FACTA UNIVERSITATIS
Series: **Automatic Control and Robotics** Vol. 7, N° 1, 2008, pp. 111 - 120

STRUCTURAL DESIGN OF THE CONTROL SYSTEM FOR MOBILE ROBOTS WITH DISTURBANCES ESTIMATION

UDC 681.51

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Abstract. *This paper describes the synthesis of autopilots for mobile objects, combining controller and estimation loop for parametric and non-measurable disturbances of given class. The results of the synthesis for an airship-based robot and of modeling, confirming the effectiveness of the proposed algorithms are included.*

Key words: *Mobile Robots, Control, Estimation, Airship*

1. INTRODUCTION

An increasing interest in creating robotized systems for mobile objects can be observed during the last few years (airship, unmanned air and underwater devices and so on). This interest is determined by several reasons, among them the need to improve the efficiency of automatically controlled mobile objects, the expanding of their functional capabilities and efforts to avoid typical faults of HID systems. On the other hand, trying to automate mobile objects raises several problems, which cannot be solved within the bounds of the traditional control structures. For example, an attempt to suppress structural and parametric uncertainties, which are the typical features of mobile objects, causes instability in small of the closed-loop system and consequently provides poorer performance of mobile objects in small deviation area for given trajectories. Besides that, the guaranteed channel-level quality of control does not necessarily lead to the desired movement pattern of the object in whole.

The specified circumstances require development of essentially new approaches to synthesis of control systems for mobile objects in general and autopilots specifically. From our point of view, the movement control processes for mobile objects should correspond with the conditions of their functioning in full, take into account their multiple connections, high dimensionality and substantial nonlinearity of mathematical models de-

Received December 12, 2007

scribing their kinematics and also be based upon new solutions in planning their trajectories and design controllers.

This paper proposes new approaches based upon principles of conformant control for dynamical objects and introducing units for estimating parametric and structural disturbances into structure of the autopilot system.

The general structure of the proposed autopilot is shown on figure 1. The mobile object is affected by the inertial and aero dynamical forces, partially defined by prior data. Navigation system is needed for measuring state variables of the mobile object – velocities and coordinates. In some cases, measuring coordinates is not necessary. The observation unit estimates unknown forces and moment. This information combined with the state variables enters the controller, which forms the control law, defined by the primary task.

2. MATHEMATICAL MODELS OF MOBILE OBJECTS

Mathematical models of mobile objects and their kinematical connections are described using the following system of differential equations, 12^{th} order in the general case:

$$\dot{x} = M^{-1}(F_u - F_d), \tag{1}$$

$$\dot{Y} = \widetilde{R}x\,, \tag{2}$$

where x – m-phase coordinate vector-projections of terrestrial and angular speed vectors of mobile object on axes of associate coordinate system OXYZ $m \leq 6$; $M(l)$ – matrix of mass-inertial parameters, where l – vector of undefined parameters, its elements are mass of mobile object, inertial moment, associated mass of mobile object $F_u(x,Y,\delta,l)$ – m-vector of driving forces and moment, $F_d(x,Y,l)$ – m-vector of nonlinear dynamical elements, measurable and non-measurable external disturbances, δ – n-vector of control coordinates (angle deflection rudder, engine thrust control lever and so on); $Y = (P,\Theta)^T = (\text{x0,y0,z0},\psi,\nu,\gamma)^T$ – associated coordinate system's m-vector of position and orientation relative to the base coordinate system, $\widetilde{R}(\Theta)$ – matrix of kinematical connections. In case the base and associated coordinate systems are arranged the way it is shown on figure 2, $\widetilde{R}(\Theta)$ matrix has the following form

$$\widetilde{R} = \begin{vmatrix} R & 0 \\ 0 & R_p \end{vmatrix}, \tag{3}$$

$$R = \begin{vmatrix} cvc\psi & -c\gamma svc\psi + s\gamma s\psi & s\gamma svc\psi + c\gamma s\psi \\ sv & c\gamma cv & -s\gamma cv \\ -cvs\psi & c\gamma svs\psi + s\gamma c\psi & -s\gamma svs\psi + c\gamma c\psi \end{vmatrix},$$

$$R_p = \begin{vmatrix} 0 & c\gamma / cv & -s\gamma / cv \\ 0 & s\gamma & c\gamma \\ 1 & -c\gamma tgv & s\gamma tgv \end{vmatrix}.$$

where $c(.) = \cos(.)$, $s(.) = \sin(.)$, ψ, ν, γ are hunt, pitch, list angles respectively.

