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BIT ERROR RATE FOR SSC/MRC COMBINER AT TWO TIME INSTANTS IN THE PRESENCE OF LOG-NORMAL FADING *

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Abstract. *In this paper, the performances of the Switch and Stay Combining/ Maximal Ratio Combining (SSC/MRC) combiner output signal at two time instants is determined. The SSC and MRC combiners with two branches are considered. The presence of the log-normal fading at the input is observed. The probability density function (PDF) at the output of the combiner is obtained. Bit error rate (BER) for the special case of the binary phase swift keying (BPSK) modulation is derived. The results are shown graphically to emphasize better performances of the SSC/MRC combiner with regard to classical SSC and MRC combiners at one time instant.*

Key words: *diversity reception, joint probability density function, log-normal fading, SSC Combining, MRC combining, two time instants, bit error rate*

1. INTRODUCTION

Fading, a variation of an instantaneous value of the received signal, the received signal envelope, is fundamental obstacle in wireless communications. Fading is one of the main causes of the receiver performance degradation. It is due to multipath propagation.

There are several ways to reduce fading effect. Diversity technique is certainly one of the most frequently used methods for combating the deleterious effect of channel fading. Particular diversity methods and combining techniques are presented in [1]. Maximal-Ratio Combining (MRC) is one of the most widely used diversity combining schemes whose SNR is the sum of the SNR's of each individual diversity branch. MRC is the optimal combining scheme, but its price and complexity are the largest, since MRC requires cognition of all fading parameters of the channel [1]. Equal Gain Combining (EGC) is the next by performances, but Selection Combining (SC) and Switch and Stay Combining

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(SSC) have lower performance, but are simpler to implement. Since the SC and SSC combining schemes do not require signal cophasing and fading envelope estimation, they are very often implemented in practice. The SC is combining technique where the strongest signal is chosen among L branches of diversity system [1].

In the case of dual branch SSC, the first branch stay selected as long as its instantaneous signal-to-noise ratio (SNR) is greater than predetermined switching threshold, even if the instantaneous SNR in the second branch may have a larger value at that time [1], [2]. The consideration of SSC systems in the literature has been restricted to low-complexity mobile units where the number of diversity antennas is typically limited to two ([3], [4], [5]). In [6] the authors have also developed, analyzed and optimized a simple form of dual-branch SSC combining.

The probability density function (PDF) of the SSC combiner output signal at one time instant and the joint probability density function of the SSC combiner output signal at two time instants in the presence of Rayleigh, Nakagami-m, Weibull and lognormal fading are determined in [7]-[10], respectively. In this paper the probability density function, the bit error rate of the SSC/MRC combiner output signal at two time instants in the presence of log-normal fading will be determined.

This paper is organized so that the second Section describes the system model and determines the probability density function and the bit error rate of the SSC/MRC combiner output signal at two time instant. Sections III presents the numerical results obtained for performances introduced in section II. Finally, the main results of the paper are given in Conclusion.

2. SYSTEM MODEL

The model of the SSC/MRC combiner with two inputs considering in this paper is shown in Fig. 1. We consider the SSC/MRC combiner with two branches at two time instants. The signals at the inputs at SSC combiner are r_{11} and r_{21} at first time moment and they are r_{12} and r_{22} at the second time moment. The output signals at SSC part are r_1 and r_2. The indexes for the input signals are: first index is the number of the branch and the other signs time instant observed. For the output signals, the index represents the time instant observed. After determining the output signals at SSC combiner r_1 and r_2, they become the inputs at MRC combiner and the overall output signal is r.

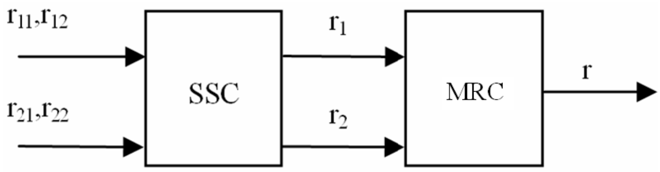

Fig. 1 Model of the SSC/SC combiner with two inputs at two time instants

The joint probability density function of correlated signals r_1 and r_2 at the output of SSC combiner at two time inputs with lognormal distribution and same standard deviation σ_i is obtained in [10] in closed form.

For $r_1 < r_T$, $r_2 < r_T$ it is:

$$
\begin{aligned}
& p_{r_1 r_2}(r_1, r_2) = P_1 \frac{1}{\sqrt{2\pi}\sigma_1 r_2} e^{-\frac{(\ln r_2 - \mu_1)^2}{2\sigma_1^2}} \cdot \\
& \cdot \left(\frac{1}{2} + erf\left(\frac{\ln r_t - (\mu_1 + \rho(\ln r_2 - \mu_1))}{\sigma_1 \sqrt{1-\rho^2}\sqrt{2}} \right) \right) \cdot \\
& \cdot \frac{1}{\sqrt{2\pi}\sigma_2 r_1} e^{-\frac{(\ln r_1 - \mu_2)^2}{2\sigma_2^2}} \left(\frac{1}{2} + erf\left(\frac{\ln r_t - (\mu_2 + \rho(\ln r_1 - \mu_2))}{\sigma_2 \sqrt{1-\rho^2}\sqrt{2}} \right) \right) + \\
& + P_2 \frac{1}{\sqrt{2\pi}\sigma_2 r_2} e^{-\frac{(\ln r_2 - \mu_2)^2}{2\sigma_2^2}} \left(\frac{1}{2} + erf\left(\frac{\ln r_t - (\mu_2 + \rho(\ln r_2 - \mu_2))}{\sigma_2 \sqrt{1-\rho^2}\sqrt{2}} \right) \right) \cdot \\
& \cdot \frac{1}{\sqrt{2\pi}\sigma_1 r_1} e^{-\frac{(\ln r_1 - \mu_1)^2}{2\sigma_1^2}} \left(\frac{1}{2} + erf\left(\frac{\ln r_t - (\mu_1 + \rho(\ln r_1 - \mu_1))}{\sigma_1 \sqrt{1-\rho^2}\sqrt{2}} \right) \right)
\end{aligned}
\tag{1}
$$

