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HYBRID PREDICTIVE CONTROL WITH A PRESCRIBED DEGREE OF STABILITY

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Abstract. *In this paper we will consider one class of hybrid controllers. Such a control strategy is a mix of continuous dynamics and discrete events philosophy. Here we consider a finite set of model predictive controllers (MPC) which is the only advanced control technique to have had a significant and widespread impact on industrial process control. There are several advantages for wide acceptance of MPC: guaranteed stability, constraints handling and easy extension to multivariable and nonlinear systems. In this paper we add one more important property: significant increasing of transient performance. Proposed controllers, also, have prescribed degree of stability as a tuning parameter.*

Key words: *Hybrid control, predictive controller, prescribed degree of stability, asymptotic stability*

1. INTRODUCTION

The model predictive control (MPC) is the only advanced control methodology that has made a significant impact in industrial control engineering.

(i) The extension to multivariable case is easy
(ii) It handles constraints. The higher performance levels are associated with pushing the limits. That frequently leads to more profitable operation
(iii) In industrial applications control update rates are relatively low and there is enough time for on-line computation

The authoritative survey papers are presented in [1]–[3]. They noticed that most control laws, for example PID, do not explicitly consider the future implication of current control actions. MPC, on the other hand, explicitly computes the predicted behaviour over some horizon. One can therefore restrict the choice of current proposed input trajectories to those that do not lead to difficulties in the future.

The key problem in MPC is the stability. It had been known from [4] that making the horizon infinite in predictive control leads to guaranteed stability, but if was not known

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how to handle constraints with infinite horizons. The paper [5] made a breakthrough. The key idea is to reparameterise the predictive control problem in terms of finite number of parameters so that the optimisation can still be performed over a finite dimensional space. So MPC remains a quadratic programming (QP) problem. In [6] is presented the thought that there is no longer any reason to use finite horizon. In this paper such ideas will be used.

Here we will consider hybrid MPC. The hybrid system can be interpreted as digital real-time systems, which are embedded in analogue environments. The paper [7] proposes a framework for modelling and controlling models of systems described by interacting physical laws, logical rules, and operating constraints. The propositional logic is transformed into linear inequations involving integer and continuous variables. So is given mixed logical dynamical (MLD) systems.

In this paper the concept of multiple models and concept of switching MPC is used. The analogue part of the system is described by finite set of discrete-time models. As a set of controllers is used a finite set of MPC with the prescribed degree of stability. Here the switching rule is based on the selection of the best performance of the closed-loop systems. In the form of theorem is proved that hybrid system is asymptotically stable in the Lyapunov sense and performance of system is no worse then the best non-switching strategy.

The MPC has applications in many areas such as discrete-event systems [8], cooperative control [9], digital electronics [10] and financial engineering [11].

Hybrid model predictive control is a perspective area of research relevant to a range of important problems such as supervisory schemes in the process industry.

2. Multiple Models

In this part of paper we consider multiple model description of processes. It will be assumed that the process model is a member of admissible process models

$$F = \underset{p \in P}{U} F_p \tag{1}$$

where P is index set which represents the range of parametric uncertainty so that for each fixed $p \in P$ the subfamily F_p accounts for unmodelled dynamics. Usually, P is compact subset of finite-dimensional normed linear vector space [12].

Here we will suppose that system for large class of structured uncertainty can be described with collection of linear time invariant systems

$$x(k+1) = A_p x(k) + B_p u(k)\,, \quad p = 1,2,...s \tag{2}$$

where $x \in R^n$ and $u \in R^m$ are state and control signal of the system respectively. Relation (2) describes the continuous part of system. The event driven part can be described in the following form

$$p^+(t) = \varphi(p(t),\ \sigma(t)) \tag{3}$$

where $p(t)$ is discrete event variable, $\sigma(t)o$ is a discrete input and $\varphi(\cdot,\cdot)$ is a function which describes behaviour of $p(t)$. It is important to note that

$$p^{+}(t) = p(t_{n+1})\ ,\ p(t) = p(t_n)\ ,\ t_n < t_{n+1} \tag{4}$$

Specific form of switching sequence will be described in the next part of the paper. Generally, logic part of the system can be described as an automata [13] , [14] or Petry net [15]. If the analysis and design of the hybrid system are based on theory of discrete event system the main tool is: representation theory, supervisory control, computer simulation and verification. In this paper we will consider hybrid system from the classical control theory point of view, i.e., as a switching control between analogue feedback loops.