From now on, without loosing generality, we shall suppose that $m = n$.

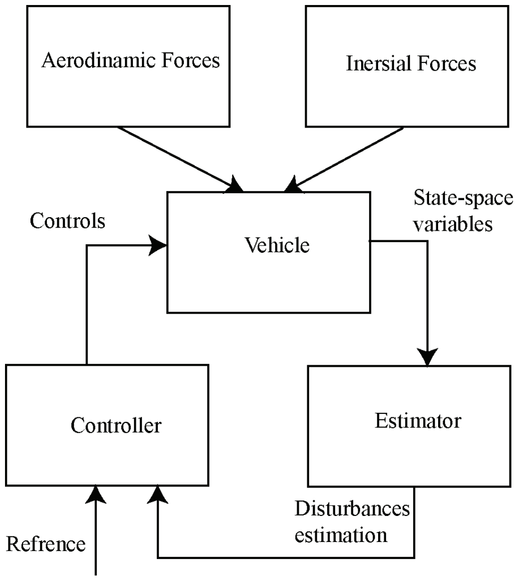

Fig. 1 Structure of autopilot with estimator

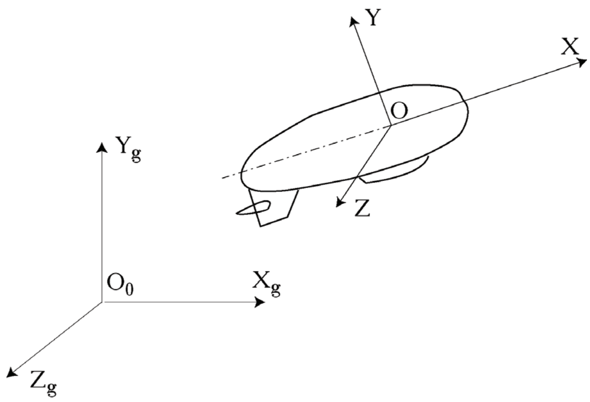

Fig. 2 MO's coordinate system

Airship dynamical models of the form (1) and (2) represent multiply connected system of nonlinear differential equations. Their elements are defined by the arrangement and parameters of the concrete MO and also by the structure and nature of external disturbances. Besides that, one of the distinctive features of MO are indeterminate elements of l-vector, which depend on the operational conditions and the nature of external disturbances. Below, kinematical and dynamical models of MO will be considered with simplifying assumptions in the synthesis procedure of the control algorithms.

3. PLANNING MOTION PATH

The specificity of using autonomous mobile robots (AMR) based upon mobile objects assumes solving of the following problems while planning their motion path: stabilization in a given point of the base coordinate space with the desired values of hunt, pitch and list angles; movement along the given trajectories in the base coordinate space with constant velocity V and given orientation of axis of the associated coordinate system; movement to a given point of the base coordinate space along the given trajectory with given orientation and without additional requirements to the speed of the MO. The multitude of given tasks can be presented as a vector-function psi of base coordinates and orientation angles and also their derivatives in the form of [1]

$$\Psi = \Psi_{tr} + A\Psi_{ck} = 0,$$

$$\Psi_{tr} = \begin{vmatrix} P^T A_{11} P + A_{12} P + A_{13} \\ P^T A_{21} P + A_{22} P + A_{23} \\ P^T A_{31} P + A_{32} P + A_{33} \\ \Phi_1(P,\Theta) \\ \Phi_2(P,\Theta) \\ \Phi_3(P,\Theta) \end{vmatrix} = 0, \tag{4}$$