For $r_1 \geq r_T$, $r_2 < r_T$:

$$
\begin{aligned}
& p_{r_1 r_2}(r_1, r_2) = P_1 \frac{1}{\sqrt{2\pi}\sigma_2 r_2} e^{-\frac{(\ln r_2 - \mu_2)^2}{2\sigma_2^2}} \cdot \\
& \cdot \frac{1}{\sqrt{2\pi}\sigma_1 r_1} e^{-\frac{(\ln r_1 - \mu_1)^2}{2\sigma_1^2}} \left(\frac{1}{2} + erf\left(\frac{\ln r_t - (\mu_1 + \rho(\ln r_1 - \mu_1))}{\sigma_1 \sqrt{1-\rho^2}\sqrt{2}} \right) \right) + \\
& + P_1 \frac{1}{\sqrt{2\pi}\sigma_1 r_2} e^{-\frac{(\ln r_2 - \mu_1)^2}{2\sigma_1^2}} \left(\frac{1}{2} + erf\left(\frac{\ln r_t - (\mu_1 + \rho(\ln r_2 - \mu_1))}{\sigma_1 \sqrt{1-\rho^2}\sqrt{2}} \right) \right) \cdot \\
& \cdot \frac{1}{\sqrt{2\pi}\sigma_2 r_1} e^{-\frac{(\ln r_1 - \mu_2)^2}{2\sigma_2^2}} \left(\frac{1}{2} + erf\left(\frac{\ln r_t - (\mu_2 + \rho(\ln r_1 - \mu_2))}{\sigma_2 \sqrt{1-\rho^2}\sqrt{2}} \right) \right) + \\
& + P_2 \frac{1}{\sqrt{2\pi}\sigma_1 r_2} e^{-\frac{(\ln r_2 - \mu_1)^2}{2\sigma_1^2}} \cdot \frac{1}{\sqrt{2\pi}\sigma_2 r_1} e^{-\frac{(\ln r_1 - \mu_2)^2}{2\sigma_2^2}} \left(\frac{1}{2} + erf\left(\frac{\ln r_t - (\mu_2 + \rho(\ln r_1 - \mu_2))}{\sigma_2 \sqrt{1-\rho^2}\sqrt{2}} \right) \right) + \\
& + P_2 \frac{1}{\sqrt{2\pi}\sigma_2 r_2} e^{-\frac{(\ln r_2 - \mu_2)^2}{2\sigma_2^2}} \left(\frac{1}{2} + erf\left(\frac{\ln r_t - (\mu_2 + \rho(\ln r_2 - \mu_2))}{\sigma_2 \sqrt{1-\rho^2}\sqrt{2}} \right) \right) \cdot \\
& \cdot \frac{1}{\sqrt{2\pi}\sigma_1 r_1} e^{-\frac{(\ln r_1 - \mu_1)^2}{2\sigma_1^2}} \left(\frac{1}{2} + erf\left(\frac{\ln r_t - (\mu_1 + \rho(\ln r_1 - \mu_1))}{\sigma_1 \sqrt{1-\rho^2}\sqrt{2}} \right) \right)
\end{aligned}
\tag{2}
$$

For $r_1 < r_T$, $r_2 \geq r_T$:

