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MATHEMATICAL MODEL AND NUMERICAL SIMULATION OF CPC-2V CONCENTRATING SOLAR COLLECTOR*

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Abstract. *Physical and mathematical model is presented, as well as numerical procedure for predicting thermal performances of the CPC-2V solar concentrator. The CPC-2V solar collector is designed for the area of middle temperature conversion of solar radiation into heat. The collector has high efficiency and low price. Working fluid is water with laminar flow through a copper pipe surrounded by an evacuated glass layer. Based on the physical model, a mathematical model is introduced, which consists of energy balance equations for four collector components. In this paper water temperatures in flow directions are numerically predicted, as well as temperatures of relevant CPC-2V collector components for various values of input temperatures and mass flow rates of the working fluid, and also for various values of direct sunlight (solar) radiation and for different collector length.*

Key words: *thermal, solar radiation, performance, model, dynamic analysis*

1. INTRODUCTION

The device which is used to transform solar energy to heat is referred to as solar collector or STC. Depending on the temperatures gained by them, STCs can be divided into low, middle and high temperature systems. Mid-temperature systems are applicable for refrigeration systems and industrial processes.

Numerous theoretical and experimental researches of solar collectors for the mid-temperature conversion of solar radiation into heat via a liquid as a working fluid have been conducted. R. Tchinda, E. Kaptouom and D. Njomo [1] have investigated heat transfer in a CPC collector and developed a model which during numerical analysis of heat transfer takes into account axial heat transfer in a CPC collector. The impact of various parameters has been

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investigated, such as input temperature and the value of the flow of working fluid on dynamical behavior of the collector. B. Norton, A. F. Kothdiwala and P. C. Eames [2] have developed a theoretical heat transfer model, which describes steady state heat behavior of a symmetric CPC collector and they have investigated the impact of the tilt angle of the collector to the CPC collector performance. R. Oommen and S. Jayaraman [3] have experimentally investigated a complex parabolic concentrator (CPC), with water as a working fluid. They have investigated efficiency of the collector for various input temperatures of water, as well as the efficiency of the collector in the conditions of steam generation. P. Gata Amaral, E. Ribeiro, R. Brites and F. Gaspar [4] have worked on the development of solar concentrating collectors based on the shape known as complex paraboilc colector of solar energy (CPC), which allow utilization of maximal solar energy ratio. By wide area of rotation movement of a SolAgua collector, it was made possible to follow the Sun. Khaled E. Albahloul, Abdullatif S. Zgalei and Omar M. Mahgjup [5] have theoretically analyzed the use of a CPC collector with refrigeration systems. They followed the efficiency of CPC collector with refrigeration systems during the year. CPC collectors have been of different length and with different concentrating ratios.

In this paper, physical and mathematical models are given, as well as the numerical calculation of a solar collector CPC-2V. This is a solar concentrator, designed for the mid-temperature conversion [6-11] (B. Nikolic, V. Stefanovic and all). The paper gives a change of fluid temperate and collector components in the direction of the flow, as well as the influence of the input fluid temperature, radiation intensity, mass flow rate to the heating behavior and current efficiency of the collector. The change of fluid temperature was given in the direction of the fluid flow for six collectors connected in a row.

2. PHYSICAL MODEL

System description

In this paper, a laminar flow of the fluid (l) through the absorber (2) is assumed. The pipe of the absorber is subjected to solar radiation. Solar rays go through the transparent cover (5) on the collector apparatus. A part of the solar radiation after going through the transparent cover of the pipe (4) and evacuated area (3), falls directly to the absorber. Another part of the solar radiation, after reflection from the reflector (6), goes through the transparent cover layer of the pipe and falls at the pipe absorber. The look of a CPC-2V module is presented in Fig. 1. (a). A scheme of the cross section of a CPC-2V module is given in Fig. 1 (b).