3. THE SWITCHING MODEL PREDICTIVE CONTROL

Generally, no single controller is capable of solving the regulation problem for the entire set of process models (1). Owing that we will use the family of controllers [16]

$$\{C_q : q \in D\} \tag{5}$$

where D is index set. It is supposed that this family is sufficiently rich so that admissible process model can be stabilized by controller C_q for some index $q \in D$. In this paper we will consider the case

$$F = D \tag{6}$$

For model (2) we will consider quadratic function on an infinite horizon

$$J_p = \sum_{k=0}^{\infty} \lambda^{-2k} [x^T(k)Qx(k) + u^T(k)Ru(k)] \tag{7}$$

in which Q and R are positive definite, symmetric matrices. The $\lambda \in (0,1]$ is a degree of stability [17] – [19]. A convenient orthonormal basis for discrete-time systems is [5]

$$V_k = [0 \quad 0 \quad .. \quad 0 \quad I_{mN_u} \quad 0 \quad 0 \quad ...]^T \tag{8}$$

where I_{mN_u} is on the k location. The V_k is complete in the space of square summable inputs. Also N_u - dimensional projection of the input into the basis is given by

$$u^{N_u} = [I_{N_u}\ 0] \sum_{k=0}^{N_u - 1} V_k u(k) \tag{9}$$

in which $u(k)$ is the control move at sample time k, $u(k) = 0$ for all $k \geq N_u$ and u^{N_u} is the mN_u vector of the nonzero inputs in the horizon. We have

$$J_p = \sum_{k=0}^{\infty} \lambda^{-2k} x^T(k)Qx(k) + \sum_{k=0}^{N_u} \lambda^{-2k} u^T(k)Ru(k) \tag{10}$$

we consider stable plants so that A_p is convergent. Let us define

$$U_p = \sum_{k=0}^{\infty} \lambda^{-2k} (A_p^T)^i QA^i \tag{11}$$

Using results from [5] and [18] we can get receding horizon control low

$$u(k) = -\lambda^{-1}(R + B_p^T P_{N_u p} B_p)^{-1} B_p^T P_{N_u p} A_p \tag{12}$$

$$P_{N_u p} = Q + \lambda^{-2} A_p^T \cdot$$
$$\cdot [P_{(N_u-1)p} - P_{(N_u-1)p} B_p (B_p^T P_{(N_u-1)p} B_p + R)^{-1} B_p^T P_{(N_u-1)p}] A_p ,$$
$$N_u > 1 , \quad P_{1p} = U_p , \quad p = 1,2,..,s \tag{13}$$

In the receding horizon framework only the first move of control signal is injected into the plant.

The optimal value of the objective function (7) is given as [18]

$$-\lambda^{-2k} x^T(k)(P_{N_u p} - Q)x(k) \tag{14}$$

The discrete feedback is designed as [20]

$$p^+(k) = \arg\min\{\lambda^{-2k} x^T(k)(P_{N_u p} - Q)x(k)\} \tag{15}$$

The relations (12) – (15) describe the receding horizon hybrid controller. When there is no discrete feedback the continuous controller is stable [5].

4. Asymptotic Stability of Switching MPC

Here we will prove that hybrid MPC is asymptotically stable in the Lyapunov sense. Results are formulated in the form of a theorem.

Theorem 1. Let us suppose that for hybrid systems (2) and (7) and hybrid MPC controller (12) - (16) is satisfied

1° Matrix Q and R are positive definite

2° For fixed p-th subsystem, couple

$[\lambda^{-1} A_p , \ B_p]$

is completely stabilisable

3° For fixed p-th subsystem, couple

$[\lambda^{-1} A_p , \ D_p]$, $D_p D_p^T = Q$

is completely detectable

4° For fixed p-th subsystem matrix

$\lambda^{-1}[A_p - B_p (R + B_p^T P_{N_u p} B_p)^{-1} B_p^T P_{N_u p} A_p],$

is Schur

5° Control horizon $H_u \geq 1$

6° For arbitrary switching sequence index of performance is bounded

$J_i \leq \eta$, $\eta \in (0,\infty)$, $p = 1,2,...,s$

Then

(i) The equilibrium point $x = 0$ of hybrid system is exponentially stable in Lyaponov sense