$$\Psi_{ck} = J_s \dot{Y} + \widetilde{V} = 0, \ \dim \Psi_{tr} = \dim \Psi_{cr} = n,$$

$$\widetilde{V} = (0, \quad 0, \quad \xi(V^2 - V^{*2}) \ \ 0, \quad 0, \quad 0),$$

$$J_s = \begin{pmatrix} 2P^T A_{11} + A_{12}, & 2P^T A_{21} + A_{22}, & 2P^T A_{31} + A_{32}, & \dfrac{\partial \Phi_1}{\partial Y^T}, \\ \dfrac{\partial \Phi_2}{\partial Y^T}, & \dfrac{\partial \Phi_3}{\partial Y^T} & & \end{pmatrix}^T.$$

where A – diagonal, positively defined matrix of set elements $dim\, A = (nxn)$, $n = 6$; A_{ij} – constant coefficients matrix of appropriate dimensionality, formed by motion path planners; Φ_k – twice differentiable function of its arguments, reflecting the requirements to orientation angle; ξ – of the airship parameter, which can be 0 or 1 (in case of moving with constant speed); V, V^* – speed of MO and its desired value.

4. PROBLEM DEFINITION AND SYNTHESIS OF CONTROL ALGORITHMS

Autopilot synthesis problem is set. It must assure realization of trajectory, positional and trajectory-positional tasks, defined by multitudes Ψ (4), in ambiguous parameters environment with measuring only phase coordinates $\mathbf{x}$ and also in the presence of external disturbances F^* which are additively included into vector $F_d(x,Y,l)$. The following algorithm, meeting the requirements of the task, will be sufficient to effectively organize movement along given trajectories.

$$F_u = -K_0^{-1}(K_1\dot{x} + K_2 x + K_3) - F*, \tag{5}$$

$$K_0 = TA(J_s\widetilde{R} + J_s^*)M^{-1},$$

$$K_1 = (T + A)J_s\widetilde{R},$$

$$K_2 = TA(\Gamma_s\widetilde{R} + J_s\dot{\widetilde{R}}),$$

$$K_3 = A\widetilde{V} + \Psi_{tr},$$

$$J_s^* = \begin{bmatrix} 0 & 0 & 0 & 000 \\ 0 & 0 & 0 & 000 \\ 2x_1 & 2x_2 & 2x_3 & 000 \\ 0 & 0 & 0 & 000 \\ 0 & 0 & 0 & 000 \\ 0 & 0 & 0 & 000 \end{bmatrix},$$

where Γ_s is first time derivative of matrix J_s, $\dot{\widetilde{R}}$ is first time derivative of matrix $\widetilde{R}$. Here $|x_1\ x_2\ x_3|^T$ terrestrial speed vector on axis of associated coordinate system; F^*_d indeterminate forces and moment vector estimation; T diagonal, positively defined matrix of given constants, $dim\ \mathrm{T} = (n \times n)$.

To implement this algorithm it is necessary to estimate non-measurable external disturbances F^*_d.

5. Parametric and external disturbances Estimation

Estimation unit equations of unknown forces and moments at any moment have the following form

$$\begin{aligned} \frac{dz_{1i}(t)}{dt} &= -l_{1i}(F_i^0 + z_{1i} + l_{1i}mV_j) + z_{2i} + l_{2i}mV_j, \\ \frac{dz_{2i}(t)}{dt} &= -l_{2i}(F_i^0 + z_{1i} + l_{1i}mV_j), \\ F^*{}_i &= z_{1i} + l_{1i}mV_j, \end{aligned} \tag{6}$$