$$p_{r_1 r_2}(r_1, r_2) = P_1\left(\frac{1}{2} + erf\left(\frac{\ln r_t - \mu_1}{\sigma_1\sqrt{2}}\right)\right)\cdot$$
$$\cdot\frac{1}{2\pi\sigma_2{}^2\sqrt{1-\rho^2}\,r_1 r_2}e^{-\frac{1}{2(1-\rho^2)}\left[\left(\frac{\ln r_1-\mu_2}{\sigma_2}\right)^2+\left(\frac{\ln r_2-\mu_2}{\sigma_2}\right)^2-2\rho\left(\frac{\ln r_1-\mu_2}{\sigma_2}\right)\left(\frac{\ln r_2-\mu_2}{\sigma_2}\right)\right]}+$$
$$+P_1\frac{1}{\sqrt{2\pi}\sigma_1 r_2}e^{-\frac{(\ln r_2-\mu_1)^2}{2\sigma_1{}^2}}\left(\frac{1}{2}+erf\left(\frac{\ln r_t-(\mu_1+\rho(\ln r_2-\mu_1))}{\sigma_1\sqrt{1-\rho^2}\sqrt{2}}\right)\right)\cdot$$
$$\cdot\frac{1}{\sqrt{2\pi}\sigma_2 r_1}e^{-\frac{(\ln r_1-\mu_2)^2}{2\sigma_2{}^2}}\left(\frac{1}{2}+erf\left(\frac{\ln r_t-(\mu_2+\rho(\ln r_1-\mu_2))}{\sigma_2\sqrt{1-\rho^2}\sqrt{2}}\right)\right)+$$
$$+P_2\left(\frac{1}{2}+erf\left(\frac{\ln r_t-\mu_2}{\sigma_2\sqrt{2}}\right)\right)\cdot$$
$$\cdot\frac{1}{2\pi\sigma_1{}^2\sqrt{1-\rho^2}\,r_1 r_2}e^{-\frac{1}{2(1-\rho^2)}\left[\left(\frac{\ln r_1-\mu_1}{\sigma_1}\right)^2+\left(\frac{\ln r_2-\mu_1}{\sigma_1}\right)^2-2\rho\left(\frac{\ln r_1-\mu_1}{\sigma_1}\right)\left(\frac{\ln r_2-\mu_1}{\sigma_1}\right)\right]}+$$
$$+P_2\frac{1}{\sqrt{2\pi}\sigma_2 r_2}e^{-\frac{(\ln r_2-\mu_2)^2}{2\sigma_2{}^2}}\left(\frac{1}{2}+erf\left(\frac{\ln r_t-(\mu_2+\rho(\ln r_2-\mu_2))}{\sigma_2\sqrt{1-\rho^2}\sqrt{2}}\right)\right)\cdot$$
$$\cdot\frac{1}{\sqrt{2\pi}\sigma_1 r_1}e^{-\frac{(\ln r_1-\mu_1)^2}{2\sigma_1{}^2}}\left(\frac{1}{2}+erf\left(\frac{\ln r_t-(\mu_1+\rho(\ln r_1-\mu_1))}{\sigma_1\sqrt{1-\rho^2}\sqrt{2}}\right)\right) \qquad (3)$$

For $r_1 \geq r_T$, $r_2 \geq r_T$:

$$p_{r_1 r_2}(r_1, r_2) = P_1\cdot\frac{1}{2\pi\sigma_2{}^2\sqrt{1-\rho^2}\,r_1 r_2}\cdot$$
$$\cdot e^{-\frac{1}{2(1-\rho^2)}\left[\left(\frac{\ln r_1-\mu_2}{\sigma_2}\right)^2+\left(\frac{\ln r_2-\mu_2}{\sigma_2}\right)^2-2\rho\left(\frac{\ln r_1-\mu_2}{\sigma_2}\right)\left(\frac{\ln r_2-\mu_2}{\sigma_2}\right)\right]}+$$
$$+P_1\frac{1}{\sqrt{2\pi}\sigma_2 r_2}e^{-\frac{(\ln r_2-\mu_2)^2}{2\sigma_2{}^2}}\cdot\frac{1}{\sqrt{2\pi}\sigma_1 r_1}e^{-\frac{(\ln r_1-\mu_1)^2}{2\sigma_1{}^2}}\cdot$$
$$\cdot\left(\frac{1}{2}+erf\left(\frac{\ln r_t-(\mu_1+\rho(\ln r_1-\mu_1))}{\sigma_1\sqrt{1-\rho^2}\sqrt{2}}\right)\right)+$$
$$+P_1\left(\frac{1}{2}+erf\left(\frac{\ln r_t-\mu_1}{\sigma_1\sqrt{2}}\right)\right)\cdot\frac{1}{2\pi\sigma_2{}^2\sqrt{1-\rho^2}\,r_1 r_2}\cdot$$

$$\cdot e^{-\frac{1}{2(1-\rho^2)}\left[\left(\frac{\ln r_1-\mu_2}{\sigma_2}\right)^2+\left(\frac{\ln r_2-\mu_2}{\sigma_2}\right)^2-2\rho\left(\frac{\ln r_1-\mu_2}{\sigma_2}\right)\left(\frac{\ln r_2-\mu_2}{\sigma_2}\right)\right]}+$$

$$+P_1\frac{1}{\sqrt{2\pi}\sigma_1 r_2}e^{-\frac{(\ln r_2-\mu_1)^2}{2\sigma_1^2}}\cdot\left(\frac{1}{2}+erf\left(\frac{\ln r_t-(\mu_1+\rho(\ln r_2-\mu_1))}{\sigma_1\sqrt{1-\rho^2}\sqrt{2}}\right)\right)$$

$$\cdot\frac{1}{\sqrt{2\pi}\sigma_2 r_1}e^{-\frac{(\ln r_1-\mu_2)^2}{2\sigma_2^2}}\cdot\left(\frac{1}{2}+erf\left(\frac{\ln r_t-(\mu_2+\rho(\ln r_1-\mu_2))}{\sigma_2\sqrt{1-\rho^2}\sqrt{2}}\right)\right)+$$

$$+P_2\frac{1}{2\pi\sigma_2^2\sqrt{1-\rho^2}\,r_1 r_2}\cdot$$

$$\cdot e^{-\frac{1}{2(1-\rho^2)}\left[\left(\frac{\ln r_1-\mu_2}{\sigma_2}\right)^2+\left(\frac{\ln r_2-\mu_2}{\sigma_2}\right)^2-2\rho\left(\frac{\ln r_1-\mu_2}{\sigma_2}\right)\left(\frac{\ln r_2-\mu_2}{\sigma_2}\right)\right]}+$$

$$+P_2\frac{1}{\sqrt{2\pi}\sigma_1 r_2}e^{-\frac{(\ln r_2-\mu_1)^2}{2\sigma_1^2}}\cdot\frac{1}{\sqrt{2\pi}\sigma_2 r_1}e^{-\frac{(\ln r_1-\mu_2)^2}{2\sigma_2^2}}\cdot$$