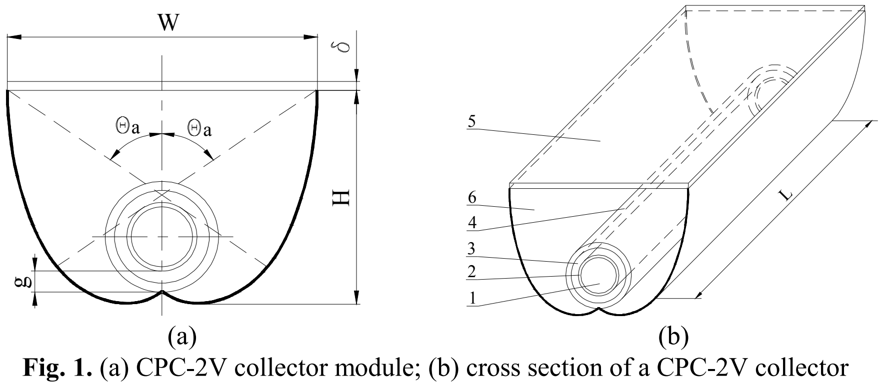


Fig. 1. (a) CPC-2V collector module; (b) cross section of a CPC-2V collector

The CPC module consists of a complex cylindrical-parabolic reflection surface and a copper tube with ø15 mm outside diameter used as the absorber. Cylindrical cooper absorber is surrounded by a glass surrounding layer for lowering convective loss from the collector pipe. The area between the pipe absorber and glass surrounding layer is evacuated. Conversion of solar energy into heat is conducted on the pipe collector. Pipe absorber is colored with selective color of high absorbing properties and low emissivity (ε_r). The area of the reflector has high reflection ratio (ρ_m). Between the pipe absorber and the reflector there is a gap (hole) which stops heat transfer from the collector pipe to the reflector. Water is the working fluid, with a laminar flow through the pipe of the collector. Apparatus of the collector is covered by transparent cover layer made of glass (Plexiglass) so the reflector area could be saved from wearing and to lower the value of heat loss from the assembly pipe absorber-surrounding pipe layer. The collector consists of six CPC modules which are connected parallel to the flow paths, so flow properties inside them may be considered equal.

3. Mathematical Model

3.1. Introduction

Based on the physical model represented by the CPC collector, through whose absorber pipe a laminar flow of water occurs, and which is subjected to solar radiation, a mathematical model is proposed. The mathematical model consists of the energy balance equations for four CPC module components: (1) working fluid, (2) collector pipe- absorber, (3) surrounding pipe layer, (4) transparent cover. During the set-up of mathematical model only one module of CPC collector was analyzed.

3.2. Basic assumption according to which the model was based

The following assumptions were introduced for the definition of mathematical model:

- The sky was treated as an absolute black object, which is the source of infrared radiation during an equivalent sky temperature
- Reflected radiation from surrounding objects was considered negligible and was not taken into account;
- Diffusive insulation on the apparatus of the cover is isotropic;
- CPC collector has permanent Sun tracking, thus Sun rays are normal to the plane of transparent cover (aperture plane);
- Radiation to the collector is unified;
- CPC module is constructed with ideal geometry, thus the concentration ratio is given by: [13]:

$$C_a = \frac{1}{\sin\theta_a} = \frac{A_c}{A_r} \tag{1}$$

- Heat transport in the transparent cover, surrounding cover layer, collector surrounding layer, pipe absorber and the fluid is transient
- Transparent cover, transparent cover layer, collector surrounding layer, and pipe absorber are homogenous and isotropic objects;
- Heat transport by conduction in the transparent cover layer and the glass layer is negligible due to small heat conductivity of glass;

- Thermo-physical properties of the CPC collector component material (ρ^*, c, λ) as well as optic properties (ρ, τ, ε, α) of the components of CPC collector do not depend on coordinates, temperature or time;
- Fluid flow is steady state and it is conducted only in the axial direction, thus the velocity field is related just to the speed component w_z;
- Fluid flow shape is the same in every axial cross sections, thus the tangential velocity component w_φ does not exist;
- Radial velocity component w_r is neglected as a value of smaller order and thus the convection in radial direction is also neglected;
- Fluid flow may be considered incompressible;
- Conduction in the axial direction has a negligibly small contribution to the resulting heat transport compared to the convection.

3.3. Energy balance equations

Mathematical model consists of equations of energy balance for all of the four components of the CPC module, relations for determining heat transfer coefficient, relations for radiation absorbed by the relevant system components. In order to gain a unified solution from the system of equations, initial and boundary conditions are defined. With predetermined assumptions, equations of energy balance may be written as:

(1) *For the working fluid*

Energy balance for an elementary fluid volume of dz length in the axial direction, after sorting, may be written as:

$$\rho_f^* c_{pf} A_f^* \frac{\partial T_f}{\partial t} = -\rho_f^* c_{pf} A_f^* w_z \frac{\partial T_f}{\partial z} + U_{r/f}(T_r - T_f) 2\pi r_{r,o} \tag{2}$$

Where ρ_f^*, c_{pf} and w_z represent density, specific heat, velocity in the axial direction of the elementary fluid volume, respectively $A_f^* = r_{r,i}^2 \pi$ is the area of the cross section of the elementary fluid particle. The second term on the right hand side of the equation (2) represents heat gained from the outer side of the collector pipe.