$$\begin{aligned} \frac{dz_{1j}(t)}{dt} &= -l_{1j}(M_j^0 + z_{1j} + l_{1j}J_j^0\omega_j) + z_{2j} + l_{2j}J_j^0\omega_j, \\ \frac{dz_{2j}(t)}{dt} &= -l_{2j}(M_j^0 + z_{1j} + l_{1j}J_j^0\omega_j), \\ M^*{}_j &= z_{1j} + l_{1j}J_j^0\omega_j. \end{aligned} \tag{7}$$

where $i, j = x, y, z, z_{1i}, z_{2i}, z_{1j}, z_{2j}$ – estimation unit state variables, F_i^0, M_j^0 – known or measured forces and moments, which affect the MO, V_j, ω_j – linear and angular velocities of the object, m, J_j^0 – mass and nominal inertia momentum, l_{1i}, l_{2i}, l_{1j}, l_{2j}, – estimation unit coefficients, assuring its processing speed, F^*_i, M^*_j unknown forces and momentum estimation.

6. AUTOPILOT SYNTHESIS ON BASIS OF ROBOT-AIRSHIP

Let us consider an autopilot synthesis of robot-airship. Movement of an airship can be described using solid body equations, which in the general case have the following form.

$$\begin{aligned}
& m\left(\frac{dV_x}{dt} - y_t\frac{d\omega_z}{dt} + z_t\frac{d\omega_y}{dt} - V_y\omega_z - V_z\omega_y +\right.\\
& \left. + y_t\omega_x\omega_y + z_t\omega_x\omega_z - x_t\omega_y^2 - x_t\omega_z^2\right) = F_{ex},\\
& m\left(\frac{dV_y}{dt} + x_t\frac{d\omega_z}{dt} - z_t\frac{d\omega_x}{dt} + V_x\omega_z - V_z\omega_x +\right.\\
& \left. + x_t\omega_x\omega_y + z_t\omega_y\omega_z - y_t\omega_x^2 - y_t\omega_z^2\right) = F_{ey},\\
& m\left(\frac{dV_z}{dt} - x_t\frac{d\omega_y}{dt} + y_t\frac{d\omega_x}{dt} - V_x\omega_y + V_y\omega_x +\right.\\
& \left. + x_t\omega_x\omega_y + y_t\omega_y\omega_z - z_t\omega_x^2 - z_t\omega_y^2\right) = F_{ez},
\end{aligned} \tag{8}$$

$$\begin{aligned}
& J_x\frac{d\omega_x}{dt} - J_{xy}\left(\frac{d\omega_y}{dt} - \omega_x\omega_z\right) + (J_z - J_y)\omega_z\omega_y +\\
& + m\left(y_t\frac{dV_z}{dt} - z_t\frac{dV_y}{dt} - y_tV_x\omega_y + y_tV_y\omega_x - z_tV_x\omega_z + z_tV_z\omega_x\right) = M_{ex},\\
& J_y\frac{d\omega_y}{dt} - J_{xy}\left(\frac{d\omega_x}{dt} + \omega_y\omega_z\right) + (J_x - J_z)\omega_x\omega_z +\\
& + m\left(z_t\frac{dV_x}{dt} - x_t\frac{dV_z}{dt} + x_tV_x\omega_y - x_tV_y\omega_x - z_tV_y\omega_z + z_tV_z\omega_y\right) = M_{ey},\\
& J_z\frac{d\omega_z}{dt} - J_{xy}\left(\frac{d\omega_x}{dt} + \omega_y^2\right) + (J_y - J_x)\omega_x\omega_y +\\
& + m\left(x_t\frac{dV_y}{dt} - y_t\frac{dV_x}{dt} + x_tV_x\omega_z - x_tV_z\omega_x + t_tV_y\omega_z - y_tV_z\omega_y\right) = M_{ez},
\end{aligned} \tag{9}$$

$$\frac{d\psi}{dt} = \frac{\omega_y \cos\gamma - \omega_z \sin\gamma}{\cos\eta},$$
$$\frac{d\eta}{dt} = \omega_z \cos\gamma - \omega_y \sin\gamma, \quad (10)$$
$$\frac{d\gamma}{dt} = \omega_x - tg\eta(\omega_y \cos\gamma - \omega_z \sin\gamma),$$

