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Editor of series: Katica (Stevanovi}) Hedrih, e-mail: katica@masfak.masfak.ni.ac.yu
Address: Univerzitetski trg 2, 18000 Niš, YU, Tel: +381 18 547-095, Fax: +381 18 547-950
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RHEONOMIC COORDINATE METHOD APPLIED TO NONLINEAR VIBRATION SYSTEMS WITH HEREDITARY ELEMENTS [1]

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Katica (Stevanović) Hedrih

Faculty of Mechanical Engineering University of Niš
Ul. Vojvode Tankosića 3/22, Yugoslavia, 18000 Niš, telefax: +381 18 41-663
e-mail: katica@masfak.masfak.ni.ac.yu

Abstract. *Results pointed out in this paper, are inspired by papers of O. A. Goroshko and N. P. Puchko (see Ref. [13] and [14]), about Lagrange's equations for the multybodies hereditary systems, and rheological models of the bodes properties presented in the monograph written by G.M. Savin and Ya.Ya. Ruschitsky (see Ref. [24]), as well as a monograph on rheonimic dynamics written by V.A. Vujičić (see [6]). By using rhelogical body models for designing deformable rheological hereditary elements with hybrid rheological elastoviscosic and/or viscoelastic properties (see Ref. [23], [24] and [18]), discrete oscillatory systems with hereditary elements as constraints, are designed, as systems with one degree of freedom as well as with many degrees of freedom. For these oscillatory hereditary systems, the integro-differential equations of the second and/or third kind are composed. The solutions of these integro-differential equations are studied.*
Equations of dynamics of a disrete system with finite constraints and standard hereditary elements are composed.
Covarinat integro-differential equations of the motion of the discrete hereditary system are composed.
The rheonomic coordinate method is applied to dicrete hereditary systems, and the modified system of the covarint integro-differential equations of motion of discrete hereditary systems with rheonomic constraints are composed.
For example, the rheological pendulum on the wool's thread with changeable length is modeled by rheonomic coordinate as well as by rheological hereditary element. By using defined rheological pendulum basic properties of the rheonomic coordinate in the sense of the Vujičić's, rheonomic coordinate are introduced. The force, as well as the power of the rate of rheological and rheonomic constraints change are determined.

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For the designed discrete hereditary systems with corresponding rheological and relaxational hereditary elements the integro-differential equations second and/or differential equations of the third order are composed. On the basis of the analysis of the discrete hereditary oscillatory systems the Goroshko's definition on dynamically determinated or indeterminated discrete hereditary systems was confirmed.

Key words: *Discrete hereditary system, standard hereditary element, oscillatory hereditary systems, rheological elements, rheonomic coordinate, rheonomic coordinate method, rheological pendulum, rheological and relaxational kernels, covariant coordinate.*

1. INTRODUCTION

The paper of academician Goroshko (see Ref. [13] and [14]) was an inspiration for research in the area of hereditary discrete systems, as well as a mutual work on the monograph: *Analytical Dynamics of the Discrete Hereditary Systems* which is to be published both in Serbian and English. Some examples were considered in the following papers: [27], [28], [32], [33] and [34].

For active constructions we can use various types of control and regulation of dynamical system parameters. In the modeling of an active construction, different kinds of active elements can be used. Some of these elements are active hereditary elements with different kinds of viscoelasticity or hereditary elasticity, with different time relaxation, as well as time retardation (see Ref. [23], [24], [25] and [18]. See also [29], and [30].). Active properties of construction can arise by active force or external excitations, active temperature fields, active electrical or optical fields, or by changeable distances between bearings, as a rheonomic coordinate, as well as by changeable rigidity. Active construction can be realized by a subsystem as an active element with external excitation.

As in the active constructions is not possible control without sensors as well sensors work by modulations of amplitude, phase as well as by modulations of frequency it is necessary to introduce the sensors parameters as an active excitation into active elements as well into active construction. Par example, optical sensors work by modulations of amplitude, of phase as well as by modulations of frequency of light waves, which arises with changeable optical parameters of material in the changes of stress and strain state in the material of construction during the way of the light waves.

Active elements are elements by the use of which we can observe and control stress and strain states in construction, as well as a temperature field state, by the use of sensor observed active parameters of the dynamical state of construction.

Active elements can be designed by the use of properties of dynamical adaptations, as an electromechanical, termomechanical or mechanical. In a mechanical way the construction rigidity of the defined sections can be made changeable.

In this paper we would like to investigate the equations of dynamics of active discrete hereditary elements as well as systems by introducing rheonomic coordinate in the standard hereditary element.

We will consider [10, [11], [12], [16], [17], [4] and [15] as well as [20], [21], [22] and [31] to be our basic literature.

2. EQUATIONS OF DYNAMICS OF A DISCRETE SYSTEM WITH FINITE CONSTRAINTS AND STANDARD HEREDITARY ELEMENTS

We will investigate a dynamical system (see Figure No. 1) of N material particles with masses m_ν, $\nu = 1,2,3,...,N$, the vector positions of which are $\vec{r}_\nu = y_\nu^i \vec{e}_i, i = 1,2,3;$ $\nu = 1,2,3,...,N$. Material particles are constrained by S finite constraints (see [1], [2], [3], [26], [16], [17], [11], [10] and [12]):

$$\bar{f}_\mu(\vec{r}_1, \vec{r}_2, ..., \vec{r}_N) = f_\mu(y^1, y^2, ..., y^{3N}) \quad \mu = 1,2,3,...,S \tag{1}$$

and where we introduce the following notations: $y_\nu^k =: y^{3\nu-(3-k)}$, $k = 1,2,3;$ $m_{3\nu-k} = m_{3\nu}$, $k = 1,2,3$, $\nu = 1,2,3,...,N$; as well as by K standard hereditary elements neglected mass and material properties parameters of which are: $n_{(\nu,\nu+1)k}$, $k = 1,2,3,...,K$, are times of relaxation, and $c_{(\nu,\nu+1)k}$ and $\widetilde{c}_{(\nu,\nu+1)k}$. are an instantaneous rigid stifness modulus as prolonged ones.

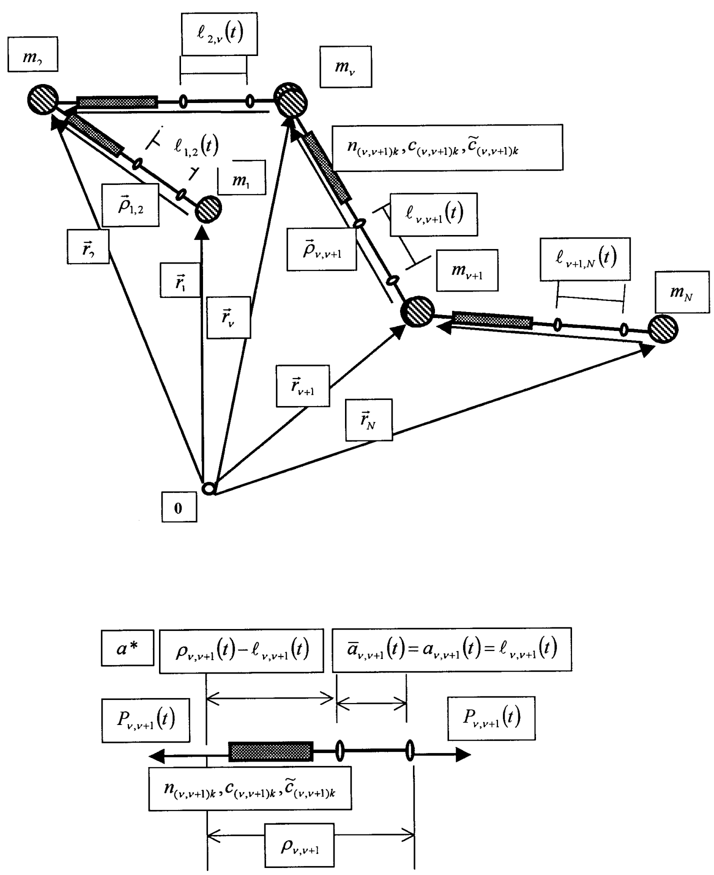

Fig. 1. Model of discrete hereditary system with rheonomic constraints and with N material particles
a^* Hereditary and rheonomic elements in series

Relations between reactions and deformations of the hereditary element in the discrete system can be defined in one of the following ways:

* in the relaxational forms by using integral stress strain state relations:

$$P_{(\nu,\nu+1)k} = c_{(\nu,\nu+1)k}[\overline{\rho}_{(\nu,\nu+1)k}(t) - \int_0^t \mathbf{R}_{(\nu,\nu+1)k}(t-\tau)\overline{\rho}_{(\nu,\nu+1)k}(\tau)d\tau], \tag{2}$$
$$\nu = 1,2,3,...,N, \quad k = 1,2,3,....,K_\nu$$

where

$$\mathbf{R}_{(\nu,\nu+1)k}(t-\tau) = \frac{c_{(\nu,\nu+1)k} - \widetilde{c}_{(\nu,\nu+1)k}}{n_{(\nu,\nu+1)k}c_{(\nu,\nu+1)k}} e^{-\frac{t-\tau}{n_{(\nu,\nu+1)k}}}, \ \nu = 1,2,3,...,N, \ k = 1,2,3,....,K_\nu \tag{3}$$

are kernels of relaxation (see Ref. [15] and [13], [14]), and

$$\rho_{(\nu,\nu+1)k} = |\vec{\rho}_{(\nu,\nu+1)k}| = |\vec{r}_{(\nu+1)k} - \vec{r}_{(\nu)k}| \quad \text{and}$$

$$\overline{\rho}_{(\nu,\nu+1)k} = |\vec{\rho}_{(\nu,\nu+1)k}| - \rho_{(\nu,\nu+1)k0} = |\vec{r}_{(\nu+1)k} - \vec{r}_{(\nu)k}| - \rho_{(\nu,\nu+1)k0}. \tag{a}$$

and $\rho_{(\nu,\nu+1)k0}$ is the natural length of a hereditary element in natural stress-strain state, when the strain and stress in the element are equal to zero.

* in the retardation forms by using integral stress strain state relations:

$$\overline{\rho}_{(\nu,\nu+1)k} = \frac{1}{c_{(\nu,\nu+1)k}}[P_{(\nu,\nu+1)k}(t) + \int_0^t \mathbf{K}_{(\nu,\nu+1)k}(t-\tau)P_{(\nu,\nu+1)k}(\tau)d\tau], \tag{4}$$
$$\nu = 1,2,3,...,N, \quad k = 1,2,3,....,K_\nu$$

where

$$\mathbf{K}_{(\nu,\nu+1)k}(t-\tau) = \frac{c_{(\nu,\nu+1)k} - \widetilde{c}_{(\nu,\nu+1)k}}{n_{(\nu,\nu+1)k}c_{(\nu,\nu+1)k}} e^{-\frac{(t-\tau)\widetilde{c}_{(\nu,\nu+1)k}}{n_{(\nu,\nu+1)k}c_{(\nu,\nu+1)k}}}, \tag{5}$$
$$\nu = 1,2,3,...,N, \quad k = 1,2,3,....,K_\nu$$

is a kernel of rheology, and $\rho_{(\nu,\nu+1)k} = |\vec{\rho}_{(\nu,\nu+1)k}| = |\vec{r}_{(\nu+1)k} - \vec{r}_{(\nu)k}|$.