$$\cdot\left(\frac{1}{2}+erf\left(\frac{\ln r_t-(\mu_2+\rho(\ln r_1-\mu_2))}{\sigma_2\sqrt{1-\rho^2}\sqrt{2}}\right)\right)+$$

$$+P_2\left(\frac{1}{2}+erf\left(\frac{\ln r_t-\mu_2}{\sigma_2\sqrt{2}}\right)\right)\cdot\frac{1}{2\pi\sigma_1^2\sqrt{1-\rho^2}\,r_1 r_2}\cdot$$

$$\cdot e^{-\frac{1}{2(1-\rho^2)}\left[\left(\frac{\ln r_1-\mu_1}{\sigma_1}\right)^2+\left(\frac{\ln r_2-\mu_1}{\sigma_1}\right)^2-2\rho\left(\frac{\ln r_1-\mu_1}{\sigma_1}\right)\left(\frac{\ln r_2-\mu_1}{\sigma_1}\right)\right]}+$$

$$+P_2\frac{1}{\sqrt{2\pi}\sigma_2 r_2}e^{-\frac{(\ln r_2-\mu_2)^2}{2\sigma_2^2}}\cdot\left(\frac{1}{2}+erf\left(\frac{\ln r_t-(\mu_2+\rho(\ln r_2-\mu_2))}{\sigma_2\sqrt{1-\rho^2}\sqrt{2}}\right)\right)\cdot$$

$$\cdot\frac{1}{\sqrt{2\pi}\sigma_1 r_1}e^{-\frac{(\ln r_1-\mu_1)^2}{2\sigma_1^2}}\left(\frac{1}{2}+erf\left(\frac{\ln r_t-(\mu_1+\rho(\ln r_1-\mu_1))}{\sigma_1\sqrt{1-\rho^2}\sqrt{2}}\right)\right) \quad (4)$$

where σ_i is standard deviation, μ_i is the mean of lognormal distribution, ρ is correlation coefficient and r_t is the threshold of the decision for SSC combiner.

The outputs of SSC combiner are used as inputs for MRC combiner.

Total conditional signal value at the output of the MRC combiner, for equally transmitted symbols of L branch MRC receiver, is given by [11]

$$r=\sum_{l=1}^{L} r_l \quad (5)$$

For coherent binary signals the conditional BER $P_b(e|\{r_l\}_{l=1}^{L})$ is given by [1]

$$P_b(e|\{r_l\}_{l=1}^{L}) = Q(\sqrt{2gr}) \tag{6}$$

where Q is the one-dimensional Gaussian Q-function [1]

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-t^2/2} dt \tag{7}$$

Gaussian Q-function can be defined using alternative form as [1,12]

$$Q(x) = \frac{1}{\pi} \int_0^{\pi/2} \exp\left(-\frac{x^2}{2\sin^2\phi}\right) d\phi \tag{8}$$

Using the alternative representation of the Gaussian-Q function (10), the conditional BER can be expressed as

$$P_b(e|\{r_l\}_{l=1}^{L}) = \frac{1}{\pi} \int_0^{\pi/2} \exp\left(-\frac{gr}{\sin^2\phi}\right) d\phi = \frac{1}{\pi} \int_0^{\pi/2} \prod_{l=1}^{L} \left(-\frac{gr_l}{\sin^2\phi}\right) d\phi \tag{9}$$

The unconditional BER can be obtained by averaging the multichannel conditional BER over the joint PDF of the signals at the input of MRC combiner

$$P_b(e) = \underbrace{\int_0^{\infty}\int_0^{\infty} \cdots \int_0^{\infty}}_{L} P_b\left(\{r_l\}_{l=1}^{L}\right) p_{r_1, r_2, \ldots r_L}(r_1, r_2, \ldots r_L) dr_1 dr_2 \ldots dr_L \tag{10}$$

Substituting (9) in (10), $P_b(e)$ is obtained as

$$P_b(e) = \underbrace{\int_0^{\infty}\int_0^{\infty} \cdots \int_0^{\infty} \frac{1}{\pi} \int_0^{\pi/2} \prod_{l=1}^{L} \left(-\frac{gr_l}{\sin^2\phi}\right) d\phi}_{L}\, p_{r_1, r_2, \ldots r_L}(r_1, r_2, \ldots r_L) dr_1 dr_2 \ldots dr_L \tag{11}$$

For dual branch MRC combiner, $P_b(e)$ is

$$P_b(e) = \frac{1}{\pi} \int_0^{\infty}\int_0^{\infty}\int_0^{\pi/2} \left(-\frac{gr_1}{\sin^2\phi}\right)\left(-\frac{gr_2}{\sin^2\phi}\right) p_{r_1, r_2}(r_1, r_2) d\phi dr_1 dr_2 \tag{12}$$