$$\frac{dx}{dt} = R(1,1)V_x + R(1,2)V_y + R(1,3)V_z,$$
$$\frac{dy}{dt} = R(2,1)V_x + R(2,2)V_y + R(2,3)V_z, \quad (11)$$
$$\frac{dz}{dt} = R(3,1)V_x + R(3,2)V_y + R(3,3)V_z,$$

where V_x, V_y, V_z – linear velocities, ω_x, ω_y, ω_z – angular velocities, m – airship's mass, $J_x = J_x^0 + \Delta J_x$, $J_y = J_y^0 + \Delta J_y$, $J_z = J_z^0 + \Delta J_z$, $J_{xy} = J_{xy}^0 + \Delta J_{xy}$ – inertia momentum, divided into known J^0 and unknown ΔJ, $x_t = x_t^0 + \Delta x_t$, $y_t = y_t^0 + \Delta y_t$, $z_t = z_t^0 + \Delta z_t$ – center of mass deviation from geometrical center (also prior given with 0 index) and unknown disturbances (denoted by Δ), F_{ex}, F_{ey}, F_{ez}, M_{ex}, M_{ey}, M_{ez} – projections of forces and momentum can be represented as

$$\begin{aligned} F_{ex} &= P_x + P_{gx} + R_{ax}, \\ F_{ey} &= P_y + P_{gy} + R_{ay}, \\ F_{ez} &= P_z + P_{gz} + R_{az}, \\ M_{ex} &= M_x + M_{gx} + M_{ax}, \\ M_{ey} &= M_y + M_{gy} + M_{ay}, \\ M_{ez} &= M_z + M_{gz} + M_{az}, \end{aligned} \quad (12)$$

where P_x, P_y, P_z, M_x, M_y, M_z – controlling forces and moments, P_{gx}, P_{gy}, P_{gz}, M_{gx}, M_{gy}, M_{gz} – moments and forces caused by gravity.

$$\begin{aligned} P_{gy} &= -\sin\eta g(m - \rho U), \\ P_{gx} &= -\cos\eta\cos\gamma g(m - \rho U), \\ P_{gz} &= -\cos\eta\sin\gamma g(m - \rho U), \\ M_{gx} &= (y_t + z_t) mg\cos\eta\sin\gamma, \\ M_{gy} &= (-z_t \sin\eta - x_t \cos\eta\sin\gamma) mg, \\ M_{gz} &= (yt\sin\eta - x_t \cos\eta\sin\gamma) mg, \end{aligned} \quad (13)$$

$$R_{ax} = -\left(\lambda_{11}\frac{dV_x}{dt} - \lambda_{22}V_y\omega_z + \lambda_{33}V_z\omega_y - \lambda_{26}\omega_z^2 + \lambda_{35}\omega_y^2 + \lambda_{34}\omega_x\omega_y\right) - 0.5c_x s\rho V^2 \quad (14)$$

$$R_{ay} = -\left(\lambda_{22}\frac{dV_y}{dt} + \lambda_{26}\frac{d\omega_z}{dt} - \lambda_{33}V_z\omega_x - \lambda_{35}\omega_x\omega_y - \lambda_{34}\omega_x^2\right) - 0.5c_y s\rho V^2 \quad (15)$$

$$R_{az} = -\left(\lambda_{33}\frac{dV_z}{dt} + \lambda_{34}\frac{d\omega_x}{dt} + \lambda_{35}\frac{d\omega_y}{dt} + \lambda_{22}V_y\omega_x + \lambda_{26}\omega_x\omega_z\right) - 0.5c_z s\rho V^2 \quad (16)$$

$$\begin{aligned} M_{ax} = &-\lambda_{44}\frac{d\omega_x}{dt} - \lambda_{34}\frac{dV_z}{dt} - \lambda_{34}V_y\omega_x - \\ &-(\lambda_{66} - \lambda_{55})\omega_y\omega_z - \\ &-(\lambda_{26} + \lambda_{35})(V_y\omega_y + V_z\omega_z) + 0.5m_x U\rho V^2, \end{aligned} \quad (17)$$