* in differential form:

$$n_{(\nu,\nu+1)k}\dot{P}_{(\nu,\nu+1)k}(t) + P_{(\nu,\nu+1)k}(t) = n_{(\nu,\nu+1)k}c_{(\nu,\nu+1)k}\dot{\overline{\rho}}_{(\nu,\nu+1)k} + \widetilde{c}_{(\nu,\nu+1)k}\overline{\rho}_{(\nu,\nu+1)k} \tag{6}$$
$$\nu = 1,2,3,...,N, \quad k = 1,2,3,....,K_\nu$$

Finite constraints (1) must satisfy the following velocity condition:

$$\dot{f}_\mu = \sum_{\alpha=1}^{\alpha=3N} \frac{\partial f_\mu}{\partial y^\alpha}\dot{y}^\alpha = 0, \quad \alpha = 1,2,3,....,3N, \quad \mu = 1,2,3,....,S \tag{7}$$

as well as the acceleration conditions:

$$\ddot{f}_\mu = \sum_{\alpha=1}^{\alpha=3N} \frac{\partial f_\mu}{\partial y^\alpha}\ddot{y}^\alpha + \sum_{\alpha=1}^{\alpha=3N}\sum_{\beta=1}^{\beta=3N} \frac{\partial^2 f_\mu}{\partial y^\alpha \partial y^\beta}\dot{y}^\alpha\dot{y}^\beta = 0 \tag{8}$$

As this finite constraints are independent the differential determinate of matrix is different from zero:

$$\Delta = |\Delta_{\mu\alpha}| = \left|\frac{\partial f_\mu}{\partial y^\alpha}\right| \neq 0\,,\ \alpha = 1,2,3,....,3N\,,\ \mu = 1,2,3,....,S \tag{9}$$

By using previous velocity conditions we can write ortogonality conditions $(grad_\nu f_\mu, \vec{v}^\nu) = 0$, $\nu = 1,2,3,...,N$, $\mu = 1,2,3,....,S$ between mass particles and gradients of the finite constraints, for ideal constraints reactions we can write the following:

$$\vec{\mathbf{R}}_\nu = \sum_{\mu=1}^{\mu=S} \lambda_\mu grad_\nu f_\mu(\vec{r}_1,...,\vec{r}_N)\,,\ \nu = 1,2,3,...,N \tag{10}$$

in which the λ_μ are Lagrange's multiplikators of the finite constraints, as well as

$$grad_\nu f_\mu = \vec{i}\,\frac{\partial f_\mu}{\partial y_\nu^{3\nu-3}} + \vec{j}\,\frac{\partial f_\mu}{\partial y_\nu^{3\nu-2}} + \vec{k}\,\frac{\partial f_\mu}{\partial y_\nu^{3\nu-1}} \tag{10*}$$

The resulting reactions of the K standard hereditary elements into ν-rd ($\nu+1$ with opposite direction) mass material particle are:

$$P_{\nu,\nu+1}(t) = \sum_{k=1}^{k=K} P_{(\nu,\nu+1)k}(t)\frac{\vec{\rho}_{(\nu,\nu+1)k}}{|\vec{\rho}_{(\nu,\nu+1)k}|} = \sum_{k=1}^{k=K} P_{(\nu,\nu+1)k}(t)\frac{\vec{r}_{(\nu+1)k} - \vec{r}_{(\nu)k}}{|\vec{r}_{(\nu+1)k} - \vec{r}_{(\nu)k}|} \tag{11}$$

Resulting reaction forces of finite constraints and hereditary elements in the observed system are:

$$\vec{\mathbf{R}}_\nu = \sum_{\mu=1}^{\mu=S} \lambda_\mu grad_\nu f_\mu(\vec{r}_1,...,\vec{r}_N) + P_{\nu,\nu+1}(t) + \vec{\mathbf{R}}_{\nu T} \tag{12*}$$

$$\vec{\mathbf{R}}_\nu = \sum_{\mu=1}^{\mu=S} \lambda_\mu grad_\nu f_\mu(\vec{r}_1,\vec{r}_2,...,\vec{r}_N) + \sum_{k=1}^{k=K_\nu} P_{(\nu,\nu+1)k}(t)\frac{\vec{r}_{(\nu+1)k} - \vec{r}_{(\nu)k}}{|\vec{r}_{(\nu+1)k} - \vec{r}_{(\nu)k}|} + \vec{\mathbf{R}}_{\nu\mathbf{T}} \tag{12}$$

From a principle of the work on the virtual system displacements the initial equation can be written in the following form:

$$\sum_{\nu=1}^{\nu=N} \{\vec{\mathrm{I}}_\nu + \vec{\mathrm{F}}_\nu + \vec{\mathrm{R}}_\nu + \vec{\mathrm{P}}_\nu + \vec{\mathrm{R}}_{\nu\mathrm{T}}\}\delta\vec{r}_\nu = 0 \tag{13*}$$

or:

$$\sum_{\nu=1}^{\nu=N}\left\{m_\nu\ddot{\vec{r}}_\nu - \vec{\mathrm{F}}_\nu(t) - \sum_{\mu=1}^{\mu=S}\lambda_\mu grad_\nu f_\mu(\vec{r}_1,...,\vec{r}_N) - \sum_{k=1}^{k=K_\nu} P_{(\nu,\nu+1)k}(t)\frac{\vec{r}_{(\nu+1)k} - \vec{r}_{(\nu)k}}{|\vec{r}_{(\nu+1)k} - \vec{r}_{(\nu)k}|} - \vec{\mathrm{R}}_{\nu\mathrm{T}}\right\}\delta\vec{r}_\nu = 0 \tag{13}$$

Dynamical Lagrange's equations of the first kind arise from previous equation in the following form

$$m_\nu\ddot{\vec{r}}_\nu = \vec{\mathrm{F}}_\nu(t) + \sum_{\mu=1}^{\mu=S}\lambda_\mu grad_\nu f_\mu(\vec{r}_1,...,\vec{r}_N) + \sum_{k=1}^{k=K_\nu} P_{(\nu,\nu+1)k}(t)\frac{\vec{r}_{(\nu+1)k} - \vec{r}_{(\nu)k}}{|\vec{r}_{(\nu+1)k} - \vec{r}_{(\nu)k}|} + \vec{\mathrm{R}}_{\nu\mathrm{T}} \tag{14}$$

$\nu = 1,2,3,...,N$

Let us define some relations by which the investigation and description of the dynamics of this problem is simplified. For this reason the equations (14) are rewritten for ν and $\nu+1$ in the forms:

$$\ddot{\vec{r}}_{\nu}=\frac{1}{m_{\nu}}\left\{\vec{\mathrm{F}}_{\nu}(t)+\sum_{\mu=1}^{\mu=S}\lambda_{\mu}grad_{\nu}f_{\mu}(\vec{r}_1,...,\vec{r}_N)+\sum_{k=1}^{k=K_{\nu}}P_{(\nu,\nu+1)k}(t)\frac{\vec{r}_{(\nu+1)k}-\vec{r}_{(\nu)k}}{|\vec{r}_{(\nu+1)k}-\vec{r}_{(\nu)k}|}+\vec{\mathrm{R}}_{\nu\mathrm{T}}\right\}$$

$$\ddot{\vec{r}}_{\nu+1}=\frac{1}{m_{\nu+1}}\left\{\vec{\mathrm{F}}_{\nu+1}(t)+\sum_{\mu=1}^{\mu=S}\lambda_{\mu}grad_{\nu+1}f_{\mu}(\vec{r}_1,...,\vec{r}_N)+\sum_{k=1}^{k=K_{\nu}}P_{(\nu+1,\nu+2)k}(t)\frac{\vec{r}_{(\nu+2)k}-\vec{r}_{(\nu+1)k}}{|\vec{r}_{(\nu+2)k}-\vec{r}_{(\nu+1)k}|}+\vec{\mathrm{R}}_{\nu+1\mathrm{T}}\right\} \quad (15)$$

$\nu=1,2,3,...,N$

By subtraction of the previous equations (15) result becomes:

$$\begin{aligned}\ddot{\vec{r}}_{\nu+1}-\ddot{\vec{r}}_{\nu}=&\frac{1}{m_{\nu+1}}\vec{\mathrm{F}}_{\nu+1}(t)-\frac{1}{m_{\nu}}\vec{\mathrm{F}}_{\nu}(t)+\frac{1}{m_{\nu+1}}\sum_{\mu=1}^{\mu=S}\lambda_{\mu}grad_{\nu+1}f_{\mu}(\vec{r}_1,...,\vec{r}_N)-\\&-\frac{1}{m_{\nu}}\sum_{\mu=1}^{\mu=S}\lambda_{\mu}grad_{\nu}f_{\mu}(\vec{r}_1,...,\vec{r}_N)+\frac{1}{m_{\nu+1}}\sum_{k=1}^{k=K_{\nu}}P_{(\nu+1,\nu+2)k}(t)\frac{\vec{r}_{\nu+2}-\vec{r}_{\nu+1}}{|\vec{r}_{\nu+2}-\vec{r}_{\nu+1}|}+\\&+\frac{1}{m_{\nu}}\sum_{k=1}^{k=K_{\nu}}P_{(\nu,\nu+1)k}(t)\frac{\vec{r}_{\nu+1}-\vec{r}_{\nu}}{|\vec{r}_{\nu+1}-\vec{r}_{\nu}|}+\frac{1}{m_{\nu+1}}\vec{\mathrm{R}}_{\nu+1\mathrm{T}}-\frac{1}{m_{\nu}}\vec{\mathrm{R}}_{\nu\mathrm{T}}\\&\nu=1,2,3,...,N\end{aligned} \quad (16)$$

The last relation may be written as:

$$\begin{aligned}\ddot{\vec{r}}_{\nu+1}-\ddot{\vec{r}}_{\nu}=&\frac{1}{m_{\nu+1}}\vec{\mathrm{F}}_{\nu+1}(t)-\frac{1}{m_{\nu}}\vec{\mathrm{F}}_{\nu}(t)+\sum_{\mu=1}^{\mu=S}\lambda_{\mu}\left[\frac{1}{m_{\nu+1}}grad_{\nu+1}f_{\mu}(\vec{r}_1,...,\vec{r}_N)-\frac{1}{m_{\nu}}grad_{\nu}f_{\mu}(\vec{r}_1,...,\vec{r}_N)\right]+\\&+\sum_{k=1}^{k=K_{\nu}}\left\{\frac{1}{m_{\nu+1}}P_{(\nu+1,\nu+2)k}(t)\frac{\vec{\rho}_{(\nu+1,\nu+2)k}}{\rho_{(\nu+1,\nu+2)k}}-\frac{1}{m_{\nu}}P_{(\nu,\nu+1)k}(t)\frac{\vec{\rho}_{(\nu,\nu+1)k}}{\rho_{(\nu,\nu+1)k}}\right\}+\frac{1}{m_{\nu+1}}\vec{\mathrm{R}}_{\nu+1\mathrm{T}}-\frac{1}{m_{\nu}}\vec{\mathrm{R}}_{\nu\mathrm{T}}\end{aligned} \quad (16)$$

$\nu=1,2,3,...,N$

By using the fact that distance, between any two material particles, from the system, is changeable, a constraint is expressed by equation of the form $\vec{\rho}^2_{\nu,\nu+1}=(\vec{r}_{\nu+1}-\vec{r}_{\nu})^2$. By two time derivatives of this constraint relation and knowing that $\vec{v}_{rel(\nu+1,\nu)}=\vec{v}_{\nu+1,\nu}=\dot{\vec{r}}_{\nu+1}-\dot{\vec{r}}_{\nu}$ is relative velocity of $\nu+1$ st material mass particle around ν-st material mass particle in the relative rotation, we can write the following relation (see Ref. [8]):

$$(\vec{r}_{\nu+1}-\vec{r}_{\nu},\ddot{\vec{r}}_{\nu+1}-\ddot{\vec{r}}_{\nu})=\dot{\rho}^2_{\nu+1,\nu}+\rho_{\nu+1,\nu}\ddot{\rho}_{\nu+1,\nu}-v^2_{\nu+1,\nu}, \quad \nu=1,2,3,...,N \quad (17)$$

Having in mind the previous relations (16*) the previous relation (17) becomes:

$$\begin{aligned}&\sum_{k=1}^{k=K_{\nu}}\left\{\frac{1}{m_{\nu+1}}P_{(\nu+1,\nu+2)k}(t)\frac{(\vec{\rho}_{(\nu+1,\nu+2)k},\vec{\rho}_{(\nu,\nu+1)k})}{\rho_{(\nu+1,\nu+2)k}}-\frac{1}{m_{\nu}}P_{(\nu,\nu+1)k}(t)\rho_{(\nu,\nu+1)k}\right\}=\\&=\dot{\rho}^2_{\nu+1,\nu}+\rho_{\nu+1,\nu}\ddot{\rho}_{\nu+1,\nu}-v^2_{\nu+1,\nu}-\\&-\left[\frac{1}{m_{\nu+1}}(\vec{\mathrm{F}}_{\nu+1}(t),\vec{\rho}_{\nu,\nu+1})-\frac{1}{m_{\nu}}(\vec{\mathrm{F}}_{\nu}(t),\vec{\rho}_{\nu,\nu+1})\right]-\left[\frac{1}{m_{\nu+1}}(\vec{\mathrm{R}}_{\nu+1\mathrm{T}},\vec{\rho}_{\nu,\nu+1})-\frac{1}{m_{\nu}}(\vec{\mathrm{R}}_{\nu\mathrm{T}},\vec{\rho}_{\nu,\nu+1})\right]-\\&-\sum_{\mu=1}^{\mu=S}\lambda_{\mu}\left[\frac{1}{m_{\nu+1}}(grad_{\nu+1}f_{\mu}(\vec{r}_1,...,\vec{r}_N),\vec{\rho}_{\nu,\nu+1})-\frac{1}{m_{\nu}}(grad_{\nu}f_{\mu}(\vec{r}_1,...,\vec{r}_N),\vec{\rho}_{\nu,\nu+1})\right]\end{aligned} \quad (18)$$

$\nu=1,2,3,...,N$

Into previous equation (18), the expression of the reaction-force of the standard hereditary element by using relations (2) of the stress-strain state or coordinates of deformation, or by using relations (4) should be introduced, or the corresponding equations (6) should join these equations (18). The resulting system of equations can be solved as an explicit independent system of equations in relation to the coordinates of vector positions of mass particles. This is because that we separate only equations with "internal" system coordinate by the use of which the internal relative positions between material mass particles are defined. We must have in mind that stress-strain relations use coordinates of hereditary element deformations $\bar{\rho}_{\nu,\nu+1} = \rho_{\nu,\nu+1} - \rho_{(\nu,\nu+1)0}$ which differ between relative coordinate $\rho_{\nu,\nu+1}$ - distance between two material particles and their distance $\rho_{(\nu,\nu+1)0}$, in hereditary element natural state if it was in such state at the initial moment.