Substituting (1-4) in (12), $P_b(e)$ of SSC/MRC combiner can be obtained as:

$$P_b(e) = \frac{1}{\pi} \int_0^{r_t}\int_0^{r_t}\int_0^{\pi/2} dr_1 dr_2 d\phi \left[\left(-\frac{gr_1}{\sin^2\phi}\right)\left(-\frac{gr_2}{\sin^2\phi}\right)\cdot\right.$$

$$\cdot P_1 \frac{1}{\sqrt{2\pi}\sigma_1 r_2} e^{-\frac{(\ln r_2 - \mu_1)^2}{2\sigma_1^2}} \cdot$$

$$\cdot\left(\frac{1}{2}+erf\left(\frac{\ln r_t-(\mu_1+\rho(\ln r_2-\mu_1))}{\sigma_1\sqrt{1-\rho^2}\sqrt{2}}\right)\right)\cdot$$

$$\cdot\frac{1}{\sqrt{2\pi}\sigma_2 r_1}e^{-\frac{(\ln r_1-\mu_2)^2}{2\sigma_2{}^2}}\left(\frac{1}{2}+erf\left(\frac{\ln r_t-(\mu_2+\rho(\ln r_1-\mu_2))}{\sigma_2\sqrt{1-\rho^2}\sqrt{2}}\right)\right)+$$

$$+P_2\frac{1}{\sqrt{2\pi}\sigma_2 r_2}e^{-\frac{(\ln r_2-\mu_2)^2}{2\sigma_2{}^2}}\left(\frac{1}{2}+erf\left(\frac{\ln r_t-(\mu_2+\rho(\ln r_2-\mu_2))}{\sigma_2\sqrt{1-\rho^2}\sqrt{2}}\right)\right)\cdot$$

$$\cdot\frac{1}{\sqrt{2\pi}\sigma_1 r_1}e^{-\frac{(\ln r_1-\mu_1)^2}{2\sigma_1{}^2}}\left(\frac{1}{2}+erf\left(\frac{\ln r_t-(\mu_1+\rho(\ln r_1-\mu_1))}{\sigma_1\sqrt{1-\rho^2}\sqrt{2}}\right)\right)\Bigg]+$$

$$+\frac{1}{\pi}\int_{r_t}^{\infty}\int_{0}^{r_t}\int_{0}^{\pi/2}dr_1dr_2d\phi\left[\left(-\frac{gr_1}{\sin^2\phi}\right)\left(-\frac{gr_2}{\sin^2\phi}\right)\cdot\right.$$

$$\cdot P_1\frac{1}{\sqrt{2\pi}\sigma_2 r_2}e^{-\frac{(\ln r_2-\mu_2)^2}{2\sigma_2{}^2}}\cdot$$

$$\cdot\frac{1}{\sqrt{2\pi}\sigma_1 r_1}e^{-\frac{(\ln r_1-\mu_1)^2}{2\sigma_1{}^2}}\left(\frac{1}{2}+erf\left(\frac{\ln r_t-(\mu_1+\rho(\ln r_1-\mu_1))}{\sigma_1\sqrt{1-\rho^2}\sqrt{2}}\right)\right)+$$

$$+P_1\frac{1}{\sqrt{2\pi}\sigma_1 r_2}e^{-\frac{(\ln r_2-\mu_1)^2}{2\sigma_1{}^2}}\left(\frac{1}{2}+erf\left(\frac{\ln r_t-(\mu_1+\rho(\ln r_2-\mu_1))}{\sigma_1\sqrt{1-\rho^2}\sqrt{2}}\right)\right)\cdot$$

$$\cdot\frac{1}{\sqrt{2\pi}\sigma_2 r_1}e^{-\frac{(\ln r_1-\mu_2)^2}{2\sigma_2{}^2}}\left(\frac{1}{2}+erf\left(\frac{\ln r_t-(\mu_2+\rho(\ln r_1-\mu_2))}{\sigma_2\sqrt{1-\rho^2}\sqrt{2}}\right)\right)+$$

$$+P_2\frac{1}{\sqrt{2\pi}\sigma_1 r_2}e^{-\frac{(\ln r_2-\mu_1)^2}{2\sigma_1{}^2}}\cdot\cdot\frac{1}{\sqrt{2\pi}\sigma_2 r_1}e^{-\frac{(\ln r_1-\mu_2)^2}{2\sigma_2{}^2}}\left(\frac{1}{2}+erf\left(\frac{\ln r_t-(\mu_2+\rho(\ln r_1-\mu_2))}{\sigma_2\sqrt{1-\rho^2}\sqrt{2}}\right)\right)+$$

$$+P_2\frac{1}{\sqrt{2\pi}\sigma_2 r_2}e^{-\frac{(\ln r_2-\mu_2)^2}{2\sigma_2{}^2}}\left(\frac{1}{2}+erf\left(\frac{\ln r_t-(\mu_2+\rho(\ln r_2-\mu_2))}{\sigma_2\sqrt{1-\rho^2}\sqrt{2}}\right)\right)\cdot$$

$$\left.\cdot\frac{1}{\sqrt{2\pi}\sigma_1 r_1}e^{-\frac{(\ln r_1-\mu_1)^2}{2\sigma_1{}^2}}\left(\frac{1}{2}+erf\left(\frac{\ln r_t-(\mu_1+\rho(\ln r_1-\mu_1))}{\sigma_1\sqrt{1-\rho^2}\sqrt{2}}\right)\right)\right]+$$

$$+\frac{1}{\pi}\int_0^{r_t}\int_{r_t}^{\infty}\int_0^{\pi/2} dr_1 dr_2 d\phi\left[\left(-\frac{gr_1}{\sin^2\phi}\right)\left(-\frac{gr_2}{\sin^2\phi}\right)\cdot\right.$$

$$\cdot P_1\left(\frac{1}{2}+erf\left(\frac{\ln r_t-\mu_1}{\sigma_1\sqrt{2}}\right)\right)\cdot$$