Boundary condition for this equation is:

$$\text{for } z = 0,\ T_f = T_{in}. \tag{3}$$

(2) *For the collector absorber pipe*

Energy balance for elementary part of the collector of dz length in the axial direction may be written as:

$$\begin{aligned}\rho_r^* c_r A_r^* \frac{\partial T_r}{\partial t} = \frac{\partial}{\partial z}\left(\lambda_r \frac{\partial T_r}{\partial z}\right) A_r^* - (h_{c,r/e} + h_{r,r/e})(T_r - T_e) 2\pi r_{r,o} - \\ -U_{r/f}(T_r - T_f) 2\pi r_{r,o} + (q_{b,r} + q_{d,r}) 2\pi r_{r,o}\end{aligned} \tag{4}$$

Where λ_r, ρ and c_r are heat conduction coefficient, density and specific heat for elementary pipe volume, respectively. $A_r^* = (r_{r,o}^2 - r_{r,i}^2)\pi$ is the area of the cross section of elementary part of the collector pipe. The first term on the right hand side of the equation (4) represents heat transport by conduction in the axial direction of the collector, the second is the heat loss due to convention and radiation between the collector pipe and the surrounding layer of the collector pipe, the third term represents heat given to the working fluid, and the fourth represents heat gained via solar radiation.

Boundary conditions for this equation are:

$$\text{for } z = 0, \ \frac{\partial T_r}{\partial z} = 0 ; \tag{5}$$

$$\text{for } z = L, \ \frac{\partial T_r}{\partial z} = 0 \tag{6}$$

(3) *For the transparent cover of the pipe collector*

Energy balance for the elementary part of the surrounding layer of the collector pipe of dz length is written as

$$\rho_e^* c_e A_e^* \frac{\partial T_e}{\partial t} = (h_{c,r/e} + h_{r,r/e})(T_r - T_e)2\pi r_{r,o} - \\ - (h_{c,e/c} + h_{r,e/c})(T_e - T_c)2\pi r_{e,o} + (q_{b,e} + q_{d,e})2\pi r_{r,o} \tag{7}$$

Where ρ_e^* and c_e are density and specific heat of the surrounding layer of the collector $A_r^* = (r_{e,o}^2 - r_{e,i}^2)\pi$ is the area of the cross section of elementary part of the surrounding layer of the collector. The second term on the right hand side of the equation (7) is the heat lost due to radiation from the surrounding layer of the collector to the transparent cover.

(4) *For the transparent cover*

Energy balance for the elementary part of the transparent cover of the concentrating collector, with dz length in the z direction and W width, may be written as:

$$\rho_c^* c_c A_c^* \frac{\partial T_c}{\partial t} = (h_{c,e/c} + h_{r,e/c})(T_e - T_c)2\pi r_{e,o} - h_{c,c/a}(T_c - T_a)W - \\ - h_{r,c/s}(T_c - T_s)W + (q_{b,c} + q_{d,c})2\pi r_{r,o} \tag{8}$$

$A_c^* = W\delta$ is the area of the cross section of elementary part of the transparent cover of the collector. The second term on the right hand side of the equation (8) is the heat lost due to convection from the transparent cover to the environment, the third term is the heat lost due to radiation between the transparent cover and the sky.

3.4. Initial conditions

It was assumed for the initial conditions that in the initial time point the temperature field in all components is equal to the environment temperature T_a, while the fluid temperature at the inlet is constant in time:

$$\text{For } t = 0, \quad T_f = T_r = T_e = T_c = T_a \tag{9}$$

$$\text{For } z = 0, \quad T_f = T_{in} \tag{10}$$

3.5. Heat transfer coefficients

Heat transfer coefficient for convection from the cover to the environment, caused by air flow (wind) $h_{c,c/a}$ is given by the following relation (Duffie and Beckman [12]):

$$h_{c,c/a} = (5.7 + 3.8v)\frac{A_c}{A_r} \tag{11}$$

Heat transfer coefficient for the radiation of heat between the transparent cover and the sky $h_{r,c/s}$ is calculated by:

$$h_{r,c/s} = \varepsilon_c \sigma (T_c^2 + T_s^2)(T_c + T_s)\frac{A_c}{A_r} \tag{12}$$

Convective heat transfer coefficient between the surrounding pipe layer and the transparent cover, $h_{c,e/c}$ may be calculated by [14]:

$$h_{c,e/c} = \left[3.25 + 0.0085\frac{T_e - T_c}{4r_e}\right]\frac{A_e}{A_r} \tag{13}$$

Heat transfer coefficient originating from the radiation form the surrounding pipe layer to the transparent cover $h_{r,e/c}$ is given by [14]:

$$h_{r,e/c} = \frac{\sigma(T_e^2 + T_c^2)(T_e + T_c)}{1/\varepsilon_c + A_e/A_c(1/\varepsilon_c - 1)}\frac{A_e}{A_r} \tag{14}$$

Heat transfer by convection through the evacuated area between the collector pipes and the surrounding collector layer may be neglected, so the heat transfer coefficient by convection is $h_{c,r/e} \approx 0$.