We accept that, at initial moment, the hereditary element was in a natural state without deformation, without stress as well as without strain. If we suppose that hereditary elements have history than into stress-strain relations boundaries of integral are different: boundaries: from zero to t changes into boundaries: from $-\infty$ to t.

$$P_{(\nu,\nu+1)k} = c_{(\nu,\nu+1)k}\{[\rho_{(\nu,\nu+1)k}(t) - \rho_{(\nu,\nu+1)k0}] - \int_0^t \mathbf{R}_{(\nu,\nu+1)k}(t-\tau)[\rho_{(\nu,\nu+1)k}(\tau) - \rho_{(\nu,\nu+1)k0}]d\tau\} \tag{19}$$
$$\nu = 1,2,3,...,N\,, \quad k = 1,2,3,....,K_\nu$$

When we observe a system in space, then it is useful, that square $v^2_{\nu+1,\nu}$ of relative velocity $\vec{v}_{rel(\nu+1,\nu)} = \vec{v}_{\nu+1,\nu} = \dot{\vec{r}}_{\nu+1} - \dot{\vec{r}}_\nu$ of relative motion $\nu + 1$ st material particle around ν-th material particle be expressed by sphere coordinate in relation to the relative pole. Then the radius $\rho_{\nu,\nu+1}$, and circular and meridional angles: $\varphi_{\nu+1,\nu}$ and $\psi_{\nu+1,\nu}$ are used. For the following coordinates of the material mass particle in relation to the previous mass particle coordinate we can write:

$$\begin{aligned}
&y^k_\nu =: y^{3\nu-(3-k)}, \quad k = 1,2,3; \quad m_{3\nu-k} = m_{3\nu}, \quad k = 1,2,3, \quad \nu = 1,2,3,...,N \\
&y^1_{\nu+1} = y^1_\nu + \rho_{\nu,\nu+1}\cos\psi_{\nu+1,\nu}\cos\varphi_{\nu+1,\nu} \\
&y^2_{\nu+1} = y^2_\nu + \rho_{\nu,\nu+1}\cos\psi_{\nu+1,\nu}\sin\varphi_{\nu+1,\nu} \\
&y^3_{\nu+1} = y^3_\nu + \rho_{\nu,\nu+1}\sin\psi_{\nu+1,\nu}
\end{aligned} \tag{20}$$

If we observe νth set of coordinates of the νth mass particle in relation to the set coordinates of first mass particle we can write:

$$\begin{aligned}
y^1_{\nu+1} &= y^1_1 + \sum_{k=1}^{k=\nu}\rho_{k,k+1}\cos\psi_{k+1,k}\cos\varphi_{k+1,k} \\
y^2_{\nu+1} &= y^2_1 + \sum_{k=1}^{k=\nu}\rho_{k,k+1}\cos\psi_{k+1,k}\sin\varphi_{k+1,k} \\
y^3_{\nu+1} &= y^3_1 + \sum_{k=1}^{k=\nu}\rho_{k,k+1}\sin\psi_{k+1,k}\,.
\end{aligned} \tag{21}$$

$$\ddot{\vec{r}}_{\nu+1} = \ddot{\vec{r}}_{\nu} + \frac{d^2}{dt^2}\{\rho_{\nu,\nu+1}[\vec{i}\cos\psi_{\nu+1,\nu}\cos\varphi_{\nu+1,\nu} + \vec{j}\cos\psi_{\nu+1,\nu}\sin\varphi_{\nu+1,\nu} + \vec{k}\sin\psi_{\nu+1,\nu}]\} \quad (22)$$

Now in the pair from the system (15) and by using system (21) we can write:

$$\ddot{\vec{r}}_{\nu} = \frac{1}{m_{\nu}}\left\{\vec{\mathrm{F}}_{\nu}(t) + \sum_{\mu=1}^{\mu=S}\lambda_{\mu}\, grad_{\nu} f_{\mu}(\vec{r}_1,...,\vec{r}_N) + \sum_{k=1}^{k=K_{\nu}} P_{(\nu,\nu+1)k}(t)\frac{\vec{\rho}_{(\nu,\nu+1)k}}{\rho_{(\nu,\nu+1)k}} + \vec{\mathrm{R}}_{\nu \mathrm{T}}\right\}$$

$$\ddot{\vec{r}}_{\nu+1} = \frac{1}{m_{\nu+1}}\left\{\vec{\mathrm{F}}_{\nu+1}(t) + \sum_{\mu=1}^{\mu=S}\lambda_{\mu}\, grad_{\nu+1} f_{\mu}(\vec{r}_1,...,\vec{r}_N) + \sum_{k=1}^{k=K_{\nu}} P_{(\nu+1,\nu+2)k}(t)\frac{\vec{\rho}_{(\nu+1,\nu+2)k}}{\rho_{(\nu+1,\nu+2)k}} + \vec{\mathrm{R}}_{\nu+1 \mathrm{T}}\right\} \quad (15^*)$$

$$\nu = 1,2,3,...,N$$

We can see that a system can be obtained from which the Descartes coordinate $y_{\nu}^{k} =: y^{3\nu-(3-k)},\ k = 1,2,3;$ by the use of which absolute positions of mass particles of the system are defined, are eliminated. These equations contain a system of generalized coordinates without absolute coordinates of the first mass particle. These chosen coordinates are internal coordinates of the system by the use of which the internal relative positions between mass particles of the system are determined.

For example, by using (16) or (16*) and the last relation (22) we can obtain:

$$\ddot{\vec{r}}_{\nu+1} - \ddot{\vec{r}}_{\nu} = \frac{d^2}{dt^2}\{\rho_{\nu,\nu+1}\vec{i}[\vec{i}\cos\psi_{\nu+1,\nu}\cos\varphi_{\nu+1,\nu} + \vec{j}\cos\psi_{\nu+1,\nu}\sin\varphi_{\nu+1,\nu} + \vec{k}\sin\psi_{\nu+1,\nu}]\} =$$

$$= \frac{1}{m_{\nu+1}}\vec{\mathrm{F}}_{\nu+1}(t) - \frac{1}{m_{\nu}}\vec{\mathrm{F}}_{\nu}(t) + \sum_{\mu=1}^{\mu=S}\lambda_{\mu}\left[\frac{1}{m_{\nu+1}} grad_{\nu+1} f_{\mu}(\vec{r}_1,...,\vec{r}_N) - \frac{1}{m_{\nu}} grad_{\nu} f_{\mu}(\vec{r}_1,...,\vec{r}_N)\right] + \quad (16^{**})$$

$$+ \sum_{k=1}^{k=K_{\nu}}\left\{\frac{1}{m_{\nu+1}} P_{(\nu+1,\nu+2)k}(t)\frac{\vec{\rho}_{(\nu+1,\nu+2)k}}{\rho_{(\nu+1,\nu+2)k}} - \frac{1}{m_{\nu}} P_{(\nu,\nu+1)k}(t)\frac{\vec{\rho}_{(\nu,\nu+1)k}}{\rho_{(\nu,\nu+1)k}}\right\} + \frac{1}{m_{\nu+1}}\vec{\mathrm{R}}_{\nu+1\mathrm{T}} - \frac{1}{m_{\nu}}\vec{\mathrm{R}}_{\nu\mathrm{T}}$$

$$v = 1,2,3,...,N$$

in which only internal coordinates of the system are contained: mutual distances of mass particles $\rho_{v,v+1}$, and circular and meridional angles: $\varphi_{v+1,v}$ and $\psi_{v+1,v}$, relative rotation motion $v+1$ st material particle around v-th mass particle. Square of the relative velocity of the relative rotation motion $v+1$ st material particle around v-th mass particle is:

$$v_{\nu+1,\nu}^2 = \dot{\rho}_{\nu,\nu+1}^2 + [\rho_{\nu,\nu+1}\dot{\varphi}_{\nu+1,\nu}\cos\psi_{\nu+1,\nu}]^2 + \rho_{\nu,\nu+1}^2\dot{\psi}_{\nu,\nu+1}^2$$
$$\nu = 1,2,3,...,N \quad (23)$$

In the case when we have plane discrete system motion the previous relation becomes:

$$y_{\nu+1}^1 = y_1^1 + \sum_{k=1}^{k=\nu}\rho_{k,k+1}\cos\varphi_{k+1,k}$$
$$y_{\nu+1}^2 = y_1^2 + \sum_{k=1}^{k=\nu}\rho_{k,k+1}\sin\varphi_{k+1,k} \quad (24)$$

$$\ddot{\vec{r}}_{\nu+1} - \ddot{\vec{r}}_{\nu} = \frac{d^2}{dt^2}\{\rho_{\nu,\nu+1}[\vec{i}\cos\varphi_{\nu+1,\nu} + \vec{j}\sin\varphi_{\nu+1,\nu}]\}$$
$$\nu = 1,2,3,...,N \quad (25)$$

In a similar way as in the case of the space system motion the relation became:

$$\ddot{\vec{r}}_{\nu+1} - \ddot{\vec{r}}_{\nu} = \frac{d^2}{dt^2}\{\rho_{\nu,\nu+1}[\vec{i}\cos\varphi_{\nu+1,\nu} + \vec{j}\sin\varphi_{\nu+1,\nu}]\} = \frac{1}{m_{\nu+1}}\vec{\mathrm{F}}_{\nu+1}(t) - \frac{1}{m_\nu}\vec{\mathrm{F}}_\nu(t)$$
$$+\sum_{\mu=1}^{\mu=S}\lambda_\mu\left[\frac{1}{m_{\nu+1}} grad_{\nu+1} f_\mu(\vec{r}_1,...,\vec{r}_N) - \frac{1}{m_\nu} grad_\nu f_\mu(\vec{r}_1,...,\vec{r}_N)\right]+ \tag{26}$$
$$+\sum_{k=1}^{k=K_\nu}\left\{\frac{1}{m_{\nu+1}}P_{(\nu+1,\nu+2)k}(t)\frac{\vec{\rho}_{(\nu+1,\nu+2)k}}{\rho_{(\nu+1,\nu+2)k}} - \frac{1}{m_\nu}P_{(\nu,\nu+1)k}(t)\right\} + \frac{1}{m_{\nu+1}}\vec{\mathrm{R}}_{\nu+1\mathrm{T}} - \frac{1}{m_\nu}\vec{\mathrm{R}}_{\nu\mathrm{T}}$$
$$\nu = 1,2,3,...,N$$

For that case we suppose that all active forces are in the same motion plane of the discrete mass particle system.

The previous relations, now, are an explicite function only of the internal coordinates: mutual distance $\rho_{\nu,\nu+1}$, between two mass particles and circular angles $\varphi_{\nu+1,\nu}$ of the relative rotation motion $\nu+1$ st material particle around ν-th mass particle in the motion plane. Square of the relative velocity of the relative rotation motion $\nu+1$ st material particle around ν-th mass particle is:

$$v_{\nu+1,\nu}^2 = \dot{\rho}_{\nu,\nu+1}^2 + [\rho_{\nu,\nu+1}\dot{\varphi}_{\nu+1,\nu}]^2, \quad \nu = 1,2,3,...,N \tag{27}$$

The system of N relations (26) is expressed in vectorial forms, and obtains the form of $2N$ scalar relation in which appear $S \le N$ reactions $P_{\nu,\nu+1}(t)$ of the stressed hereditary elements and $(N-2)$ internal system coordinattes: mutual distance between mass particles $\rho_{\nu,\nu+1}$, and circular angles $\varphi_{\nu+1,\nu}$ of the relative rotation motion $\nu+1$ st material particle around ν-th mass particle in the motion plane. The square of the relative velocity of the relative rotation motion $\nu+1$ st material particle around ν-th mass particle in the motion plane is the same as (27).

If we have a dynamicaly determined system it is possible to eliminate reactions $P_{\nu,\nu+1}(t)$ of the stressed hereditary elements and obtain independent system of the internal system coordinates. By these equations we obtain functional relations between internal coordinates: angle rotation motion $\varphi_{\nu+1,\nu}$ and distance $\rho_{\nu,\nu+1}$, between mass particles.

EXAMPLE 1. For the system of two mass particles (see Figure No.2) constrained by one hereditary element in the space from system of the relations (16**) by elimitation reaction of the stressed hereditary element we can obtain the following relations between internal system coordinate ρ, φ and ψ in the following forms:

$$\frac{1}{\cos\psi\sin\varphi}\left\{\frac{d^2}{dt^2}\{\rho\cos\psi\sin\varphi\} + \frac{1}{m_1}F_{01y}\cos\Omega t\right\} = \frac{1}{\cos\psi\cos\varphi}\left\{\frac{d^2}{dt^2}\{\rho\cos\psi\cos\varphi\} + \frac{1}{m_1}F_{01x}\cos\Omega t\right\} \tag{b}$$
$$\frac{1}{\cos\psi\sin\varphi}\left\{\frac{d^2}{dt^2}\{\rho\cos\psi\sin\varphi\} + \frac{1}{m_1}F_{01y}\cos\Omega t\right\} = \frac{1}{\sin\psi}\left\{\frac{d^2}{dt^2}\{\rho\sin\psi\} + \frac{1}{m_1}F_{01z}\cos\Omega t\right\}$$

from which we obtain:

$$[\rho(t)\ddot{\varphi}(t) + 2\dot{\rho}(t)\dot{\varphi}(t)]\cos\psi - 2\rho(t)\dot{\varphi}(t)\dot{\psi}(t)\sin\psi = \frac{1}{m_1}[F_{01x}\sin\varphi - F_{01y}\cos\varphi]\cos\Omega t \tag{c}$$
$$[\rho(t)\ddot{\psi}(t) + 2\dot{\rho}(t)\dot{\psi}(t)] + \rho(t)[\dot{\varphi}(t)]^2\cos\psi\sin\psi = \frac{1}{m_1\sin\varphi}[F_{01z}\cos\psi\sin\varphi - F_{01y}\sin\psi]$$

The last system equations yield relation between angles velocities components $\dot{\psi}$ meridional and $\dot{\varphi}$ circular of the relative rotation motion of the second material particle around the first mass particle in the space of the dynamic of this discrete mass particles system. We can see that these angle velocity components of the relative rotation motion of the second material particle around first mass particle in the space are coupled as functions of distance ρ between these mass particles.