$$\cdot\frac{1}{2\pi\sigma_2{}^2\sqrt{1-\rho^2}\,r_1r_2}e^{-\frac{1}{2(1-\rho^2)}\left[\left(\frac{\ln r_1-\mu_2}{\sigma_2}\right)^2+\left(\frac{\ln r_2-\mu_2}{\sigma_2}\right)^2-2\rho\left(\frac{\ln r_1-\mu_2}{\sigma_2}\right)\left(\frac{\ln r_2-\mu_2}{\sigma_2}\right)\right]}+$$

$$+P_1\frac{1}{\sqrt{2\pi}\sigma_1 r_2}e^{-\frac{(\ln r_2-\mu_1)^2}{2\sigma_1{}^2}}\left(\frac{1}{2}+erf\left(\frac{\ln r_t-(\mu_1+\rho(\ln r_2-\mu_1))}{\sigma_1\sqrt{1-\rho^2}\sqrt{2}}\right)\right)\cdot$$

$$\cdot\frac{1}{\sqrt{2\pi}\sigma_2 r_1}e^{-\frac{(\ln r_1-\mu_2)^2}{2\sigma_2{}^2}}\left(\frac{1}{2}+erf\left(\frac{\ln r_t-(\mu_2+\rho(\ln r_1-\mu_2))}{\sigma_2\sqrt{1-\rho^2}\sqrt{2}}\right)\right)+$$

$$+P_2\left(\frac{1}{2}+erf\left(\frac{\ln r_t-\mu_2}{\sigma_2\sqrt{2}}\right)\right)\cdot$$

$$\cdot\frac{1}{2\pi\sigma_1{}^2\sqrt{1-\rho^2}\,r_1r_2}e^{-\frac{1}{2(1-\rho^2)}\left[\left(\frac{\ln r_1-\mu_1}{\sigma_1}\right)^2+\left(\frac{\ln r_2-\mu_1}{\sigma_1}\right)^2-2\rho\left(\frac{\ln r_1-\mu_1}{\sigma_1}\right)\left(\frac{\ln r_2-\mu_1}{\sigma_1}\right)\right]}+$$

$$+P_2\frac{1}{\sqrt{2\pi}\sigma_2 r_2}e^{-\frac{(\ln r_2-\mu_2)^2}{2\sigma_2{}^2}}\left(\frac{1}{2}+erf\left(\frac{\ln r_t-(\mu_2+\rho(\ln r_2-\mu_2))}{\sigma_2\sqrt{1-\rho^2}\sqrt{2}}\right)\right)\cdot$$

$$\left.\cdot\frac{1}{\sqrt{2\pi}\sigma_1 r_1}e^{-\frac{(\ln r_1-\mu_1)^2}{2\sigma_1{}^2}}\left(\frac{1}{2}+erf\left(\frac{\ln r_t-(\mu_1+\rho(\ln r_1-\mu_1))}{\sigma_1\sqrt{1-\rho^2}\sqrt{2}}\right)\right)\right]+$$

$$+\frac{1}{\pi}\int_{r_t}^{\infty}\int_{r_t}^{\infty}\int_0^{\pi/2} dr_1 dr_2 d\phi\left[\left(-\frac{gr_1}{\sin^2\phi}\right)\left(-\frac{gr_2}{\sin^2\phi}\right)\right.$$

$$\cdot P_1\frac{1}{2\pi\sigma_2{}^2\sqrt{1-\rho^2}\,r_1r_2}\cdot$$

$$\cdot e^{-\frac{1}{2(1-\rho^2)}\left[\left(\frac{\ln r_1-\mu_2}{\sigma_2}\right)^2+\left(\frac{\ln r_2-\mu_2}{\sigma_2}\right)^2-2\rho\left(\frac{\ln r_1-\mu_2}{\sigma_2}\right)\left(\frac{\ln r_2-\mu_2}{\sigma_2}\right)\right]}+$$

$$+P_1\frac{1}{\sqrt{2\pi}\sigma_2 r_2}e^{-\frac{(\ln r_2-\mu_2)^2}{2\sigma_2{}^2}}\cdot\frac{1}{\sqrt{2\pi}\sigma_1 r_1}e^{-\frac{(\ln r_1-\mu_1)^2}{2\sigma_1{}^2}}.$$

$$\cdot\left(\frac{1}{2}+erf\left(\frac{\ln r_t-(\mu_1+\rho(\ln r_1-\mu_1))}{\sigma_1\sqrt{1-\rho^2}\sqrt{2}}\right)\right)+$$

$$+P_1\left(\frac{1}{2}+erf\left(\frac{\ln r_t-\mu_1}{\sigma_1\sqrt{2}}\right)\right)\cdot\frac{1}{2\pi\sigma_2{}^2\sqrt{1-\rho^2}\,r_1r_2}\cdot$$

$$\cdot e^{-\frac{1}{2(1-\rho^2)}\left[\left(\frac{\ln r_1-\mu_2}{\sigma_2}\right)^2+\left(\frac{\ln r_2-\mu_2}{\sigma_2}\right)^2-2\rho\left(\frac{\ln r_1-\mu_2}{\sigma_2}\right)\left(\frac{\ln r_2-\mu_2}{\sigma_2}\right)\right]}+$$

$$+P_1\frac{1}{\sqrt{2\pi}\sigma_1 r_2}e^{-\frac{(\ln r_2-\mu_1)^2}{2\sigma_1{}^2}}\cdot\left(\frac{1}{2}+erf\left(\frac{\ln r_t-(\mu_1+\rho(\ln r_2-\mu_1))}{\sigma_1\sqrt{1-\rho^2}\sqrt{2}}\right)\right)$$