$$\begin{aligned} M_{ay} = &-\lambda_{55}\frac{d\omega_y}{dt} + \lambda_{35}\frac{dV_z}{dt} - \lambda_{34}(V_x\omega_x - V_z\omega_z) + \\ &+(\lambda_{44} - \lambda_{66})\omega_x\omega_z - \\ &-\lambda_{26}V_y\omega_x + 0.5m_y U\rho V^2, \end{aligned} \quad (18)$$

$$\begin{aligned} M_{ay} = &-\lambda_{66}\frac{d\omega_z}{dt} + \lambda_{26}\frac{dV_y}{dt} + (\lambda_{55} - \lambda_{44})\omega_x\omega_y - \\ &-\lambda_{35}V_z\omega_x - \lambda_{34}V_z\omega_y + 0.5m_z U\rho V^2, \end{aligned} \quad (19)$$

where $\lambda_{ij} = \lambda_{ij}^0 + \Delta\lambda_{ij}$ – associated masses, $c_x = c_x^0 + \Delta c_x$, $c_y = c_y^0 + \Delta c_y$, $c_z = c_z^0 + \Delta c_z$, $m_x = m_x^0 + \Delta m_x$, $m_y = m_y^0 + \Delta m_y$, $m_z = m_z^0 + \Delta m_z$ – aero dynamical coefficients (nominal values are denoted by ^{0}, disturbances are denoted by Δ), s – effective airship's area, ρ – density of air, U – airship's volume, $V^2 = V_x^2 + V_y^2 + V_z^2$.

In this case control matrix takes the form of $T = diag(s_i)_{6\times6}$, s_i define the processing speed of the controller, $A = T$.

$$M = \begin{bmatrix} m & 0 & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 \\ 0 & 0 & 0 & J_x^0 & 0 & 0 \\ 0 & 0 & 0 & 0 & J_y^0 & 0 \\ 0 & 0 & 0 & 0 & 0 & J_z^0 \end{bmatrix},$$

$$\mathbf{\Psi}_{tr} = \begin{bmatrix} a_{11}^1x^2 + a_{12}^2y^2 + a_{13}^3z^2 + a_{13}^1x + a_{14}^1y + a_{15}^1z + a_{16}^1 \\ a_{21}^1x^2 + a_{22}^2y^2 + a_{23}^3z^2 + a_{23}^1x + a_{24}^1y + a_{25}^1z + a_{26}^1 \\ 0 \\ \psi + \arctan\left(\dfrac{2a_{11}^2x + a_{13}^1}{2a_{12}^1z + a_{14}^1}\right) \\ \eta \\ \gamma \end{bmatrix}.$$

Vector F_d^* is derived from the right sides of the equations, and that is why it has a lengthy appearance and is not included. It can be noted that the vector is divided into two parts. The first part holds the nominal values and enters the controller without estimation. The second part consists of unknown forces, and that is why they are substituted with their estimations.

Figures 3-8 show the results of modeling for a closed loop system with the following parameters $m = 230$, $\Delta\lambda_{11} = 0.05m$, $\Delta\lambda_{22} = 0.05m$, $\Delta\lambda_{33} = 0.05m$, $\Delta\lambda_{26} = 0.01m$, $\Delta\lambda_{44} = 0.05m$, $\Delta\lambda_{55} = 0.05m$, $\Delta\lambda_{66} = 0.05m$, $s = 100$, $\rho = 1$, $\Delta y_t = 0.5$, $\Delta J_{xy} = 100$, $J_x^0 = 1800$, $J_y^0 = 4500$, $J_z^0 = 5500$, $\Delta c_x = 0.1 - 0.0002V$, $g = 9.8$, $\Delta c_y = 0.6\omega_z$, $\Delta c_z = -0.6\omega_y$, $s = 0.5$, $\Delta m_x = -0.114\omega_x$, $\Delta m_y = -0.114\omega_y$, $\Delta m_z = -1.15\omega_z$, $U = 400$, $l_{1i} = 200$, $l_{2i} = 10000$, $a_{11}^1 = 1$, $a_{12}^1 = 1$, $a_{13}^1 = 1$, $a_{16}^1 = -100$, $a_4^1 = 1$, $a_{26}^1 = -5$, $\nu_k = 20$.