Heat transfer coefficient for radiation from the collector pipe to the collector surrounding layer $h_{r,r/e}$, is given as:

$$h_{r,r/e} = \frac{\sigma(T_e^2 + T_r^2)(T_e + T_r)}{1/\varepsilon_r + A_r/A_e(1/\varepsilon_e - 1)} \tag{15}$$

Total heat transfer coefficient $U_{r/f}$ between the outside surface of the collector pipe and the working fluid is [14]:

$$U_{r/f} = \frac{1}{\dfrac{r_{r,o}\ln(r_{r,o}/r_{r,i})}{\lambda_r} + \dfrac{A_{r,o}}{h_f A_{r,i}}} \tag{16}$$

Where $r_{r,o}$ and $r_{r,i}$ are outside and inside radius of the pipe of the collector, λ_r heat conduction coefficient of the collector pipe h_f coefficient of heat transfer by convection from the internal surface of the pipe of the collector to the working fluid.

Coefficient of heat transfer by convection, from the internal pipe side to the fluid h_f, is given as:

$$h_f = \frac{N_{uf}\lambda_f}{d} \tag{17}$$

Where N_{uf} is the Nusselt number calculated according to Sieder and Tate relation [16, 17]:
For laminar fluid flow through the pipe Re<2300:

$$N_{uf} = 1{,}86\left[\frac{R_{ef}P_{rf}d}{L}\right]^{1/3}\left(\frac{\mu_f}{\mu_{fs}}\right)^{0.14} \tag{18}$$

For the valid range: $(R_{ef}P_{rf}d/L)^{1/3}\left(\frac{\mu_f}{\mu_{fs}}\right)^{0.14} \geq 2$,

where: $R_{ef} = \frac{4\dot{m}}{\mu_f \pi d}$ and $P_{rf} = \frac{\mu_f C_{pf}}{\lambda_f}$. the relation stands for $0.7 \leq P_{rf} \leq 16700$.

The terms for direct and diffusive radiation (reduced to a unit of absorber area) which falls on the transparent cover and is absorbed by it - $q_{b,c}$ and $q_{d,c}$, direct and diffusive radiation absorbed by the glass layer of the collector $q_{b,e}$ and $q_{d,e}$ and direct and diffusive radiation absorbed by the collector pipe - $q_{b,r}$ and $q_{d,r}$, are taken according to [14] (Hsieh).

A model set this way can be solved numerically, and for solving a complex model finite volume method was used.

3.6. Heat efficiency of the CPC-2V collectors

Heat efficiency of the collector (η) represents its thermal, or energy efficiency, which is basically its ability to convert a certain amount of radiated solar energy into heat, and is given as:

$$\eta = \frac{\dot{m}C_{pf}(T_{out} - T_{in})}{I_{tot} \cdot A_c} \tag{19}$$

4. Numerical Model

4.1. Dividing the calculation area into control volumes

The numerical model is based on dividing every component of the CPC collector, with L length, on a number of control volumes, thus making each control volume surrounding a single node. Differential balance equations are integrated for every control volume of the relevant component [15]. N number of nodes in z direction is taken. Nodes 1 and N are positioned on the system boundary. Control volumes surrounding these nodes are of Δz/2 length, whilst control volumes surrounding internal nodes are of Δz length. A divided working fluid with N control volumes is presented in Fig 2. Such division was made for other CPC collector components.

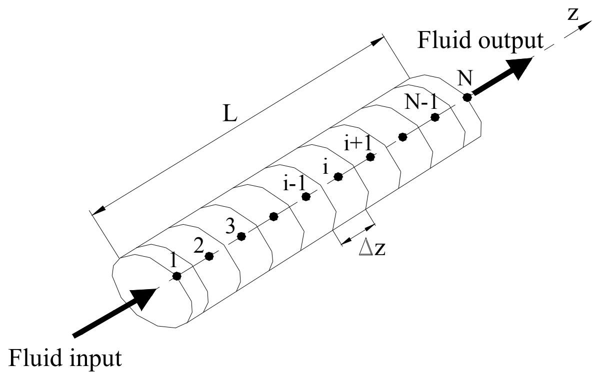


Fig. 2. Grid for the working fluid of CPC collector

4.1.1. *Discretization energy balance equations for the working fluid*

In order to create a discretized equation for the working fluid, a control volume around node i is taken, as shown in Fig 3. Node i is marked by *P*, and the neighboring nodes. Node *i*–1 and *i*+1 are marked by *W* and *E*, respectively. Since the time is a one way coordinate the solution is obtained by "marching" through time beginning with initial temperature field. Temperature T is set for the nodes in the t time step, and *T* values in the time step *t*+Δ*t* should be found. "Old" (set) values for T in the nodes will be marked T_{fP}^0, T_{fE}^0, T_{fW}^0 and "new" (unknown) values in *t*+Δ*t* timestep are marked as T_{fP}^1, T_{fE}^1, T_{fW}^1. Discretization of the differential equations of energy balance for the working fluid is possible to conduct by its integration for the control volume as in Fig. 3. In the time interval *t* to *t*+Δ*t*:

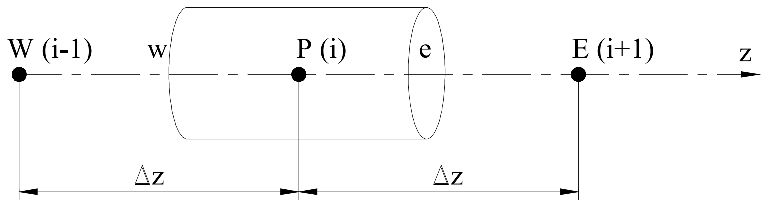


Fig. 3. Grid for the working fluid of the CPC collector

$$\rho_f^* c_{pf} A_f^* \int_w^e \int_t^{t+\Delta t} \frac{\partial T_f}{\partial t} dt dz = -c_{pf} A_f^* \int_t^{t+\Delta t} \int_w^e \rho_f^* w_z \frac{\partial T_f}{\partial z} dz dt + \int_t^{t+\Delta t} \int_w^e U_{r/f} (T_r - T_f) 2\pi r_{r,o} dz dt \quad (20)$$

The order of the integration was chosen according the toe nature of the term. For the representation of $\partial T/\partial t$, prevailing value for *T* in a node for "stepwise" scheme in the control volume. This leaves:

$$\rho_f^* c_{pf} A_f^* \int_w^e \int_t^{t+\Delta t} \frac{\partial T_f}{\partial t} dt dz = \rho_f^* c_{pf} A_f^* \Delta z (T_{fP}^1 - T_{fP}^0) \tag{21}$$

For the integration of the convective term upwind scheme is used, thus the value of temperature at the mid surface is equal to the value of temperature in the node of the upwind side of the node:

$$\begin{aligned} c_{pf} A_f^* \int_t^{t+\Delta t} \int_w^e \rho_f^* w_z \frac{\partial T_f}{\partial z} dz dt &= c_{pf} A_f^* \int_t^{t+\Delta t} (\rho_{fe}^* w_{ze} T_{fe} - \rho_{fw}^* w_{zw} T_{fw}) dt = \\ &= c_{pf} A_f^* (F_e T_{fe}^0 - F_w T_{fw}^0) \Delta t = \\ &= c_{pf} A_f^* \Delta t \left[(T_{fP}^0 \| F_e, 0 \| - T_{fE}^0 \| -F_e, 0 \|) - (T_{fW}^0 \| F_w, 0 \| - T_{fP}^0 \| -F_w, 0 \|) \right] \end{aligned} \tag{22}$$

$$T_{fe}^0 = T_{fP}^0 \quad \text{za} \quad w_z > 0 \tag{23}$$

$$T_{fe}^0 = T_{fE}^0 \quad \text{za} \quad w_z < 0 \tag{24}$$

According to the assumption of the model that this is an uncompressible $\rho_{fe}^* = \rho_{we}^*$ and $A_{fe}^* = A_{fw}^*$ we have that $w_{ze} = w_{zw}$. Further, due to more simple writing, a new operator is introduced $\|A,B\|$ standing for the greater value from A and B. A new sign F is introduced which stands for the convection intensity (flow): $F_e = \rho_{fe}^* w_{ze}$ and $F_w = \rho_{fw}^* w_{zw}$.

$$F_e T_{fe}^0 = T_{fP}^0 \| F_e, 0 \| - T_{fE}^0 \| -F_e, 0 \| \tag{25}$$

$$F_w T_{fw}^0 = T_{fW}^0 \| F_w, 0 \| - T_{fP}^0 \| -F_w, 0 \| \tag{26}$$

By integration of the third term:

$$\begin{aligned} \int_t^{t+\Delta t} \int_w^e U_{r/f} (T_r - T_f) 2\pi r_{r,o} dz dt &= U_{r/f} 2\pi r_{r,o} \Delta z \int_t^{t+\Delta t} (T_{rP} - T_{fP}) dt = \\ U_{r/f} (T_{rP}^0 - T_{fP}^0) 2\pi r_{r,o} \Delta z \Delta t \end{aligned} \tag{27}$$

For the temperature change T_{fP}, T_{rP}, T_{fE} and T_{fW} with time t to $t+\Delta t$, an assumption of explicit scheme was introduced.