$$\cdot\frac{1}{\sqrt{2\pi}\sigma_2 r_1}e^{-\frac{(\ln r_1-\mu_2)^2}{2\sigma_2{}^2}}\cdot\left(\frac{1}{2}+erf\left(\frac{\ln r_t-(\mu_2+\rho(\ln r_1-\mu_2))}{\sigma_2\sqrt{1-\rho^2}\sqrt{2}}\right)\right)+$$

$$+P_2\frac{1}{2\pi\sigma_2{}^2\sqrt{1-\rho^2}\,r_1r_2}\cdot$$

$$\cdot e^{-\frac{1}{2(1-\rho^2)}\left[\left(\frac{\ln r_1-\mu_2}{\sigma_2}\right)^2+\left(\frac{\ln r_2-\mu_2}{\sigma_2}\right)^2-2\rho\left(\frac{\ln r_1-\mu_2}{\sigma_2}\right)\left(\frac{\ln r_2-\mu_2}{\sigma_2}\right)\right]}+$$

$$+P_2\frac{1}{\sqrt{2\pi}\sigma_1 r_2}e^{-\frac{(\ln r_2-\mu_1)^2}{2\sigma_1{}^2}}\cdot\frac{1}{\sqrt{2\pi}\sigma_2 r_1}e^{-\frac{(\ln r_1-\mu_2)^2}{2\sigma_2{}^2}}\cdot$$

$$\cdot\left(\frac{1}{2}+erf\left(\frac{\ln r_t-(\mu_2+\rho(\ln r_1-\mu_2))}{\sigma_2\sqrt{1-\rho^2}\sqrt{2}}\right)\right)+$$

$$+P_2\left(\frac{1}{2}+erf\left(\frac{\ln r_t-\mu_2}{\sigma_2\sqrt{2}}\right)\right)\cdot\frac{1}{2\pi\sigma_1{}^2\sqrt{1-\rho^2}\,r_1r_2}\cdot$$

$$\cdot e^{-\frac{1}{2(1-\rho^2)}\left[\left(\frac{\ln r_1-\mu_1}{\sigma_1}\right)^2+\left(\frac{\ln r_2-\mu_1}{\sigma_1}\right)^2-2\rho\left(\frac{\ln r_1-\mu_1}{\sigma_1}\right)\left(\frac{\ln r_2-\mu_1}{\sigma_1}\right)\right]}+$$

$$+P_2\frac{1}{\sqrt{2\pi}\sigma_2 r_2}e^{-\frac{(\ln r_2-\mu_2)^2}{2\sigma_2{}^2}}\cdot\left(\frac{1}{2}+erf\left(\frac{\ln r_t-(\mu_2+\rho(\ln r_2-\mu_2))}{\sigma_2\sqrt{1-\rho^2}\sqrt{2}}\right)\right)\cdot$$

$$\cdot\frac{1}{\sqrt{2\pi}\sigma_1 r_1}e^{-\frac{(\ln r_1-\mu_1)^2}{2\sigma_1{}^2}}\left(\frac{1}{2}+erf\left(\frac{\ln r_t-(\mu_1+\rho(\ln r_1-\mu_1))}{\sigma_1\sqrt{1-\rho^2}\sqrt{2}}\right)\right)\Bigg]\qquad(13)$$

3. NUMERICAL RESULTS

Some values of bit error rate for different types of combiners and correlation parameters are presented in Figs. 2- 4. where it is assumed that both input branches have the same channel parameters. It is adopted that r_t is optimal threshold of the SSC decision [1]:

$$r_t = \exp(\mu + \sigma^2 / 2) \tag{14}$$

The family of curves for the BER for one channel receiver and for MRC combiner at one time instant and SSC/MRC combiner at two time instants for uncorrelated case and for very strong correlation is shown in Fig. 2 and 4. versus different distribution parameters.

We can see that SSC/MRC combiner has significantly better performances for uncorrelated case compared to MRC combiner at one time instant and for $\rho = 1$ the BER of SSC/MRC combiner follows the results for MRC combiner.

Also, it is obvious that the use of this complex SSC/MRC combiner is not economical in the case of strongly correlated signals because it does not give better performance than MRC combiner.

In Fig. 3. the influence of correlation to outage probability of SSC/MRC combiner is presented. The benefits of using this type of combiner increase with the decrease of the correlation between input signals.

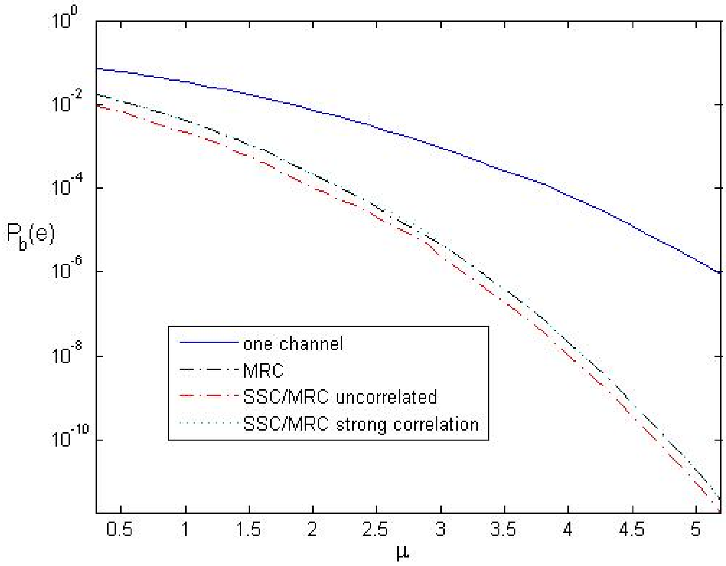

Fig. 2 Bit error rate for different types of combiners versus parameter μ for σ = 1