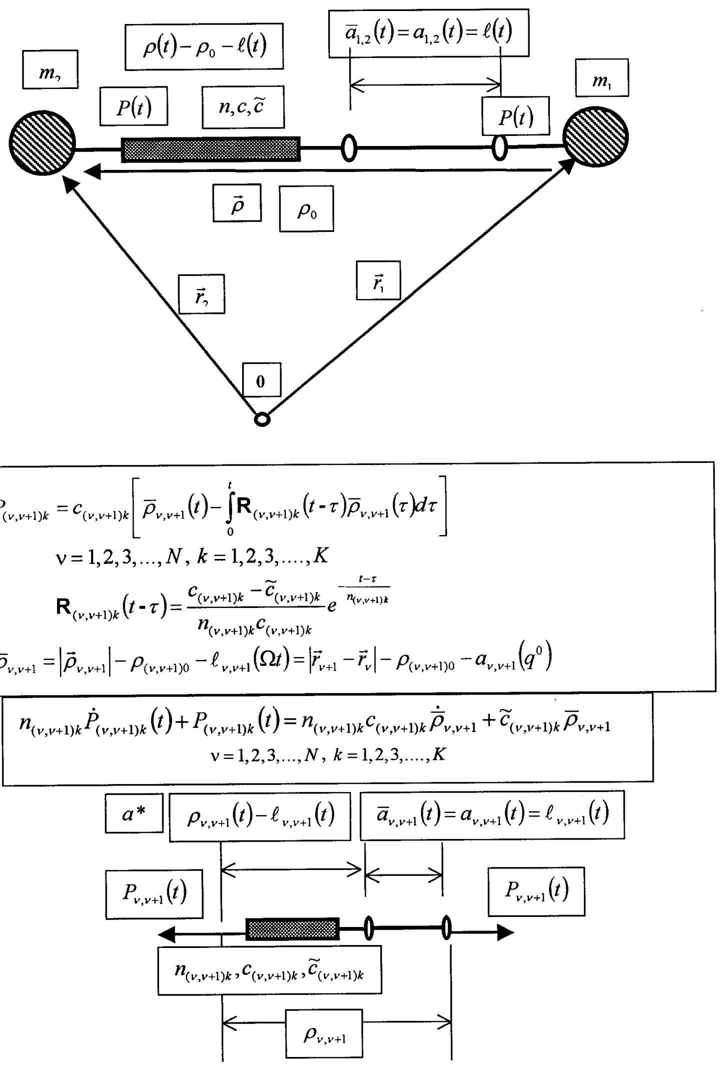

Fig. 2. Model of a discrete hereditary system with rheonomic constraint and with two materials particles
a^* Hereditary and rheonomic elements in series

Square of relative motion velocity of the relative rotation motion second material particle around first mass particle in the space is:

$$v_r^2=\dot{\rho}^2+[\rho\dot{\varphi}\cos\psi]^2+\rho^2\dot{\psi}^2 \tag{e}$$

By introducing expression (e) of the square relative velocity motion into expression (18) for the rheological reaction $P_{1,2}(t) = P(t)$ as a force of the internal influence between mass particles, we can obtain the following expression:

$$P(t) = -\frac{m_1 m_2}{m_1+m_2}\left\{\frac{\dot{\vec{\rho}}^2 + (\vec{\rho},\ddot{\vec{\rho}}) - \vec{v}_r^2}{\rho} - \frac{1}{m_1\rho}(\vec{\rho},\vec{F}_1)\right\} = \qquad \text{(f)}$$
$$-\frac{m_1 m_2}{m_1+m_2}\left\{\ddot{\rho} - \rho\dot{\varphi}^2\cos^2\psi - \rho\dot{\psi}^2 - \frac{1}{m_1\rho}(\vec{\rho},\vec{F}_1)\right\}$$

By introducing the previous expression (f) of the mutual influence force into integral relation (2) rheological-hereditary relation for the case of the forced motion of the system in space rheological relation becomes the following integro-diferential form:

$$\rho(t) - \rho_0 - \int_0^t \mathrm{R}\,(t-\tau)[\rho(\tau)-\rho_0]d\tau + \frac{m_1 m_2}{c(m_1+m_2)}\{\ddot{\rho}(t) - \rho(t)\dot{\varphi}^2\cos^2\psi - \rho(t)\dot{\psi}^2\} = \qquad \text{(g)}$$
$$= \frac{m_2}{(m_1+m_2)}\frac{1}{c}[F_{01x}\cos\psi\cos\varphi + F_{01y}\cos\psi\sin\varphi + F_{01z}\sin\psi]\sin\Omega t$$

The system of coupled differential and integrodiferential equations can be transformed into the following system of equations of the first kind:

$$\frac{d\rho(t)}{dt} = u(t) \qquad \frac{d\varphi(t)}{dt} = v(t) \qquad \frac{d\psi(t)}{dt} = w(t)$$

$$\frac{du(t)}{dt} = \frac{c(m_1+m_2)}{m_1 m_2}\left\{-\rho(t) + \rho_0 + \int_0^t \mathrm{R}\,(t-\tau)[\rho(\tau)-\rho_0]d\tau\right\} + c\{\rho(t)[v(t)]^2\cos^2\psi(t) + \rho(t)[w(t)]^2\} +$$
$$+\frac{1}{m_1}[F_{01x}\cos\psi(t)\cos\varphi(t)F_{01y}\cos\psi(t)\sin\varphi(t) + F_{01z}\sin\psi(t)]\sin\Omega t$$

$$\frac{dv(t)}{dt} = \frac{1}{\rho(t)\cos\psi(t)}\{2\rho(t)v(t)w(t)\sin\psi(t) - 2u(t)v(t)\cos\psi(t) +$$
$$+\frac{1}{m_1}[F_{01x}\sin\varphi(t) - F_{01y}\cos\varphi(t)]\cos\Omega t\} \qquad \text{(l)}$$
$$\frac{dw(t)}{dt} = \frac{1}{\rho(t)}\{-2u(t)w(t) - \rho(t)[v(t)]^2\cos\psi(t)\sin\psi(t) +$$
$$+\frac{1}{m_1\sin\varphi(t)}[F_{01z}\cos\psi(t)\sin\varphi(t) - F_{01y}\sin\psi(t)]\cos\Omega t\}$$

EXAMPLE 2. For the system of a mass material particles couple by one hereditary element in a plane (see Figure No. 3), by using relation (16**) and by eliminating the reaction of the hereditary element we can obtain the following relation between internal system coordinates ρ and φ:

$$\cos\varphi\frac{d^2}{dt^2}\{\rho\sin\varphi\} = \sin\varphi\frac{d^2}{dt^2}\{\rho\cos\varphi\} - \frac{m_2}{m_1}F_{01}\sin\beta\cos\Omega_1 t \qquad \text{(a)}$$

$$[2\dot{\rho}\dot{\varphi} + \rho\ddot{\varphi}] = -\frac{m_2}{m_1}F_{01}\sin\beta\cos\Omega_1 t \qquad \text{(b)}$$

The solution of the previous equation (b), which has the form of differential equation of the first kind of $\dot{\varphi}(t)$ depending of the internal coordinate $\rho(t)$, takes the following form:

$$\dot{\varphi}(t) = \dot{\varphi}_0\frac{\rho_0^2}{[\rho(t)]^2} - \frac{m_2}{m_1}F_{01}\sin\beta\left[\frac{1}{\rho(t)}\int_0^t \rho(\tau)\cos\Omega_1\tau d\tau\right] \qquad \text{(c)}$$

The last differential equation (c) gives a relation between angular velocity $\dot{\varphi}(t)$ of the relative rotation second material particle around first of the system dynamics of these mass particles and internal coordinate $\rho(t)$-distance between mass particles, in the motion plane.

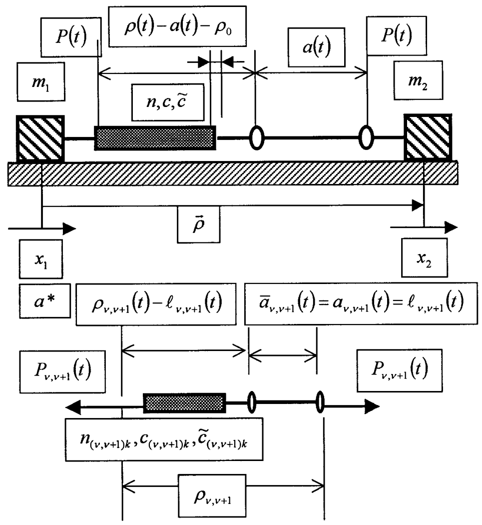

Fig. 3. Model of a discrete hereditary system with rheonomic constraint and with two materials particles
$a*$ Hereditary and rheonomic elements in series

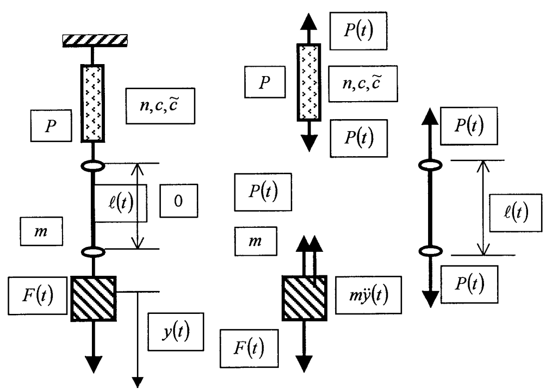

Fig. 4. Hereditary-rheonomic oscillator, excited by $F(t)$

We can see that the angular velocity $\dot{\varphi}(t)$ of relative rotation of the second material particle around first of the system dynamics is composed of two members: one is opposite proportional to the square of the distance ρ, and the second member is in the integral form and is dependent on the external excitation force. The first componet of velocity that corresponds to proper free motion, arises as the result of the initial conditions perturbation of the equilibrium position.

The square relative velocity of the relative rotation of the second mass particle around first is:

$$v_r^2 = \dot{\rho}^2 + \rho^2\omega^2 = [\dot{\rho}(t)]^2 + [\rho(t)]^2 \left\{ \dot{\varphi}_0 \frac{\rho_0^2}{[\rho(t)]^2} - \frac{m_2}{m_1} F_{01} \sin\beta \left[\frac{1}{\rho(t)} \int_0^t \rho(\tau) \cos\Omega_1 \tau d\tau \right] \right\}^2 \tag{d}$$

By introducing the expression (d) of the square relative velocity of the relative rotation of the second mass particle around first into expression (18) for the force $P_{1,2}(t) = \mathrm{P}(t)$ of the mutual influence between mass particles, and after this result has been introduced in relation (2) of the rheological-hereditary relation for the case of the forced system motion in the plane, and the rheological relation becomes:

$$\rho(t) - \rho_0 - \int_0^t \mathrm{R}\,(t-\tau)[\rho(\tau) - \rho_0]\,d\tau =$$
$$-\frac{m_1 m_2}{c(m_1+m_2)}\left\{\ddot{\rho}(t) - [\rho(t)]\left\{\dot{\varphi}_0 \frac{\rho_0^2}{[\rho(t)]^2} - \frac{m_2}{m_1} F_{01}\sin\beta\left[\frac{1}{\rho(t)}\int_0^t \rho(\tau)\cos\Omega_1\tau\, d\tau\right]\right\}^2\right\} \tag{e}$$

This rheological relation (e) is integro-differntial equation from which we can determine relative distance $\rho(t)$ between mass particles as a time function and as a soluttion of this equation. After solving the previous equation we have the main part of defined problem solution.

