^{10}))^{-1} \quad (56)$$

$$\mathrm{M}_{\mathrm{L},5}(p) = \left(2 - \left(W_0(p)\, R_{2,2}^{(400)}(p) + \sum_{k=1}^{10} W_k(p)\, (R_{2,2}^{(400)}(p))^{10}\right)\right)^{-1} \quad (57)$$

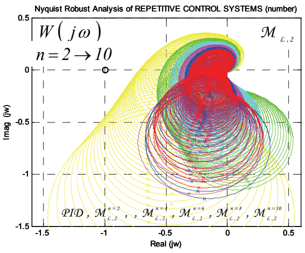


Fig. 20.a

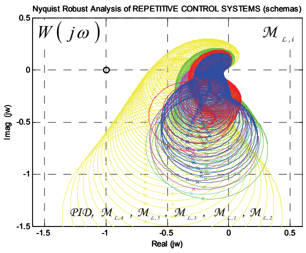


Fig. 20.b

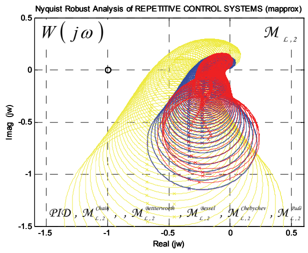


Fig. 20.c

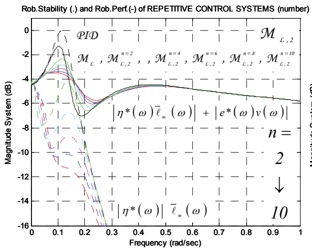


Fig. 21.a

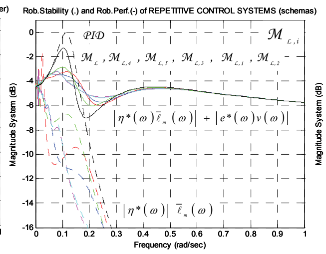


Fig.21. b

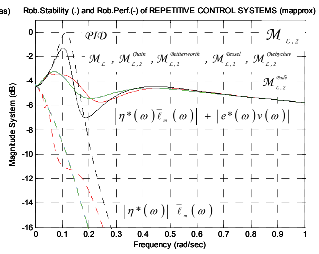


Fig. 21.c

7. Conclusions

The systematization of the explored frequency methods for robust analysis in this work demonstrates their analytical binding and specific features.

The ***functional set*** method visualizes the influence of restructuring and/or repameterization in the plants models upon control systems stability. The work presents a possibility to analytically express and visualize the requirements of the frequency methods for robust analysis.

In the examples given, the applicability and the working capacity of the methods are confirmed and, also, the equivalence of the obtained qualitative estimates for the concrete conditions according to the three robust analysis methods. The equivalence is obvious from the robust analysis results after: the ***sensitivity functions method*** the (28), (36) (Fig.7.a and Fig.8.a); ***Nyquist-analysis*** (Fig.7.b, Fig.8.b) and ***Black-Nichols-analysis*** (Fig.7.c, Fig.7.d, Fig.8.c, Fig.8.d) (24), (30).

The achievements in the present work impose the following basic conclusions:

● The estimation of the investigated systems performance, obtained using any of the presented frequency analysis methods is an analytical proof for reaching the criteria for robust stability and robust performance. This is shown for the systems from the explored numerical examples;

● The comparative analysis of the characteristics from Fig.7 and Fig.8 confirms the equivalence of the obtained estimates and the equally situated applicability of the presented frequency robust analysis methods using ***sensitivity functions, Nyquist-*** and ***Black-Nichols-analysis***;

● The results obtained from using any of the presented frequency methods give rich visual possibilities for estimation of systems performance. The results from the ***Nyquist-***

and ***Black-Nichols-analysis*** show in a direct form the influence of the restructuring and/or repameterization in the plant model upon the system's characteristics;

● The results from the application of the ***sensitivity functions*** method give also possibilities for quantitive estimates of the robust properties of the analyzed control (such as robust stability reserve, for example);

● The results in the present work confirm that the use of the three frequency methods for robust analysis ensures complete notion for the behaviour and the properties of the control systems under a priori uncertainty conditions. The methods are used [1,5,6,7, 8,9,10] as a proof of the robust performance during the analysis of industrial plants control systems under uncertainty conditions. Their requirements serve also as criteria for the design of robust control systems.

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FREKVENTNE METODE ZA ANALIZU ROBUSNOSTI

E. Nikolov

Cilj ovog rada je da analitički predstavi i nedvosmisleno potvrdi rezultate dobijene frekventnim metodama za robusnu analizu stabilnosti i karakteristike industrijskog sistema upravljanja u uslovima nesigurnosti. Prikazan je numerički primer analize robusnosti sistema sa unutrašnjim modelom. Ispitivani su metodi Nyqust-ove i Black-Nichols-ove analize sistema upravljanja i analiza pomoću funkcije osetljivosti za evaluaciju karakteristike robusnosti sistema upravljanja.

Key words: *robusna stabilnost, karakteristike robusnosti, frekventne metode za analizu robusnosti*