(ii) The performance of hybrid MPC system J_0 is never worse then the performance of non-switching MPC controller where

$\eta_0 = \min\{x^T(0)\,(P_{N_u p} - Q)\,x(0)\}$ ▪

Proof: Let us introduce the next transformation

$$\hat{x}(k) = \lambda^{-k} x(k) \tag{16}$$

$$\hat{u}(k) = \lambda^{-k} u(k) \tag{17}$$

Relation (6), (11) and (13) will take the form

$$\dot{\hat{x}}(k+1) = \lambda^{-1} A_p \hat{x}(k) + B_p \hat{u}(k) \tag{18}$$

$$\hat{u}(k) = -\lambda^{-1}(R + B_p^T P_{N_u p} B_p)^{-1} B_p^T P_{N_u p} A_p \hat{x}(k) \tag{19}$$

$$J_i = \sum_{k=i}^{\infty} [\hat{x}^T(k)\,Q\hat{x}(k) + \hat{u}^T(k) R\hat{u}(k)] \tag{20}$$

From condition 4° of the theorem it follows

$$\sum_{k=i}^{\infty} \hat{x}^T(k)\,Q\hat{x}(k) \le \eta \tag{21}$$

and using condition 1° of the theorem we have

$$\sum_{k=0}^{\infty} \left\|\hat{x}(k)\right\|^2 \le \frac{\eta}{\lambda_{\min}\{Q\}} = \eta_1 \tag{22}$$

From relation (18) and (19), for p-th closed-loop subsystem, we have

$$\dot{\hat{x}}(k+1) = \widetilde{A}_p \hat{x}(k) \tag{23}$$

where

$$\widetilde{A}_p = \lambda^{-1}[A_p - B_p (R + B_p^T P_{N_u p} B_p)^{-1} B_p^T P_{N_u p} A_p] \tag{24}$$

and using condition 3° of theorem one can conclude that subsystem (23) is asymptotically stable. Hence, all eigen values $\lambda_j\{\widetilde{A}_p\}$ of $\widetilde{A}_p$ are inside unit circle.

From relation (23) it follows

$$\dot{\hat{x}}(k) = (\widetilde{A}_p)^{\,k}\,\hat{x}(0) \tag{25}$$

Let us introduce

$$\sigma_p = \max_{p=1,\ldots,s} \left| \lambda_j\{\widetilde{A}_p\} \right| \tag{26}$$

It is possible to find a constant $c_p > 0$ such that

$$\|\hat{x}(k)\| \geq (c_p \sigma_p)^{k} \|x(0)\| \tag{27}$$

Let us define

$$\sigma = \max_{p=1,\dots,s} \quad \sigma_p > 0 \tag{28}$$

$$c = \min_{p=1,\dots,s} \quad c_p > 0 \tag{29}$$

Then

$$\|\hat{x}(k)\|^2 \geq (c\sigma)^k \|\hat{x}(0)\| \tag{30}$$

From (23) we have

$$\hat{x}(k) = (\widetilde{A}_p)^{k-i} \hat{x}(i) \tag{31}$$

and then

$$\|\hat{x}(k)\|^2 \geq (c\sigma)^{2(k-i)} \|\hat{x}(i)\|^2 \tag{32}$$

Suppose that for some $k > 0$

$$\|\hat{x}(k)\|_\infty = M \tag{33}$$

We have according to the (30) and (33)

$$\sum_{k=i}^{i+1} \|\hat{x}(k)\|^2 = \|x(i)\|^2 + \|x(i+1)\|^2 \geq \|x(i)\|^2 + (c\sigma)^2 \|x(i)\|^2 = M^2 + M(c\sigma)^2 \tag{34}$$

From (22) and (34) it follows

$$M \leq \sqrt{\frac{\eta_1}{1+(c\sigma)^2}} \tag{35}$$

and using relation (33) and (35) we have

$$\|x(k)\|_\infty \leq \sqrt{\frac{\eta_1}{1+(c\sigma)^2}} \tag{36}$$

We can always find the constant c_1 such that is

$$\eta_1 \leq \frac{c_1 \|\hat{x}(0)\|^2}{\lambda_{\min}\{Q\}} \quad , \quad \exists c_1 > 0 \tag{37}$$