EXAMPLE 3. For the case of the three mass particles connected by the three hereditary elements in the plane, for the relations between internal system coordinate we obtain the following:

$$\begin{aligned}
&\left(\frac{1}{m_1}+\frac{1}{m_2}\right)\left\{\sin(\varphi_{13}-\varphi_{23})\left[\sin\varphi_{12}\frac{d^2}{dt^2}(\rho_{12}\cos\varphi_{12}) - \cos\varphi_{12}\frac{d^2}{dt^2}(\rho_{12}\sin\varphi_{12})\right]\right\}+\\
&+\left(\frac{1}{m_1}+\frac{1}{m_2}\right)\left\{\sin(\varphi_{12}-\varphi_{23})\left[\sin\varphi_{13}\frac{d^2}{dt^2}(\rho_{13}\cos\varphi_{13}) - \cos\varphi_{13}\frac{d^2}{dt^2}(\rho_{13}\sin\varphi_{13})\right]\right\}=\\
&=\frac{1}{m_1}\left\{\sin(\varphi_{13}-\varphi_{23})\left[\sin\varphi_{12}\frac{d^2}{dt^2}(\rho_{13}\cos\varphi_{13}) - \cos\varphi_{12}\frac{d^2}{dt^2}(\rho_{13}\sin\varphi_{13})\right]\right\}+\\
&+\frac{1}{m_1}\left\{\sin(\varphi_{12}-\varphi_{23})\left[\sin\varphi_{13}\frac{d^2}{dt^2}(\rho_{12}\cos\varphi_{12}) - \cos\varphi_{13}\frac{d^2}{dt^2}(\rho_{12}\sin\varphi_{12})\right]\right\}
\end{aligned} \tag{f}$$

$$\begin{aligned}
&\left(\frac{1}{m_1}+\frac{1}{m_3}\right)\left\{\sin(\varphi_{12}-\varphi_{23})\left[\sin\varphi_{13}\frac{d^2}{dt^2}(\rho_{13}\cos\varphi_{13}) - \cos\varphi_{13}\frac{d^2}{dt^2}(\rho_{13}\sin\varphi_{13})\right]\right\}+\\
&+\left(\frac{1}{m_1}+\frac{1}{m_3}\right)\left\{\sin(\varphi_{13}-\varphi_{23})\left[\sin\varphi_{12}\frac{d^2}{dt^2}(\rho_{12}\cos\varphi_{12}) - \cos\varphi_{12}\frac{d^2}{dt^2}(\rho_{12}\sin\varphi_{12})\right]\right\}=\\
&=\frac{1}{m_1}\left\{\sin(\varphi_{12}-\varphi_{23})\left[\sin\varphi_{13}\frac{d^2}{dt^2}(\rho_{12}\cos\varphi_{12}) - \cos\varphi_{13}\frac{d^2}{dt^2}(\rho_{12}\sin\varphi_{12})\right]\right\}+\\
&+\frac{1}{m_1}\left\{\sin(\varphi_{13}-\varphi_{23})\left[\sin\varphi_{12}\frac{d^2}{dt^2}(\rho_{13}\cos\varphi_{13}) - \cos\varphi_{12}\frac{d^2}{dt^2}(\rho_{13}\sin\varphi_{13})\right]\right\}
\end{aligned} \tag{g}$$

The third relation is similar to the previous, for permuted index. The following relations are obtained as well as:

$$\begin{aligned}
tg\varphi_{23} &= \frac{\rho_{12}\sin\varphi_{12} - \rho_{13}\sin\varphi_{13}}{\rho_{13}\cos\varphi_{13} - \rho_{12}\cos\varphi_{12}}\\
\rho_{23}^2 &= \rho_{12}^2 + \rho_{13}^2 - 2\rho_{12}\rho_{13}\cos(\varphi_{12}-\varphi_{13})
\end{aligned} \tag{f}$$

These equations must be solved as a system of coupled equations expressed by internal coordinates and must be solved together with corresponding of (2) (or (4) or (6)) and (18). This is a system with complete equations with coresponding number to the unknown coordinates.

3. COVARIANT INTEGRO-DIFFERENTIAL EQUATIONS OF THE MOTION OF THE DISCRETE HEREDITARY SYSTEM

By using principle of the work on the virtual displacemet we can write (see Ref. [5]):

$$\sum_{\nu=1}^{\nu=N}\{\vec{\mathrm{I}}_\nu+\vec{\mathrm{F}}_\nu+\vec{\mathrm{R}}_\nu+\vec{\mathrm{P}}_\nu+\vec{\mathrm{R}}_{\nu\mathrm{T}}\}\delta\vec{r}_\nu=0 \qquad (13^*)$$

or:

$$\sum_{\nu=1}^{\nu=N}\left\{m_\nu\ddot{\vec{r}}_\nu-\vec{\mathrm{F}}_\nu(t)-\sum_{\mu=1}^{\mu=S}\lambda_\mu grad_\nu f_\mu(\vec{r}_1,...,\vec{r}_N)-\sum_{k=1}^{k=K_\nu}P_{(\nu,\nu+1)k}(t)\frac{\vec{r}_{(\nu+1)k}-\vec{r}_{(\nu)k}}{|\vec{r}_{(\nu+1)k}-\vec{r}_{(\nu)k}|}-\vec{\mathrm{R}}_{\nu\mathrm{T}}\right\}\delta\vec{r}_\nu=0 \quad (13)$$

Now, the virtual displacement can be expressed by using generalized coordinates in the form: $\delta\vec{r}_\nu=\sum_{\alpha=1}^{\alpha=n}\frac{\partial\vec{r}_\nu}{\partial q^\alpha}\delta q^\alpha$ and introduced into the previous equation (13) for the work of the active and reactive forces on the virtual displacements, and we obtain the following:

$$\sum_{\nu=1}^{\nu=N}\left\{m_\nu\ddot{\vec{r}}_\nu-\vec{\mathrm{F}}_\nu(t)-\sum_{\mu=1}^{\mu=S}\lambda_\mu grad_\nu f_\mu(\vec{r}_1,...,\vec{r}_N)-\sum_{k=1}^{k=K_\nu}P_{(\nu,\nu+1)k}(t)\frac{\vec{r}_{(\nu+1)k}-\vec{r}_{(\nu)k}}{|\vec{r}_{(\nu+1)k}-\vec{r}_{(\nu)k}|}-\vec{\mathrm{R}}_{\nu\mathrm{T}}\right\}\sum_{\alpha=1}^{\alpha=n}\frac{\partial\vec{r}_\nu}{\partial q^\alpha}\delta q^\alpha=0 \quad (27)$$

Now, by changing the order of sumarizing we can obtain:

$$\begin{aligned}\sum_{\alpha=1}^{\alpha=n}\delta q^\alpha\sum_{\nu=1}^{\nu=N}\Bigg\{&m_\nu\left(\ddot{\vec{r}}_\nu,\frac{\partial\vec{r}_\nu}{\partial q^\alpha}\right)-\left(\vec{\mathrm{F}}_\nu(t,\frac{\partial\vec{r}_\nu}{\partial q^\alpha}\right)-\sum_{\mu=1}^{\mu=S}\lambda_\mu\left(grad_\nu f_\mu(\vec{r}_1,...,\vec{r}_N),\frac{\partial\vec{r}_\nu}{\partial q^\alpha}\right)-\\&-\sum_{k=1}^{k=K_\nu}P_{(\nu,\nu+1)k}(t)\frac{\left(\vec{r}_{(\nu+1)k}-\vec{r}_{(\nu)k},\frac{\partial\vec{r}_{(\nu)k}}{\partial q^\alpha}\right)}{|\vec{r}_{(\nu+1)k}-\vec{r}_{(\nu)k}|}-\left(\vec{\mathrm{R}}_{\nu\mathrm{T}},\frac{\partial\vec{r}_\nu}{\partial q^\alpha}\right)\Bigg\}=0\end{aligned} \quad (27^*)$$

By analysing the member from previous expression we have the following fictive, active and reactive forces:

$$\mathrm{I}_\alpha=-\sum_{\nu=1}^{\nu=N}m_\nu\left(\ddot{\vec{r}}_\nu,\frac{\partial\vec{r}_\nu}{\partial q^\alpha}\right)=-\sum_{\nu=1}^{\nu=N}m_\nu\left(\frac{d\vec{v}_\nu}{dt},\frac{\partial\vec{r}_\nu}{\partial q^\alpha}\right)=-\sum_{\nu=1}^{\nu=N}m_\nu\left(\frac{d}{dt}\sum_{\beta=0}^{\beta=n}\frac{\partial\vec{r}_\nu}{\partial q^\beta}\dot{q}^\beta,\frac{\partial\vec{r}_\nu}{\partial q^\alpha}\right)=$$

$$=-\sum_{\nu=1}^{\nu=N}m_\nu\left(\sum_{\gamma=1}^{\gamma=n}\sum_{\beta=1}^{\beta=n}\frac{\partial^2\vec{r}_\nu}{\partial q^\beta\partial q^\gamma}\dot{q}^\beta\dot{q}^\gamma+\sum_{\beta=1}^{\beta=n}\frac{\partial\vec{r}_\nu}{\partial q^\beta}\ddot{q}^\beta,\frac{\partial\vec{r}_\nu}{\partial q^\alpha}\right)=-[a_{\alpha\beta}(\ddot{q}^\beta+\Gamma^\alpha_{\gamma\delta}\dot{q}^\gamma\dot{q}^\delta)]=-a_{\alpha\beta}\frac{D\dot{q}^\beta}{dt} \quad (28)$$

$\alpha=1,2,3,...,n;\quad n=3N-S$

$$\mathrm{Q}_\alpha=\sum_{\nu=1}^{\nu=N}\left(\vec{\mathrm{F}}_\nu(t),\frac{\partial\vec{r}_\nu}{\partial q^\alpha}\right) \quad (29)$$

$$\mathrm{Q}^f_\alpha=\sum_{\nu=1}^{\nu=N}\sum_{\mu=1}^{\mu=S}\lambda_\mu\left(grad_\nu f_\mu(\vec{r}_1,...,\vec{r}_N),\frac{\partial\vec{r}_\nu}{\partial q^\alpha}\right)=0 \quad (30)$$

$$\mathrm{P}_\alpha=\sum_{\nu=1}^{\nu=N}\sum_{k=1}^{k=K_\nu}P_{(\nu,\nu+1)k}(t)\frac{\left(\vec{r}_{(\nu+1)k}-\vec{r}_{(\nu)k},\frac{\partial\vec{r}_{(\nu)k}}{\partial q^\alpha}\right)}{|\vec{r}_{(\nu+1)k}-\vec{r}_{(\nu)k}|}=\sum_{\nu=1}^{\nu=N}\sum_{k=1}^{k=K_\nu}P_{(\nu,\nu+1)k}(t)\frac{\left(\vec{\rho}_{(\nu,\nu+1)k},\frac{\partial\vec{r}_{(\nu)k}}{\partial q^\alpha}\right)}{|\vec{\rho}_{(\nu,\nu+1)k}|} \quad (31)$$

$$\mathrm{Q}^*_\alpha = \sum_{\nu=1}^{\nu=N} \left(\vec{\mathrm{R}}_{\nu T}(t), \frac{\partial \vec{r}_\nu}{\partial q^\alpha} \right) \tag{32}$$

$$\sum_{\alpha=1}^{\alpha=n} [\mathrm{I}_\alpha + \mathrm{Q}_\alpha + \mathrm{P}_\alpha + \mathrm{Q}^*_\alpha] \delta q^\alpha = 0 \tag{33}$$

A system of dynamic equations in the covariant coordinates can be written in the following form:

$$\mathrm{I}_\alpha + \mathrm{Q}_\alpha + \mathrm{P}_\alpha + \mathrm{Q}^*_\alpha = 0 \qquad \alpha = 1,2,3,...,n; \quad n = 3N - S \tag{34}$$

or:

$$a_{\alpha\beta} \frac{D\dot{q}^\beta}{dt} = \mathrm{Q}_\alpha + \mathrm{Q}^*_\alpha + \mathrm{P}_\alpha \qquad \alpha = 1,2,3,...,n; \quad n = 3N - S \tag{34*}$$

4. THE RHEONOMIC COORDINATE METHOD APPLIED TO DISCRETE HEREDITARY SYSTEMS. MODIFIED SYSTEM OF THE COVARIANT INTEGRO-DIFFERENTIAL EQUATIONS OF MOTION OF A DISCRETE HEREDITARY SYSTEM WITH RHEONOMIC CONSTRAINTS

Let us consider K, $K = \sum_{\nu=1}^{\nu=N} K_\nu$ standard hereditary elements of neglected mass and rheological properties defined by material parameters: $n_{(\nu,\nu+1)k}$, $k = 1,2,3,...,K_\nu$ times of relaxations; coefficients of rigidity $c_{(\nu,\nu+1)k}$ and $\tilde{c}_{(\nu,\nu+1)k}$ are an instanteneous rigidity and a prolonged one. Relations between reaction and deformation of the stressed and strained hereditary element in the discrete system can be expressed by the following different forms:

* in the relaxational forms by using integral relation:

$$P_{(\nu,\nu+1)k} = c_{(\nu,\nu+1)k} \left[\overline{\rho}_{(\nu,\nu+1)k}(t) - \int_0^t \mathbf{R}_{(\nu,\nu+1)k}(t-\tau) \overline{\rho}_{(\nu,\nu+1)k}(\tau) d\tau \right], \tag{35}$$
$$\nu = 1,2,3,...,N, \quad k = 1,2,3,....,K_\nu,$$

where:

$$\boldsymbol{R}_{(\nu,\nu+1)k}(t-\tau) = \frac{c_{(\nu,\nu+1)k} - \tilde{c}_{(\nu,\nu+1)k}}{n_{(\nu,\nu+1)k} c_{(\nu,\nu+1)k}} e^{-\frac{t-\tau}{n_{(\nu,\nu+1)k}}} \tag{36}$$
$$\nu = 1,2,3,...,N, \quad k = 1,2,3,....,K_\nu,$$

a kernel of relaxation, and

$$\begin{aligned} \rho_{(\nu,\nu+1)k} &= |\vec{\rho}_{(\nu,\nu+1)k}| = |\vec{r}_{(\nu+1)k} - \vec{r}_{(\nu)k}| \\ \overline{\rho}_{(\nu,\nu+1)k} &= |\vec{\rho}_{(\nu,\nu+1)k}| - \rho_{(\nu,\nu+1)k0} = |\vec{r}_{(\nu+1)k} - \vec{r}_{(\nu)k}| - \rho_{(\nu,\nu+1)k0}\,. \end{aligned} \tag{a}$$

$\rho_{(\nu,\nu+1)0}$ is the natural state length of the hereditary element without stress and strain, and without history of stresed and strained states.