By sorting the equation (20) an equation for a node (i) for the fluid in the time step (1) is, $T_{f,i}^1$:

$$T_{f,i}^1 = \frac{\Delta t}{\rho_f^* c_{pf} A_f^* \Delta z} \left[a_1 T_{f,i-1}^0 + a_2 T_{f,i}^0 + a_3 T_{f,i+1}^0 + a_4 T_{r,i}^0 \right] \tag{28}$$

For constant input temperature of the fluid T_{in}, for the first node is $T_{f,1}^0 = T_{in}$ for any time step.

4.1.2. *Discretization of the energy balance equation for collector pipe*

By integrating differential equation of the energy balance for the collector pipe for the control volume in the time interval from t to $t+\Delta t$, and then sorting, the equation for the node (i) temperature of the pipe in the time step (1) is $T_{r,i}^1$:

$$T_{r,i}^1 = \frac{\Delta t}{\rho_r^* c_r^* A_r \Delta z}[b_1 T_{r,i-1}^0 + b_2 T_{r,i}^0 + b_3 T_{r,i+1}^0 + b_4 T_{f,i}^0 + b_5 T_{e,i}^0 + b_6] \tag{29}$$

Equation (29) is a general equation set for any internal control volume of the collector pipe. Boundary nodes $i = 1$ and $i = M$ are on adiabatic boundary. For the adiabatic boundary $\partial T / \partial z = 0$. Equation (29) for the boundary nodes ($i = 1$ and $i = M$) may be written as (30) and (31):

$$T_{r,1}^1 = \frac{\Delta t}{\rho_r^* c_r^* A_r \Delta z}[b_2' T_{r,1}^0 + b_3 T_{r,2}^0 + b_4 T_{f,1}^0 + b_5 T_{e,1}^0 + b_6] \tag{30}$$

$$T_{r,N}^1 = \frac{\Delta t}{\rho_r^* c_r^* A_r \Delta z}[b_1 T_{r,N-1}^0 + b_2' T_{r,N}^0 + b_4 T_{f,i}^0 + b_5 T_{e,i}^0 + b_6] \tag{31}$$

4.1.3. *Discretization of the energy balance equation for the transparent cover of the collector*

In the same fashion, by integrating differential equation for the energy balance equation for transparent cover collector for the control volume in the time interval t to $t+\Delta t$, an equation for temperature of the node (i) is gained for the cover of the collector in the time step (1), $T_{e,i}^1$:

$$T_{e,i}^1 = \frac{\Delta t}{\rho_e^* c_e^* A_e \Delta z}[c_1 T_{e,i}^0 + c_2 T_{r,i}^0 + c_3 T_{c,i}^0 + c_4] \tag{32}$$

4.1.4. *Discretization of the energy balance equation for transparent cover*

By integrating the energy balance differential equation for the transparent cover for the control volume in the time interval t to $t+\Delta t$, an equation for the temperature of the node (i) is obtained of the transparent cover for the time step (1), $T_{c,i}^1$:

$$T_{c,i}^1 = \frac{\Delta t}{\rho_c^* c_c^* A_c \Delta z}[d_1 T_{c,i}^0 + d_2 T_{e,i}^0 + d_3] \tag{33}$$

4.2. The method for solving algebraic equations

Discretization equation of energy balance components for the corresponding receiver modules CPC gets a set of coupled linear algebraic equations of the form (28), (29), (32) and (33), with the total number of equations equals the number of nodal points in the axial direction for all four components. Since the thermophysical properties of working fluid (water) depend on the temperature, there is a need for an iterative method of solving.

During the solving of this system of algebraic equations, two temperature fields, the previous time instant (0) and the next moment in time (1) were introduced. Before solving each new temperature field (1), thermophysical properties of water, the corresponding coefficient of heat transfer coefficients and equations themselves shall be converted. For the initial temperature field it is taken that the temperature of all components of nodal points of equal temperature environment. Nodal point $i = 1$ for the working fluid at the entrance to the pipe receiver has a constant temperature T_{in} for each time instant. After solving the system of algebraic equations, the new temperature field (1) now becomes the temperature field for the previous time instant (0). Solving procedure continues until the temperature difference between two successive iterations, for all nodal points is not less than the pre-ser errors ε_e. After several test, we found that the $\varepsilon_e=10^{-5}$ (°C) and $\Delta t=0.0005$ s. These values were used for all the results of the budget, which are presented in this paper.