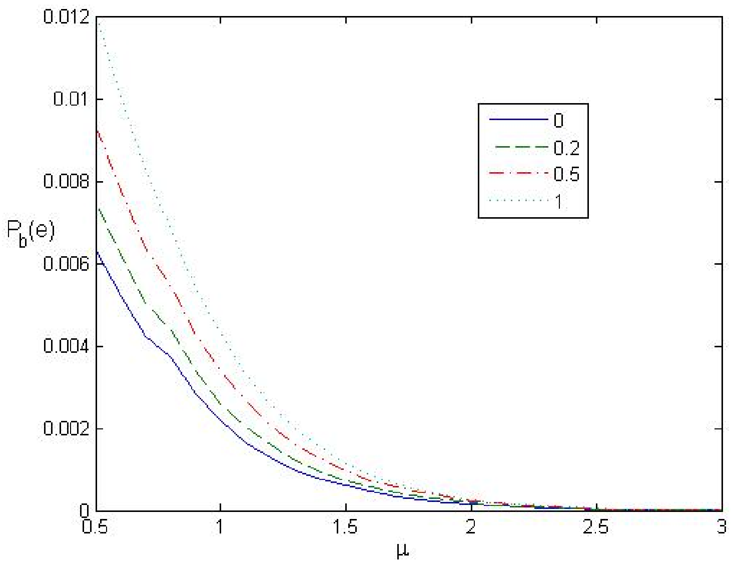

Fig. 3 Bit error rate for SSC/MRC combiner versus parameter μ for σ = 1, for different values of ρ

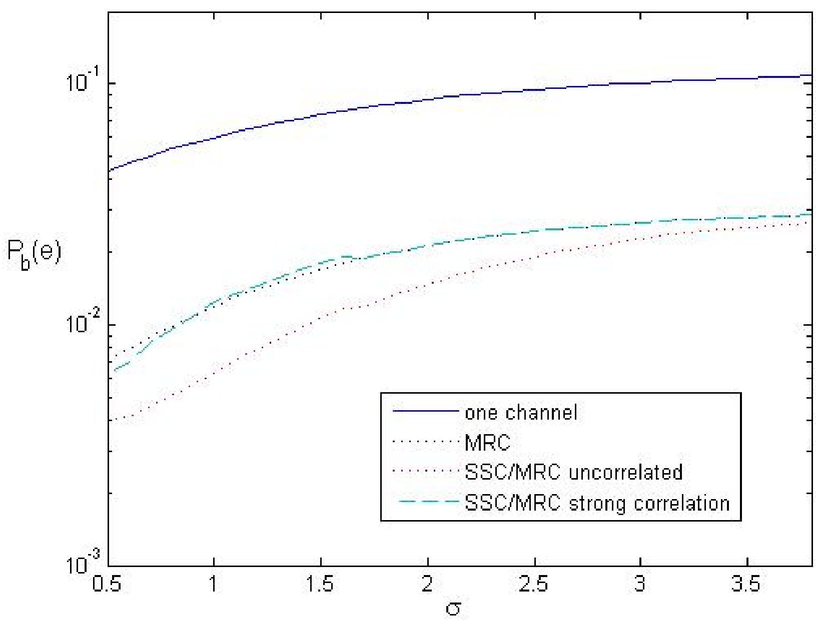

Fig. 4 Bit error rate for different types of combiners versus parameter σ for μ =0.5

4. Conclusions

The SSC and MRC are simple and frequently used techniques for combining signals in diversity systems. In this paper, the probability density function of the SSC/MRC combiner output signal with two branches at two time instants is determined and based on it, the bit error probability is expressed.

The system performances deciding by two samples can be determined by the joint probability density function of the SSC combiner output signal at two time instants and putting them as inputs of MRC combiner. The obtained results are shown graphically and depict the melioration of characteristics of SSC/MRCC combiner at two time instants compared to classical SSC and MRC combiners.

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VEROVATNOĆA GREŠKE SSC/MRC KOMBINERA U DVA TRENUTKA VREMENA U PRISUSTVU LOG-NORMALNOG FEDINGA

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U ovom radu razmatrane su performanse SSC/MRC kombinera (Switch and Stay Combining/ Maximal Ratio Combining) u dva vremenska trenutka. Razmatrani su SSC i MRC kombineri sa dve grane u prisustvu log-normalnog fedinga. Određena je funkcija gustine verovatnoće (PDF) na izlazu kombinera i na osnovu nje verovatnoća greške (BER) za slučaj binarne fazne modulacije (BPSK). Rezultati su prikazani grafički da bi se istakle bolje performanse SSC/MRC kombinera u odnosu na klasične SSC i MRC kombineri u jednom vremenskom trenutku.

Ključne reči: *diversiti prijem, združena funkcija gustine verovatnoće, log-normalni feding, SSC kombinovanje, MRC kombinovanje, dva vremenska trenutka, verovatnoća greške*