If now, between of hereditary elements and one of the two material particles of the end of hereditary element we put a rheonomic constraint in the form of the exactly defined length segment as a function of time in the form $\ell_{\nu,\nu+1}(t) = a_{\nu,\nu+1}(\Omega t) = a_{\nu,\nu+1}(q^0)$ we defined a hereditary discrete system with rheonomic constraints. In this case we made a new hereditary element with rheonomic modifications. Hereditary element and rheonomic element are connected in series.

This rheonomic modification we can introduce into hereditary element simply in parallel, or serial conection, as well as in other ways, as an element introduced into the complex system of the conected hereditary elements.

For that reason, into reserch of the system defined in that way, we introduce a time function as a rheonomic coordinate in the sence of V. Vuji~i}. As a rheonomic coordinate we can choose for example the following: $q^0 = \Omega t$.

Follows the idea of V. Vuji~i} (see Ref. [6], [7], for the description of system dynamics we choose the generalized coordinate, of which number $n = 3N - S$ is a difference between number of $3N$ positions coordinates and number S of the finite constraints, and we join a rheonomic coordinate q^0 as a more them $n = 3N - S$. This rheonomic coordinate q^0 must be chosen depending on functions introduced by rheonomic segment of lenght, as it is possible to see in the figure.

In this case we can choose as a rheonomic coordinate q^0: time, or lenght, or for example $q^0 = \Omega t$ where Ω is parameter as a frequency. This rheonomic coordinate can be chosen as dimensionless, or lenth, or an angle in radians. This depends on the concrete introduced rheonomic constraints into hereditary system. The next research is for different kinds of rheonomic coordinates and we have not defined the type of chosen rheonomic coordinate.

In acordance with introduced rheonomic lenth in series connected with hereditary element we must correct some relations for the stress-strain state of the hereditary element. For that reason we composed the following relations:

$$\rho_{\nu,\nu+1} = |\vec{\rho}_{\nu,\nu+1}| = |\vec{r}_{\nu+1} - \vec{r}_\nu| \quad \text{and}$$
$$\bar{\rho}_{\nu,\nu+1} = |\vec{\rho}_{\nu,\nu+1}| - \rho_{(\nu,\nu+1)0} - \ell_{\nu,\nu+1}(\Omega t) = |\vec{r}_{\nu+1} - \vec{r}_\nu| - \rho_{(\nu,\nu+1)0} - a_{\nu,\nu+1}(q^0)\,. \tag{a*}$$

For the compositon of dynamical equations of the discrete hereditary system with rheonomic constraints -lenght as it is defined, we can use the equation of virtual work in the form:

$$\sum_{\nu=1}^{\nu=N} \{\vec{\mathrm{I}}_\nu + \vec{\mathrm{F}}_\nu + \vec{\mathrm{R}}_\nu + \vec{\mathrm{P}}_\nu + \vec{\mathrm{R}}_{\nu\mathrm{T}}\}\delta\vec{r}_\nu = 0 \tag{13*}$$

or:

$$\sum_{\nu=1}^{\nu=N} \left\{ m_\nu \ddot{\vec{r}}_\nu - \vec{\mathrm{F}}_\nu(t) - \sum_{\mu=1}^{\mu=S} \lambda_\mu grad_\nu f_\mu(\vec{r}_1,...,\vec{r}_N) - \sum_{k=1}^{k=K_\nu} P_{(\nu,\nu+1)k}(t) \frac{\vec{r}_{(\nu+1)k} - \vec{r}_{(\nu)k}}{|\vec{r}_{(\nu+1)k} - \vec{r}_{(\nu)k}|} - \vec{\mathrm{R}}_{\nu\mathrm{T}} \right\} \delta\vec{r}_\nu = 0 \tag{13}$$

Now, the virtual displacement must be expressed by n generalized coordinates with joined rheonomic coordinate q^0, as a system of $n + 1$ coordinates q^α, $\alpha = 0,1,2,3,...,n$, $n = 3N - S$.

Now we have a modified - extended system of $n + 1$ coordinates, and virtual displacement is: $\delta\vec{r}_\nu = \sum_{\alpha=0}^{\alpha=n} \frac{\partial \vec{r}_\nu}{\partial q^\alpha} \delta q^\alpha$. By introducing this expression into initial vector equation (13), this equation takes the following form:

$$\sum_{\nu=1}^{\nu=N}\left\{m_\nu\ddot{\vec{r}}_\nu-\vec{\mathrm{F}}_\nu(t)-\sum_{\mu=1}^{\mu=S}\lambda_\mu grad_\nu f_\mu(\vec{r}_1,...,\vec{r}_N,t)-\sum_{k=1}^{k=K_\nu}P_{(\nu,\nu+1)k}(t)\frac{\vec{r}_{(\nu+1)k}-\vec{r}_{(\nu)k}}{|\vec{r}_{(\nu+1)k}-\vec{r}_{(\nu)k}|}-\vec{\mathrm{R}}_{\nu\mathrm{T}}\right\}\cdot \cdot\sum_{\alpha=0}^{\alpha=n}\frac{\partial\vec{r}_\nu}{\partial q^\alpha}\delta q^\alpha=0 \tag{38}$$

Now by changing the order of summarizing we obtain the following:

$$\sum_{\alpha=0}^{\alpha=n}\delta q^\alpha\sum_{\nu=1}^{\nu=N}\left\{m_\nu\left(\ddot{\vec{r}}_\nu,\frac{\partial\vec{r}_\nu}{\partial q^\alpha}\right)-\left(\vec{\mathrm{F}}_\nu(t),\frac{\partial\vec{r}_\nu}{\partial q^\alpha}\right)-\sum_{\mu=1}^{\mu=S}\lambda_\mu\left(grad_\nu f_\mu(\vec{r}_1,...,\vec{r}_N,t),\frac{\partial\vec{r}_\nu}{\partial q^\alpha}\right)- -\sum_{k=1}^{k=K_\nu}P_{(\nu,\nu+1)k}(t)\frac{\left(\vec{r}_{(\nu+1)k}-\vec{r}_{(\nu)k},\frac{\partial\vec{r}_{(\nu)k}}{\partial q^\alpha}\right)}{\left|\vec{r}_{(\nu+1)k}-\vec{r}_{(\nu)k}\right|}-\left(\vec{\mathrm{R}}_{\nu\mathrm{T}},\frac{\partial\vec{r}_\nu}{\partial q^\alpha}\right)\right\}=0 \tag{39}$$

For the generalised active and reactive forces in the extended forms we must vrite:

$$\mathrm{I}_\alpha=-\sum_{\nu=1}^{\nu=N}m_\nu\left(\ddot{\vec{r}}_\nu,\frac{\partial\vec{r}_\nu}{\partial q^\alpha}\right)=-\sum_{\nu=1}^{\nu=N}m_\nu\left(\frac{d\vec{v}_\nu}{dt},\frac{\partial\vec{r}_\nu}{\partial q^\alpha}\right)=-\sum_{\nu=1}^{\nu=N}m_\nu\left(\frac{d}{dt}\sum_{\beta=0}^{\beta=n}\frac{\partial\vec{r}_\nu}{\partial q^\beta}\dot{q}^\beta,\frac{\partial\vec{r}_\nu}{\partial q^\alpha}\right)= =-\sum_{\nu=1}^{\nu=N}m_\nu\left(\sum_{\gamma=0}^{\gamma=n}\sum_{\beta=0}^{\beta=n}\frac{\partial^2\vec{r}_\nu}{\partial q^\beta\partial q^\gamma}\dot{q}^\beta\dot{q}^\gamma+\sum_{\beta=0}^{\beta=n}\frac{\partial\vec{r}_\nu}{\partial q^\beta}\ddot{q}^\beta,\frac{\partial\vec{r}_\nu}{\partial q^\alpha}\right)=-[a_{\alpha\beta}(\ddot{q}^\beta+\Gamma^\alpha_{\gamma\delta}\dot{q}^\gamma\dot{q}^\delta)]=-a_{\alpha\beta}\frac{D\dot{q}^\beta}{dt} \tag{40}$$

$\alpha=0,1,2,3,...,n;\quad n=3N-S$

$$\mathrm{Q}_\alpha=\sum_{\nu=1}^{\nu=N}\left(\vec{\mathrm{F}}_\nu(t),\frac{\partial\vec{r}_\nu}{\partial q^\alpha}\right) \tag{41}$$

$$\mathrm{Q}^f_\alpha=\sum_{\nu=1}^{\nu=N}\sum_{\mu=1}^{\mu=S}\lambda_\mu\left(grad_\nu f_\mu(\vec{r}_1,...,\vec{r}_N,t),\frac{\partial\vec{r}_\nu}{\partial q^\alpha}\right) \tag{42}$$

$$\mathrm{P}_\alpha=\sum_{\nu=1}^{\nu=N}\sum_{k=1}^{k=K_\nu}P_{(\nu,\nu+1)k}(t)\frac{\left(\vec{r}_{(\nu+1)k}-\vec{r}_{(\nu)k},\frac{\partial\vec{r}_{(\nu)k}}{\partial q^\alpha}\right)}{|\vec{r}_{(\nu+1)k}-\vec{r}_{(\nu)k}|}=\sum_{\nu=1}^{\nu=N}\sum_{k=1}^{k=K_\nu}P_{(\nu,\nu+1)k}(t)\frac{\left(\vec{\rho}_{(\nu,\nu+1)k},\frac{\partial\vec{r}_{(\nu)k}}{\partial q^\alpha}\right)}{|\vec{\rho}_{(\nu,\nu+1)k}|} \tag{43}$$

$$\mathrm{Q}^*_\alpha=\sum_{\nu=1}^{\nu=N}\left(\vec{\mathrm{R}}_{\nu T}(t)\frac{\partial\vec{r}_\nu}{\partial q^\alpha}\right) \tag{44}$$

$$\sum_{\alpha=0}^{\alpha=n}[\mathrm{I}_\alpha+\mathrm{Q}_\alpha+\mathrm{Q}^f_\varepsilon+\mathrm{P}_\alpha+\mathrm{Q}^*_\alpha]\delta q^\alpha=0 \tag{44}$$

The equations of system dynamics into covariant coordinates are written in the following form:

$$\mathrm{I}_\alpha+\mathrm{Q}_\alpha+\mathrm{Q}^f_\varepsilon+\mathrm{P}_\alpha+\mathrm{Q}^*_\alpha=0\qquad\alpha=0,1,2,3,...,n;\ \ n=3N-S \tag{45}$$

or,

$$a_{\alpha\beta}\frac{D\dot{q}^\beta}{dt}=\mathrm{Q}_\alpha+\mathrm{Q}^f_\alpha+\mathrm{Q}^*_\alpha+\mathrm{P}_\alpha\qquad\alpha=0,1,2,3,...,n;\ \ n=3N-S \tag{45*}$$

The generalized inertia force is

$$\mathrm{I}_\alpha = -\sum_{\nu=1}^{\nu=N} m_\nu \left(\ddot{\vec{r}}_\nu , \frac{\partial \vec{r}_\nu}{\partial q^\alpha} \right) = -\sum_{\nu=1}^{\nu=N} m_\nu \left(\frac{d\vec{v}_\nu}{dt} , \frac{\partial \dot{\vec{r}}_\nu}{\partial \dot{q}^\alpha} \right) = -\left\{ \frac{d}{dt}\frac{\partial E_k}{\partial \dot{q}^\alpha} - \frac{\partial E_k}{\partial q^\alpha} \right\} \tag{46}$$

and kinetic energy is

$$E_k = \frac{1}{2}\sum_{\nu=1}^{\nu=N} m_\nu \vec{v}_\nu^2 = \frac{1}{2}\sum_{\nu=1}^{\nu=N} m_\nu \left[\sum_{\alpha=0}^{\alpha=n} \frac{\partial \vec{r}_\nu}{\partial q^\alpha} \dot{q}^\alpha \right]^2 = \frac{1}{2}\sum_{\alpha=0}^{\alpha=n}\sum_{\beta=0}^{\beta=n} \dot{q}^\alpha \dot{q}^\beta \sum_{\nu=1}^{\nu=N} m_\nu \left(\frac{\partial \vec{r}_\nu}{\partial q^\alpha} , \frac{\partial \vec{r}_\nu}{\partial q^\beta} \right) \tag{47}$$

The basic initial equation of the motion is in the form:

$$\sum_{\alpha=0}^{\alpha=n} \left[\frac{d}{dt}\frac{\partial E_k}{\partial \dot{q}^\alpha} - \frac{\partial E_k}{\partial q^\alpha} - \mathrm{Q}_\alpha - \mathrm{Q}_\varepsilon^f - \mathrm{P}_\alpha - \mathrm{Q}_\alpha^* \right] \delta q^\alpha = 0 \tag{48}$$

From the last equation we can write the extended and modified system of Lagrange's differential equations of the first kind with $n+1$ equations. These equation contain generalized rheological reaction of hereditary elements, rheonomic constraints reactions, and other usually active and reactive forces.