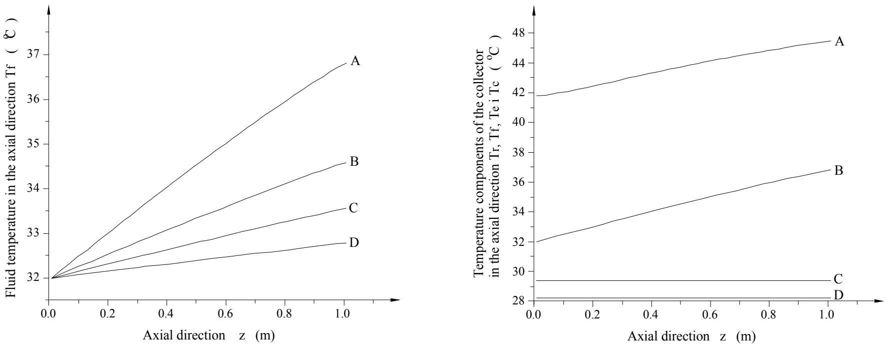
For the implementation of numerical procedures of solving the mathematical model of fluid flow in the pipe CPC receiver, and thermal behavior of other components of the receiver in the program has been developed in FORTRAN-77. The procedure begins with entering the calculation of basic geometric characteristics of CPC receiver, the number of nodal point, thermophysical and optical properties of system components, working fluid inlet temperature, intensity of solar radiation, and wind velocity.

4.3. The results of numerical experiments

During the calculation, it was assumed that the solar rays fall normal to the aperture plane. It was assumed that all off the radiation falls to the aperture directly. Dimensions, thermophysical properties for the CPC module, for which the numerical simulation was conducted, are given in Table 1.

Table 1. CPC module data

Dimensions
$r_{r,i} = 0.0065[m]$, $r_{r,o} = 0.0075[m]$,
$r_{e,i} = 0.01[m]$ $r_{e,o} = 0,012[m]$, $\delta = 0.005[m]$ $W = 0.065[m]$, $L = 1[m]$,
$H = 0.0465[m]$
Optical properties
$\varepsilon_r = 0.05$, $\varepsilon_e = \varepsilon_c = 0.85$, $\rho_e = \rho_c = 0.05$, $\rho_r = 0.15$, $\rho_m = 0.85$,
$\tau_e = \tau_c = 0.90$, $\alpha_r = 0.95$, $\alpha_e = \alpha_c = 0.05$
Thermo-physical properties
$\rho_e = \rho_c = 2700[kg/m^3]$, $d_r = 8930[kg/m^3]$, $c_e = c_c = 840[J/kgK]$,
$c_r = 383[J/kgK]$, $\lambda_r = 395[W/mK]$

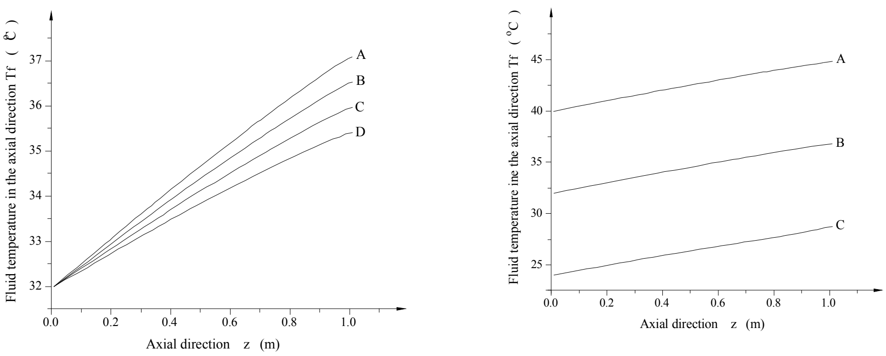


(a) (b)

Fig. 4. (a) Impact of the mass flow rate to local fluid temperature in the flow direction T_{ulf}=32 º C, *A* – $\dot{m}$ =0.00162 kg/s, B – $\dot{m}$ =0.003 kg/s, C – $\dot{m}$ =0.005 kg/s, D – $\dot{m}$ = 0.01 kg/s; (b) Change of local temperature of the collector components in the flow direction: A – collector pipe, B – working fluid, C - surrounding layer of the collector, D – transparent cover

During the calculations, environment temperature T_a = 28 ºC, wind speed v = 2 m/s were kept constant. For direct solar radiation I_b=950 W/m^2, inlet fluid temperature T_{ul}=32 ºC and different flow values, the values of local temperature of the working fluid in the direction of the flow are given in Fig. 4. (a) which shows that by increasing fluid flow rate output temperature is reduced. Fig. 4. (b) shows the change of temperature of the collector pipe, working fluid, surrounding collector pipe layer, and transparent cover in the flow direction, for the direct solar radiation I_b=950 W/m^2, inlet temperature of the fluid T_{ul}=32 ºC and working fluid flow 0.00162 kg/s.