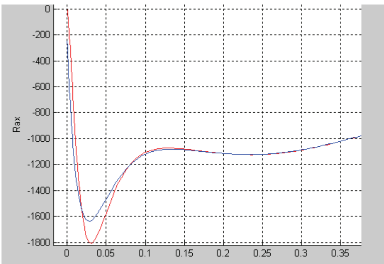

Fig. 3 Estimation of frontal resistance

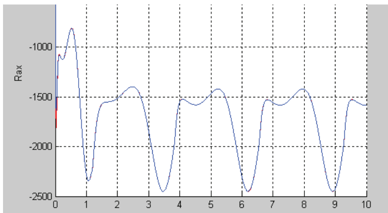

Fig. 4 Alternation of frontal resistance

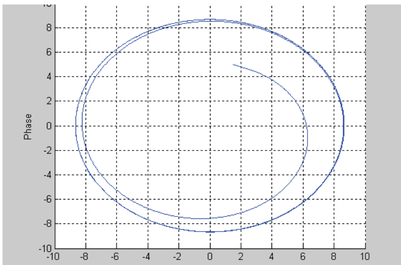

Fig. 5 Airship's trajectory

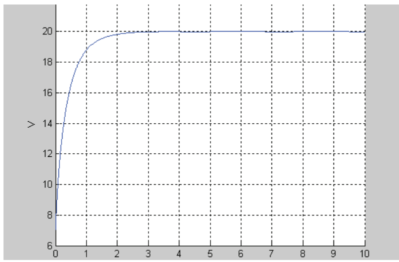

Fig. 6 Path velocity

Figure 3 shows estimation of frontal resistance (red) and the force itself (blue), figure 4 shows alternation of frontal resistance along the motion path, other pictures are airship's trajectory, path velocity, directing force Px and airship's height.

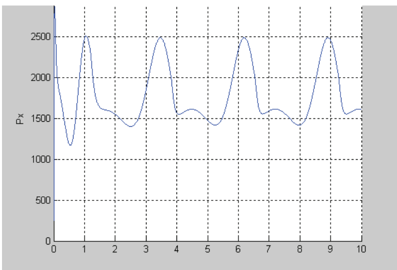

Fig. 7 Directing force Px

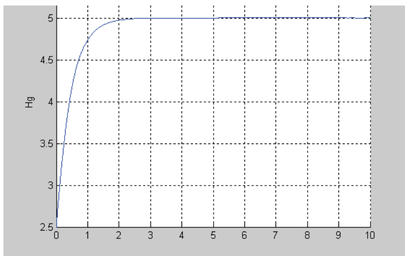

Fig. 8 Airship's height

The obtained results of the modeling confirm the effectiveness and correctness of the proposed algorithms.

Acknowledgement: The paper was supported by Russian Foundation of Basic Researches. Grant N 07-08-00373-a.

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STRUKTURALNI DIZAJN KONTROLNIH SISTEMA MOBILNOG ROBOTA SA ESTIMACIJOM POREMEĆAJA

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Ovaj rad opisuje sintezu autopilota za mobilne objekte, koji kombinuju kontrolere petlju estimacije namenjenu za parametarske i poremecaje odredjenog tipa koje je tesko meriti .U radu su ukljuceni rezultati sinteze mobilnog robota – bespilotne letilice, kao i rezultati modelovajna koji potvrdjuju efikasnost predlozenog algoritma.

Ključne reči: *mobilni roboti, kontrola, procena, letelica*