From last two relations we have

$$\|\hat{x}(k)\|_\infty \leq \sqrt{\frac{c_1}{\lambda_{\min}\{Q\}[1+(c\sigma)^2]}} \|\hat{x}(0)\| \tag{38}$$

and according to the last relation

$$\lim_{k\to\infty} \hat{x}(k) = 0 \tag{39}$$

Having in mind relation (16) we can write

$$\lim_{k\to\infty}\lambda^{-k}x(k)=0 \tag{40}$$

and so the first statement of the theorem is proven.
Let us introduce

$$T(k)=\hat{x}^T(k)Q\hat{x}(k)+\hat{u}^T(k)R\hat{u}(k) \tag{41}$$

Let us suppose that the sequence of discrete state

$$p=\{p(t_j)\ ,\ j=0,1...\},\ t_0=0 \tag{42}$$

where t_j is integer multiple of sampling period.

The fixed switching sequence is

$$p_N=\{p(0),p(t_1)...\ ,\ p(t_N)\}\ ,\ \exists N>0 \tag{43}$$

and index of performance, according to (11) and (41)

$$\begin{aligned}J_N&=\sum_{k=0}^{t_1}\lambda^{-2k}T(k)+...+\sum_{k=t_{N-1}}^{t_N}\lambda^{-2k}T(k)+\sum_{k=t_N}^{\infty}\lambda^{-2k}T(k)=\\&\sum_{k=0}^{t_1}\lambda^{-2k}T(k)+...+\sum_{k=t_{N-1}}^{t_N}\lambda^{-2k}T(k)+\lambda^{-2t_N}x^T(t_N)(P_{N_u p(t_N)}-Q)x(t_N)\end{aligned} \tag{44}$$

From switching criterion (15) it follows

$$\begin{aligned}J_N&\le\sum_{k=0}^{t_1}\lambda^{-2k}T(k)+...+\sum_{k=t_{N-1}}^{t_N}\lambda^{-2k}T(k)+\lambda^{-2t_N}x^T(t_N)(S_{p(t_{N-1})}-Q)x(t_N)=\\&\sum_{k=0}^{t_1}\lambda^{-2k}T(k)+\cdots+\sum_{k=t_{N-2}}^{t_{N-1}}\lambda^{-2k}T(k)+\lambda^{-2t_{N-1}}x^T(t_{N-1})\cdot(P_{N_u p(t_{N-1})}-Q)x(t_{N-1})\end{aligned} \tag{45}$$

Using the same procedure we finally have

$$J_N\le x^T(0)(P_{N_u p(0)}-Q)x(0)=\eta_0 \tag{46}$$

and from that follows

$$J=\lim_{N\to\infty}J_N\le\eta_0 \tag{47}$$

The theorem is proved ▪

5. Conclusion

In this paper the problem of design of hybrid model predictive controllers is considered. The main motivation for such type of controllers is performance improvement of feedback system. Also, a very important fact is that in practice there exist systems that are impossible to control with the controllers with fixed structure. In this paper is shown that using the index of performance, which is uniformly bounded, it is possible to design MPC switching controllers, which guarantee stability of feedback system. Also, the transient performance is better. Further investigations are directed to the case when in the system description exists unmodelled dynamics.

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HIBRIDNI PREDIKTIVNI REGULATORI SA ZADATIM STEPENOM STABILNOSTI

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U ovom radu je razmatrana klasa hibridnih regulatora. Takva strategija upravljanja je kombinacije kontinualne dinamike i diskretnih događaja. Ovde se razmatra konačan skup prediktivnih regulatora koji, posle PID regulatora, imaju najveću primenu u upravljanju realnih industrijskhm procesa. Razlozi za to su: garantovana stabilnost, jednostavno uključivanje ograničenja i jednostavna ekstenzija na multivarijabilne i nelinearne sisteme. Hibridne strategije, s druge strane, značajno poboljšavaju prelazne performanse sistema. Predloženi regulatori, takođe, imaju zadati stepen stabilnosti kao podešavajući parametar.

Key words: *Hibridno upravljanje, prediktivni regulator, zadati stepen stabilnosti, asimptotska stabilnost.*