By using extended coordinate system which consists of the $\vec{n}$ generalized coordinates extended with rheonomic coordinate, the extended and modified system of Lagrange's differential equations of the first kind with $n+1$ equations. must be named as a Lagrage-Vujičić-Goroshko system of differential equations for the discrete hereditary system with rheonomic constraints. These equations are in the form:

$$\frac{d}{dt}\frac{\partial E_k}{\partial \dot{q}^\alpha} - \frac{\partial E_k}{\partial q^\alpha} - \mathrm{Q}_\alpha - \mathrm{Q}_\varepsilon^f - \mathrm{P}_\alpha - \mathrm{Q}_\alpha^* = 0 , \ \alpha = 0,1,2,3,...,n ; \quad n = 3N - S \tag{49}$$

Now, we are focused on the analysis of generalized forces work. Generalized reactions work of the rheonomic constraints (42) is not equal to zero, as in the case of the scleronomic finite constraints. By using velocity conditions we can write:

$$\sum_{\nu=1}^{\nu=N} (grad_\nu f_\mu(\vec{r}_1, \vec{r}_2, ..., \vec{r}_N, t), \vec{v}_\nu) + \frac{\partial f_\mu(\vec{r}_1, \vec{r}_2, ..., \vec{r}_N, t)}{\partial t} = 0 \tag{50}$$

where:

$$\vec{v}_\nu = \sum_{\beta=0}^{\beta=n} \frac{\partial \vec{r}_\nu}{\partial q^\beta} \dot{q}^\beta \tag{51}$$

and

$$\sum_{\nu=1}^{\nu=N} \left(grad_\nu f_\mu(\vec{r}_1, \vec{r}_2, ..., \vec{r}_N, q^0), \sum_{\beta=0}^{\beta=n} \frac{\partial \vec{r}_\nu}{\partial q^\beta} \dot{q}^\beta \right) + \frac{\partial f_\mu(\vec{r}_1, \vec{r}_2, ..., \vec{r}_N, t)}{\partial q^0} \dot{q}^0 = 0 \tag{52}$$

We can see that generalized rheonomic reaction work different them zero, and that is the function of rheonomic coordinates in the form:

$$\mathrm{Q}_\alpha^f = \sum_{\nu=1}^{\nu=N} \sum_{\mu=1}^{\mu=S} \lambda_\mu \left(grad_\nu f_\mu(\vec{r}_1, ..., \vec{r}_N, t), \frac{\partial \vec{r}_\nu}{\partial q^\alpha} \right) \neq 0 \tag{53}$$

The generalized rheological constraint reaction can be expressed by using (43) and rheological reaction of the stress-strain state of the hereditary element (35):

$$\mathrm{P}_{\alpha}=\sum_{\nu=1}^{\nu=N}\sum_{k=1}^{k=K_{\nu}}c_{(\nu,\nu+1)k}\{\rho_{(\nu,\nu+1)k}(t)-\rho_{(\nu,\nu+1)k0}-a_{(\nu,\nu+1)k}(q^{0})-$$
$$-\int_{0}^{t}\mathbf{R}_{(\nu,\nu+1)k}(t-\tau)[\rho_{(\nu,\nu+1)k}(\tau)-\rho_{(\nu,\nu+1)k0}-a_{(\nu,\nu+1)k}(q^{0}(\tau))]d\tau\}\cdot\frac{\left(\vec{\rho}_{(\nu,\nu+1)k},\frac{\partial\vec{r}_{(\nu)k}}{\partial q^{\alpha}}\right)}{\left|\vec{\rho}_{(\nu,\nu+1)k}\right|}\tag{54}$$

The generalized rheological constraint reaction can be expressed by two componets: one as an elastic properties reaction, and one as a rheological properties reaction of the hereditary element:

$$\mathrm{P}_{\alpha}^{c}=\sum_{\nu=1}^{\nu=N}\sum_{k=1}^{k=K_{\nu}}c_{(\nu,\nu+1)k}\{\rho_{(\nu,\nu+1)k}(t)-\rho_{(\nu,\nu+1)k0}-a_{(\nu,\nu+1)k}(q^{0})\}\frac{\left(\vec{\rho}_{(\nu,\nu+1)k},\frac{\partial\vec{r}_{(\nu)k}}{\partial q^{\alpha}}\right)}{|\vec{\rho}_{(\nu,\nu+1)k}|}\tag{55}$$

$$\mathrm{P}_{\alpha}^{\ddot{c}}=-\sum_{\nu=1}^{\nu=N}\sum_{k=1}^{k=K_{\nu}}c_{(\nu,\nu+1)k}\{\int_{0}^{t}\mathbf{R}_{(\nu,\nu+1)k}(t-\tau)[\rho_{(\nu,\nu+1)k}(\tau)-\rho_{(\nu,\nu+1)k0}-a_{(\nu,\nu+1)k}(q^{0}(\tau))]d\tau\}\cdot$$
$$\cdot\frac{\left(\vec{\rho}_{(\nu,\nu+1)k},\frac{\partial\vec{r}_{(\nu)k}}{\partial q^{\alpha}}\right)}{|\vec{\rho}_{(\nu,\nu+1)k}|}\tag{56}$$

By multiplying with dq^{α} and by summarizing by index α, an after integrations by $d\vec{r}_{\nu}$, we can write the following expressions:

$$\sum_{\alpha=0}^{\alpha=n}\mathrm{P}_{\alpha}^{c}\,dq^{\alpha}=\sum_{\alpha=0}^{\alpha=n}\sum_{\nu=1}^{\nu=N}\sum_{k=1}^{k=K_{\nu}}c_{(\nu,\nu+1)k}\{\rho_{(\nu,\nu+1)k}(t)-\rho_{(\nu,\nu+1)k0}-a_{(\nu,\nu+1)k}(q^{0})\}\frac{\left(\vec{\rho}_{(\nu,\nu+1)k},\frac{\partial\vec{r}_{(\nu)k}}{\partial q^{\alpha}}\right)}{|\vec{\rho}_{(\nu,\nu+1)k}|}dq^{\alpha}\tag{57}$$

$$\sum_{\alpha=0}^{\alpha=n}\mathrm{P}_{\alpha}^{c}\,dq^{\alpha}=\sum_{\nu=1}^{\nu=N}\sum_{k=1}^{k=K_{\nu}}c_{(\nu,\nu+1)k}\{\rho_{(\nu,\nu+1)k}(t)-\rho_{(\nu,\nu+1)k0}-a_{(\nu,\nu+1)k}(q^{0})\}\frac{\left(\vec{\rho}_{(\nu,\nu+1)k},\sum_{\alpha=0}^{\alpha=n}dq^{\alpha}\frac{\partial\vec{r}_{(\nu)k}}{\partial q^{\alpha}}\right)}{|\vec{\rho}_{(\nu,\nu+1)k}|}\tag{58}$$

Now we introduce a function as a rheological potential which corresponds to elastic properties of the hereditary element:

$$\Pi^{c}=\sum_{\alpha=0}^{\alpha=n}\int_{0}^{q^{\alpha}}\mathrm{P}_{\alpha}^{c}\,dq^{\alpha}=$$
$$=\sum_{\nu=1}^{\nu=N}\sum_{k=1}^{k=K_{\nu}}c_{(\nu,\nu+1)k}\int_{0}^{\vec{r}_{\nu}}\{\rho_{(\nu,\nu+1)k}(t)-\rho_{(\nu,\nu+1)k0}-a_{(\nu,\nu+1)k}(q^{0})\}\frac{(\vec{\rho}_{(\nu,\nu+1)k},d\vec{r}_{(\nu)k})}{|\vec{\rho}_{(\nu,\nu+1)k}|}\tag{59}$$

Also, we introduce a function as a rheological potential which corresponds to the rheological properties of the hereditary element

$$\begin{aligned}\Pi^{\overleftrightarrow{c}} &= \sum_{\alpha=0}^{\alpha=n}\int_0^{q^\alpha} \mathrm{P}_\alpha^{\overleftrightarrow{c}}\, dq^\alpha = \\ &-\sum_{\nu=1}^{\nu=N}\sum_{k=1}^{k=K_\nu} c_{(\nu,\nu+1)k}\int_0^{q^\alpha}\{\int_0^t \mathbf{R}_{(\nu,\nu+1)k}(t-\tau)[\rho_{(\nu,\nu+1)k}(\tau)-\rho_{(\nu,\nu+1)k0}-a_{(\nu,\nu+1)k}(q^0(\tau))]d\tau\}\frac{(\vec{\rho}_{(\nu,\nu+1)k}, d\vec{r}_{(\nu)k})}{|\vec{\rho}_{(\nu,\nu+1)k}|}\end{aligned} \tag{60}$$

By using derivatives of these funtions as a rheological potential we can express generalized rheological reactions:

$$\mathrm{P}_\alpha^c = -\frac{\partial \Pi^c}{\partial q^\alpha} \qquad \mathrm{P}_\alpha^{\overleftrightarrow{c}} = -\frac{\partial \Pi^{\overleftrightarrow{c}}}{\partial q^\alpha} \tag{61}$$

EXAMPLE 4. Rheological rheonomic oscillator [27] is presented in figure. 4 and 5. Generalized coordinte is $x(t)$, and rheonomic coordinte is change $x_0(t)$ of lenght:

The initial equation is:

$$m\ddot{x} + P(t) = F(t)\,.$$

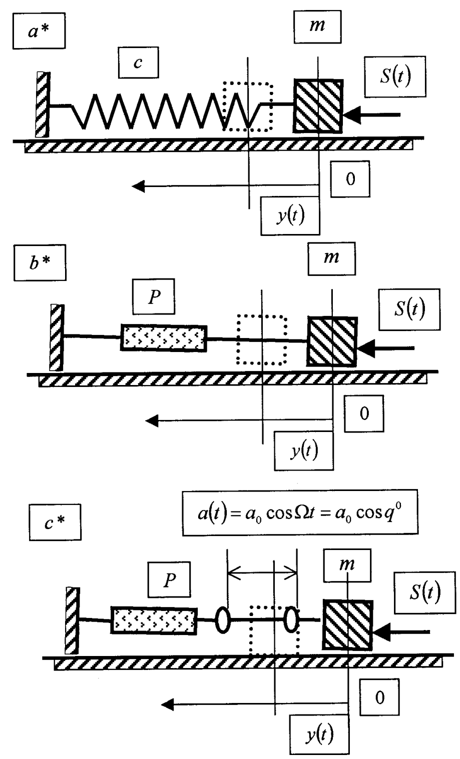

Fig. 5. Classical (a*), hereditary (b*) and hereditary-rheonomic (c*) oscillator, excited by $S(t)$

For the standard hereditary element stress-strain relation is:

$$n\dot{P}(t) + P(t) = nc[\dot{x}(t) - \dot{x}_0(t)] + \tilde{c}[x(t) - x_0(t)]$$

Dynamic equation of the rheological-rheonomic oscillator is:

$$nm\dddot{x}(t) + m\ddot{x}(t) + nc\dot{x}(t) + \tilde{c}x(t) = n[\dot{F}(t) + c\dot{x}_0(t)] + [F(t) + \tilde{c}x_0(t)]$$

and rheonomic reaction is:

$$P(t) = c\{[x(t) - x_0(t)] - \int_0^t \mathbf{R}\,(t-\tau)[x(\tau) - x_0(\tau)]d\tau\}$$

Potential function of the hereditary element and rheological reactions which correspond to elastic properties are:

$$\Pi^c = \frac{1}{2}c[x(t) - x_0(t)]^2$$

$$\mathrm{P}_0^c = -\frac{\partial \Pi^c}{\partial x_0} = c[x(t) - x_0(t)] \qquad \mathrm{P}_1^c = -\frac{\partial \Pi^c}{\partial x_1} = -c[x(t) - x_0(t)]$$

Rheological potential function of the hereditary element and rheological reactions which correspond to the rheological properties are:

$$\Pi^{\tilde{c}} = c\int_0^{x-x_0}\{\int_0^t \mathbf{R}\,(t-\tau)[x(\tau) - x_0(\tau)]d\tau\}dd[x(t) - x_t(\tau)]$$

$$Q_0^{\tilde{c}}(t) = -\frac{\partial \Pi^{\tilde{c}}}{\partial x_0} = -c\{\int_0^t \mathbf{R}\,(t-\tau)[x(\tau) - x_0(\tau)]d\tau\}$$

$$Q_1^{\tilde{c}}(t) = -\frac{\partial \Pi^{\tilde{c}}}{\partial x} = c\{\int_0^t \mathbf{R}\,(t-\tau)[x(\tau) - x_0(\tau)]d\tau\}$$

Kinetic Energy is:

$$E_k = \frac{1}{2}m\dot{x}^2$$

EXAMPLE 5. Rheological pendulum (see Figure No.6) [28] with a thread which increases its length on one of its segments (for example the unfolding of unstreachable segment and tied to the segment by a hereditary thread). The system has three degrees of motions which are defined by coordiantes: rheonomic coordinate $\rho_0(t)$ and generalized coordinates, ρ and φ. Rheological connection does not decrease the number of degrees of freedom. The rheonomic constraint decreases the number of degrees of motion freedom and that is because it assigns one degree of motion as enforcement.