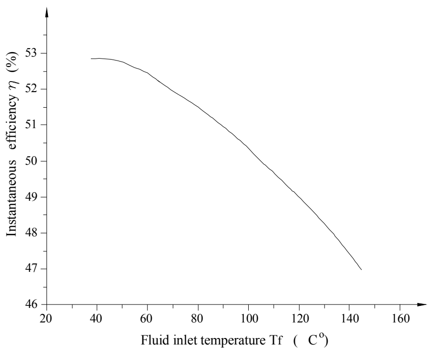
Fig 5 (a) shows the impact of the direct radiation intensity to the working temperature change in the flow direction, for the inlet fluid temperature T_{ul} =32 ºC and fluid flow 0.00162 kg/s. Fig 5. (b) depicts the impact of the value of local inlet temperature of the fluid to the local fluid temperature for the flow rate of 0.00162 kg/s and direct radiation of I_b=950 W/m^2.



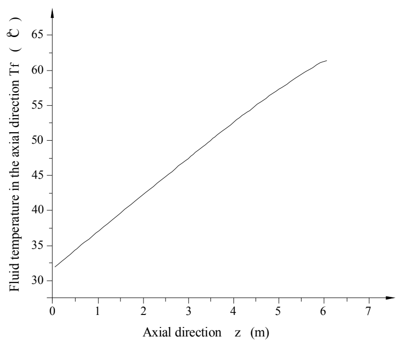
(a) (b)

Fig. 5. (a) Impact of the direct radiation value to the local temperature in the flow direction: A – I_b=1000 W/m^2, B – I_b =900 W/m^2, C – I_b =800 W/m^2, D – I_b =700 W/m^2; (b) Impact inlet fluid temperature value to the local fluid temperature in direction flow: A – T_{ulf}=40 º C, B – T_{ulf}=32 º C, C – T_{ulf}=24 º C.

Fig. 6 (a) depicts the impact of inlet fluid temperature to the current heat efficiency, for direct radiation I_b=950 W/m^2 and flow rate 0.00162 kg/s. The graphs show that by raising inlet temperature of the fluid, heat efficiency drops. This is explained by the fact that solar flux and environment temperature remain constant, and the difference between inlet fluid temperatures is reduced, thus useful energy is reduced. Fig 6(b) shows the impact of collector length to the output fluid temperature, for the CPC module length L=6m, I_b=950 W/m^2, input fluid temperature T_{ul}=32 ºC and flow rate 0.00162 kg/s where length increase compared to the collector 1 m long, a higher output temperature is achieved.



(a)



(b)

Fig. 6. (a) Impact of the input fluid temperature value to the current heat efficiency of the CPC collector, (b) Impact of the length to the value of the output temperature

5. Conclusions

This paper gives numerical estimated changes of temperature in the direction of fluid flow for different flow rates, different solar radiation intensity and different inlet fluid temperatures. By fluid flow increase output temperature is reduced, whilst by solar radiation intensity increase and inlet water temperature – output temperature of water is increased. A prediction of the change of temperature of components of the CPC 2V module in the flow direction is also given. The change in efficiency is predicted with the change in input temperature of water. With the increase of the inlet water temperature current efficiency of the collector is decreased. All of the predictions are given for a 1m long module. Also, numerical estimation of the change of the fluid temperature in the direction of the flow for a 6m long module connected in a row, where a higher output temperature is achieved compared to the 1m long module.

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MATEMATIČKI MODEL I NUMERIČKI PRORAČUN KONCENTRIŠUĆEG SOLARNOG PRIJEMNIKA CPC-2V

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Predstavljeni su fizički i matematički model, kao i numerička procedura za predviđanje termičkih performansi Složenog Paraboličnog Koncentratora CPC-2V. Solarni prijemnik CPC-2V je predviđen za oblast srednjetemperaturne konverzije sunčevog zračenja u toplotu. Prijemnik CPC-2V ima visoki učinak i nisku cenu. Radni fluid (voda) laminarno struji kroz bakarnu cev okruženu evakuisanim staklenim omotačem. Na osnovu fizičkog modela postavljen je matematički model, koga sačinjavaju jednačine bilansa energije za četiri komponente prijemnika. U ovom radu numerički su predviđane promene temperature vode u pravcu strujanja, kao i temperature odgovarajućih komponenti prijemnika CPC-2V za različite vrednosti ulazne temperature i masenog protoka radnog fluida, zatim za različite vrednosti direktnog sunčevog zračenja kao i za različitu dužinu prijemnika.

Ključne reči: *solarno zračenje, karakteristike, numerički model, dinamička analiza*