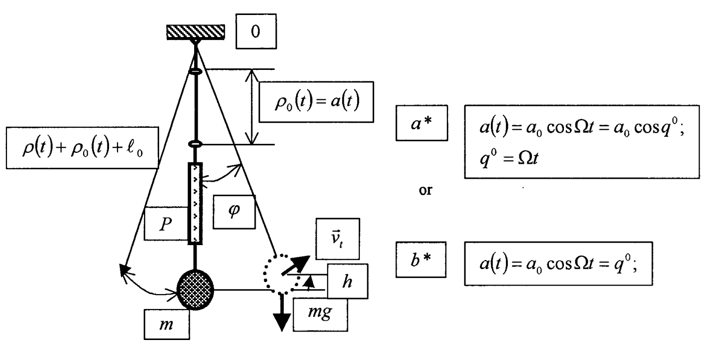

Fig. 6. Rheological-rheonomic hereditary pendulum (Pendulum with a thread which increases its length on one of its segments (for example the unfolding of unstreachable segment and tied to the segment by a hereditary thread))

The potential function of the herediatary element and rheological reaction component which correspond to elastic properties are:

$$\Pi^{c} = \frac{1}{2}c[\rho(t) - \rho_0(t)]^2$$

$$\mathrm{P}_0^c = -\frac{\partial \Pi^c}{\partial \rho_0} = c[\rho(t) - \rho_0(t)] \quad \mathrm{P}_1^c = -\frac{\partial \Pi^c}{\partial \rho} = -c[\rho(t) - \rho_0(t)] \quad \mathrm{P}_\varphi^c = -\frac{\partial \Pi^c}{\partial \varphi} = 0$$

Rheological potential as a function of the hereditary element and rheological reactions which correspond to the rheological properties of the hereditary element are:

$$\Pi^{\tilde{c}} = c\int_0^{\rho-\rho_0}\{\int_0^t \mathbf{R}\,(t-\tau)[\rho(\tau) - \rho_0(\tau)]d\tau\}d[\rho(t) - \rho_0(\tau)]$$

$$Q_{\rho_0}^{\tilde{c}}(t) = -\frac{\partial \Pi^{\tilde{c}}}{\partial \rho_0} = -c\{\int_0^t \mathbf{R}\,(t-\tau)[\rho(\tau) - \rho_0(\tau)]d\tau\}$$

$$Q_{\rho}^{\tilde{c}}(t) = -\frac{\partial \Pi^{\tilde{c}}}{\partial \rho} = c\{\int_0^t \mathbf{R}\,(t-\tau)[\rho(\tau) - \rho_0(\tau)]d\tau\}$$

$$Q_{\varphi}^{\tilde{c}}(t) = -\frac{\partial \Pi^{\tilde{c}}}{\partial \varphi} = 0$$

Kinetic energy is:

$$E_k = \frac{1}{2}m\{[\ell_0 + \rho_0(t) + \rho(t)]^2\dot{\varphi}^2 + [\dot{\rho}_0(t) + \dot{\rho}(t)]^2\}$$

The expanded system of equations is:

$$\frac{d}{dt}\frac{\partial E_k}{\partial \dot{q}^\alpha} - \frac{\partial E_k}{\partial q^\alpha} - \mathrm{Q}_\alpha - \mathrm{Q}_\varepsilon^f - \mathrm{P}_\alpha - \mathrm{Q}_\alpha^* = 0, \quad \alpha = \rho_0, \rho, \varphi;$$

By using previous equations for each of the coordiantes we determine an equation:

EXAMPLE 6. Motion in the plane of two material particles tied to one another by a hereditary element and a rheonomical constraint as a connection in a que (series). We are using the earlier discussed example.

For a system of two material particles on interdistance of ρ in the plane of the system motion of relations (16**), by eliminatng the reaction of the hereditary element we obtain the following relation between internal coordinates ρ and φ of the system.

$$\cos\varphi\frac{d^2}{dt^2}\{\rho\sin\varphi\} = \sin\varphi\frac{d^2}{dt^2}\{\rho\cos\varphi\} - \frac{m_2}{m_1}F_{01}\sin\beta\cos\Omega_1 t \tag{a}$$

$$[2\dot{\rho}\dot{\varphi} + \rho\ddot{\varphi}] = -\frac{m_2}{m_1}F_{01}\sin\beta\cos\Omega_1 t \tag{b}$$

The solution of the last differential equation (b) which can be observed as a differential equation of the first order by $\dot{\varphi}(t)$, so that we obtain:

$$\dot{\varphi}(t) = \dot{\varphi}_0\frac{\rho_0^2}{[\rho(t)]^2} - \frac{m_2}{m_1}F_{01}\sin\beta\left[\frac{1}{\rho(t)}\int_0^t \rho(\tau)\cos\Omega_1\tau d\tau\right] \tag{c}$$

The last differential equation (c) gives the connection between angular velocity $\dot{\varphi}(t)$ of the relative rotating of the second material particle around the first one in the plane of dynamics of that system of material particles and distance ρ between those particles. We can see that angular velocity $\dot{\varphi}(t)$ of the relative rotating of the second material paritcle around the first consists of two parts, and that one of the components is opposite proportional to the square of their interdistance ρ, and that the second component is in an integral form and depends on the external enforcement excitation force.The first component corresponds to the case of the own-free motion of material particles connected with a hereditary element, when there is no external enforcement force, while the other is the result of enforced relative rotation as a consequence of external enforcement force.

The square relative velocity of the relative rotation of the second mass particle around first one is:

$$v_r^2 = \dot\rho^2 + \rho^2\omega^2 = [\dot\rho(t)]^2 + [\rho(t)]^2\left\{\dot\varphi_0 \frac{\rho_0^2}{[\rho(t)]^2} - \frac{m_2}{m_1}F_{01}\sin\beta\left[\frac{1}{\rho(t)}\int_0^t \rho(\tau)\cos\Omega_1\tau d\tau\right]\right\}^2 \tag{d}$$

By introducing the expression (d) for the square of the relative velocity of the second material particle in relation to the first one in the expression (18) for force $P_{1,2}(t) = P(t)$ of interaction of material particles, and then the obtained expression we introduce into the expression for force of interaction of material particles into the integral realtion (2) of rheological hereditary connection in series with the rheonomic costraint for the case of enforced motion of the system in the plane of rhological -rheonomic connection which comes down to the following integro-differential relation which has the form:

$$\begin{aligned}&[\rho(t) - \ell_0 - \rho_0(t)] - \int_0^t \mathrm{R}\,(t-\tau)[\rho(\tau) - \ell_0 - \rho_0(\tau)]d\tau = \\ &= -\frac{m_1m_2}{c(m_1+m_2)}\left\{\ddot\rho(t) - [\rho(t)]^2\left\{\dot\varphi_0 \frac{\rho_0^2}{[\rho(t)]^2} - \frac{m_2}{m_1}F_{01}\sin\beta\left[\frac{1}{\rho(t)}\int_0^t \rho(\tau)\cos\Omega_1\tau d\tau\right]\right\}^2\right\}\end{aligned} \tag{e}$$

This rheological-rheonomic connection is an integro-differential nonlinear equation from which we determine the relative distance $\rho(t)$ of material particles as the function of time, in which way we in fact solved the basic problem.

Everything else is simple.

Instead of the previous connection we can write:

* for the standard hereditary element in series (que) with a reheonomical segment of the thread:

$$n\,\dot{\mathrm{P}}(t) + \mathrm{P}(t) = nc[\dot\rho(t) - \dot\rho_0(t)] + \tilde c[\rho(t) - \ell_0 - \rho_0(t)]$$

While from dynamic equations of force of material particles interaction it is:

$$\mathrm{P}(t) = -\frac{m_1m_2}{(m_1+m_2)}\left\{\ddot\rho(t) - \rho(t)\left\{\dot\varphi_0 \frac{\rho_0^2}{[\rho(t)]^2} - \frac{m_2}{m_1}F_{01}\sin\beta\left[\frac{1}{\rho(t)}\int_0^t \rho(\tau)\cos\Omega_1\tau d\tau\right]\right\}^2\right\}$$

By eliminating the force $\mathrm{P}(t)$ from the last two realtions we obtain:

$$\begin{aligned}&-\frac{m_1m_2}{(m_1+m_2)}\left\{n\dddot\rho(t) + \ddot\rho(t) - n\dot\rho(t)\left\{\dot\varphi_0 \frac{\rho_0^2}{[\rho(t)]^2} - \frac{m_2}{m_1}F_{01}\sin\beta\left[\frac{1}{\rho(t)}\int_0^t \rho(\tau)\cos\Omega_1\tau d\tau\right]\right\}^2\right\} + \\ &+\frac{m_1m_2}{(m_1+m_2)}\left\{-[\rho(t)]\left\{\dot\varphi_0 \frac{\rho_0^2}{[\rho(t)]^2} - \frac{m_2}{m_1}F_{01}\sin\beta\left[\frac{1}{\rho(t)}\int_0^t \rho(\tau)\cos\Omega_1\tau d\tau\right]\right\}^2\right\} + \\ &+\frac{m_1m_2}{(m_1+m_2)}2n[\rho(t)]\left\{\dot\varphi_0 \frac{\rho_0^2}{[\rho(t)]^2} - \frac{m_2}{m_1}\frac{1}{\rho(t)}F_{01}\sin\beta\int_0^t \rho(\tau)\cos\Omega_1\tau d\tau\right\}\cdot \\ &\cdot\left\{2\dot\varphi_0 \frac{\rho_0^2}{[\rho(t)]^3}\dot\rho(t) + \frac{m_2}{m_1}F_{01}\sin\beta\frac{d}{dt}\left[\frac{1}{\rho(t)}\int_0^t \rho(\tau)\cos\Omega_1\tau d\tau\right]\right\} = \\ &= nc[\dot\rho(t) - \dot\rho_0(t)] + \tilde c[\rho(t) - \ell_0 - \rho_0(t)]\end{aligned}$$

For the case of free motion without the effects of an external force, only under the influence of the initial disturbance of equilibrium system position, and the excitation by a rheonomic constraint-conncection the previous equation for determining the distance between the material particles is:

$$-\frac{m_1 m_2}{(m_1+m_2)}\left\{ n\dddot{\rho}(t) + \ddot{\rho}(t) + [\rho(t) - 3n\dot{\rho}(t)]\left\{\dot{\varphi}_0 \frac{\rho_0^2}{[\rho(t)]^2}\right\}^2 \right\} =$$

$$= nc[\dot{\rho}(t) - \dot{\rho}_0(t)] + \tilde{c}[\rho(t) - \ell_0 - \rho_0(t)]$$

or

$$n\dddot{\rho}(t) + \ddot{\rho}(t) + n\frac{m_1+m_2}{m_1 m_2}c[\dot{\rho}(t) - \dot{\rho}_0(t)] + \frac{m_1+m_2}{m_1 m_2}[\rho(t) - \ell_0 - \rho_0(t)] + \dot{\varphi}_0^2 \frac{\rho_0^4}{[\rho(t)]^4}[3n\dot{\rho}(t) - \rho(t)] = 0$$

5. CONCLUDING REMARKS

In this paper we derive the covariant integro-differential equations of the discrete hereditary systems with rheonomic constraints between mass particles and hereditary element. In this paper we show applications of the rheonomic coordinate method to the dynamics of the discrete hereditary system. By introducing rheonomic coordinate in the sense of V. Vuji~i} we derive an extended system of differenttial equations in covariant form and coresponding generalizer reaktive forces for generalized coordinates in extendend form. We consider some special examples of the discrete hereditary systems. We introduce expressions of the rheological potential which correspond to elastic and hereditary properties.

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METODA REONOMNE KOORDINATE U PRIMENI NA NELINEARNE OSCILATORNE SISTEME SA NASLEDNIM ELEMENTIMA

Katica (Stevanović) Hedrih

Rezultati prikazani u ovom radu inspirisani su radovima O. A. Goroshko i N. P. Puchko (vidi Ref. [13] i [14]), o Lagrange-ovim jednačinama za nasledne diskretne sisteme (više tela) i reološkim modelima tela koji su prikazani u monografiji G.M. Savin-a i Ya.Ya. Ruschitsky (vidi Ref. [24]), kao i monografijom V.A. Vujičić-a (vidi [6]). Koristeći modele reoloških tela za opisivanje deformabilnih reoloških, naslednih elemenata sa hibridnim reološkim-elasto-viskoznim i/ ili visko-elastičnim svojstvima, postavljeni su modeli diskretnih naslednih sistema sa jednim i više stepeni slobode kretanja. Za takve oscilatorne nasledne sisteme integro-diferencijalne jednačine drugog, i/ili diferencijalne jednačine trećeg reda su sastavljene.

Sastavljene su jednačine dinamike diskretnog sistema sa konačnim vezama i standardrnim naslednim elementima. Sastavljene su integro-diferencijalne jednačine kretanja diskretnog naslednog sistema u kovarijantnom obliku. Prikazana je metoda reonomne koordinate u primeni na diskretne nasledne sisteme. Izveden je prošireni sistem integro-diferencijalnih jednačina kretanja, u kovarijantnom obliku, naslednog sistema sa reonomnim vezama.

Dat je veći broj primera reološko-reonomnog oscilatora, kao i reološko-reonomnih klatna.

Ključne reči: *Diskretni nasledni sistem, standardni nasledni element, oscilatorni nasledni sistem, reonomna koordinata, reološka koordinata, metoda reonomne koordinate, reološko-reonomno klatno, reološko i relaksaciono jezgro, kovarijantne koordinate.*