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PARAMETERS AFFECTING THE DYNAMIC RESPONSE OF LIGHT (STEEL) BRIDGES

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Abstract. *This paper brings together and examines a lot of parameters that, usually, are not taken into account either during the design of a bridge or because some assumptions are supposed by the designer to hold true for the design and the calculations of a bridge. Some of the more serious of these parameters are the irregularities of the surface of the bridge's deck, the vehicle model selected, or the neglect of some forces that arise by the movement of the vehicle on the bridge. We will find out that the effect of those parameters is, sometimes, very remarkable.*

1. HISTORICAL REVIEW

The study and the determination of the influence of dynamic loads on elastic structures is a very old and complicated problem. Especially the determination of the dynamic effect of moving loads on elastic structures and particularly on bridges is a very old and complicated problem. Many systems in civil engineering and mainly on the design of bridges can be idealized as a flexible beam under a moving mass. The existence of a moving mass makes the problem very difficult because of its non-linearity, which arises exactly by the moving mass.

A number of works has been reported during the last hundred years trying to present reliable solutions for such a *multi-parameter* problem by using *two* different methods: The first one is to perform *tests* and the second is that of pure *theoretical* investigation. In recent years *transport engineering* has experienced serious advances characterized by increasingly higher *speeds* and *weights* of vehicles, as a result of which vibrations and dynamic stresses larger than ever before have been developed. From the historical viewpoint, the problem of moving loads was first considered approximately for the case where the *mass* of the *girder* was negligible compared to the *mass* of a single *moving load* of constant magnitude [1], [2] and [3]. The other extreme case in which the mass of the moving load was negligible compared to the mass of the girder was originally studied by Krylov [4] and later on by Timoshenko [5] and Lowan [6]. The more complicated

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problem including both the parameters, the load mass and the mass of the girder, was studied by other investigators among whom we should mention Steuding [7], Schallemcamp [8], and Bolotin [9]. A very thorough treatise on the dynamic response of several types of *railway* bridges traversed by steam locomotives was presented by Inglis [10] using harmonic analysis. Interesting analyses were also presented by Hillerborg [11] using Fourier's analysis and by Biggs, Suer and Louw [12] using Ingliss technique. The problem of the dynamic response of bridges under moving loads is reviewed in detail by Timoshenko [13], and later on by Kolousek in1956, 1961, and 1967 [14]. We should also mention the extended review reported by Fryba [15] in his excellent monograph on this subject. These analyses have been extended to simple frames subjected to moving loads by Karaolides and Kounadis [16] and thereafter to a two-bar frame under a moving load in which the effect of axial motion has been taken into account by Sophianopoulos and Kounadis [17]. Some partial results regarding the effects of the mass of a moving load on the dynamic response of a simply supported beam were recently presented by Michaltsos, Sophianopoulos and Kounadis [18].

From the above short historical viewpoint, one can see that the problem of moving loads was first considered for the case where the *mass* of the *girder* was negligible compared to the *mass* of a single *moving load* of constant magnitude. The second also extreme case in which the mass of the moving load was negligible compared to the mass of the girder was the second step in the study of the moving loads. The more complicated problem, including both the parameters, the load mass and the mass of the girder, was the third step in the study.

Today, the investigation on the dynamic behaviour of the bridges has moved on from this infantile period and more complicated models and also a difficult non-linear mathematical analysis for the studied problems is used.

2. Description of parameters

The most significant parameters that affect the dynamic response of light (steel) bridges can be classified in two great categories:

a. Those which are connected to the design, the shape of the bridge and the characteristics of the vehicle and which can be called physical parameters, and
b. Those which depend on the acceptances and on the necessary simplifications connected to the difficulties met, each time, during the forming of the mathematical models of the problem. The above can be called design parameters.

2.1. Physical parameters.

In the first category are classified the characteristics of the cross-section of the bridge, the static system, the roughness of the deck's surface, and the probable existing irregularities of the deck that are accidental or purposely constructed by the designer (they are often desired for a lot of different reasons).

All the above parameters are taken into account by the designers except for the two last ones. We, on the other hand, will focus our examination on the two last ones. By doing this we do not mean to say that the influence of the first group of parameters is not significant (as one can see in figure 1 the form of the bridge's axis, for example, has a very serious influence on its dynamic behaviour [19]) but because there are not many studies about the two last ones.

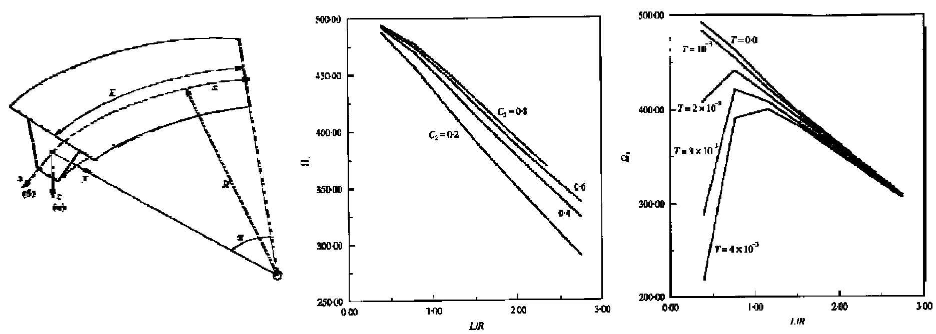

Fig. 1. The influence of the curvature of bridge's axis on the 1st dimensionless eigenfrequency: Case a: $C_1 = 0$ (warping parameter), T = 0 (mass rotatory param.)
Case b: $C_1 = 10^{-3}$ (warping param.), T = 0.2 (tortional param.)

2.2. Design parameters

In this second category are contained the model of the considered vehicle and the assumptions of the design.

A real vehicle can be simulated from the simplest up to the most complicated model. This depends on the design's phase (preliminary study or final study) and, as for the investigation, on the ratio of the vehicle mass and dimensions to the mass and dimensions of the bridge.

The assumptions of the design presuppose simplifications. By design simplifications we mean the completeness of the used equations of motion.

In all the above in paragraph 1 studies, the response of laterally vibrating beams and frames due to moving loads was established on the basis of *standard dynamic analysis* which neglects the effects of both C*entripetal force*, C*oriolis force* and the *Rotatory Inertia,* associated with the *mass* of the moving load which follows the *transverse motion* of the flexural vibrating beam.

3. THE CHOSEN MODEL

It is determinative, as for the exactitude of the results, to choose each time the right model. The choice depends on various parameters. Among them we can refer to the use of the bridge (highway or railway), the material from which it is made (concrete or steel) and therefore its weight, the static system, the density of the traffic, and the most serious, the length of the bridge.

For long-length bridges we prefer the one-axle models (see figure 2 from a to f). For shorter bridges one can choose one of the models g, h or i (fig. 2). For long-length bridges and heavy traffic, there are still the models j and k .

Models g, h, i, and k take into account the influence of the vehicle's mass-rotatory inertia $J = Md^2e$, where *M* is the mass of the vehicle, *d* is the length of the vehicle and *e* the so called shape factor of the vehicle, which usually, extends from 0.08 to 0.10.

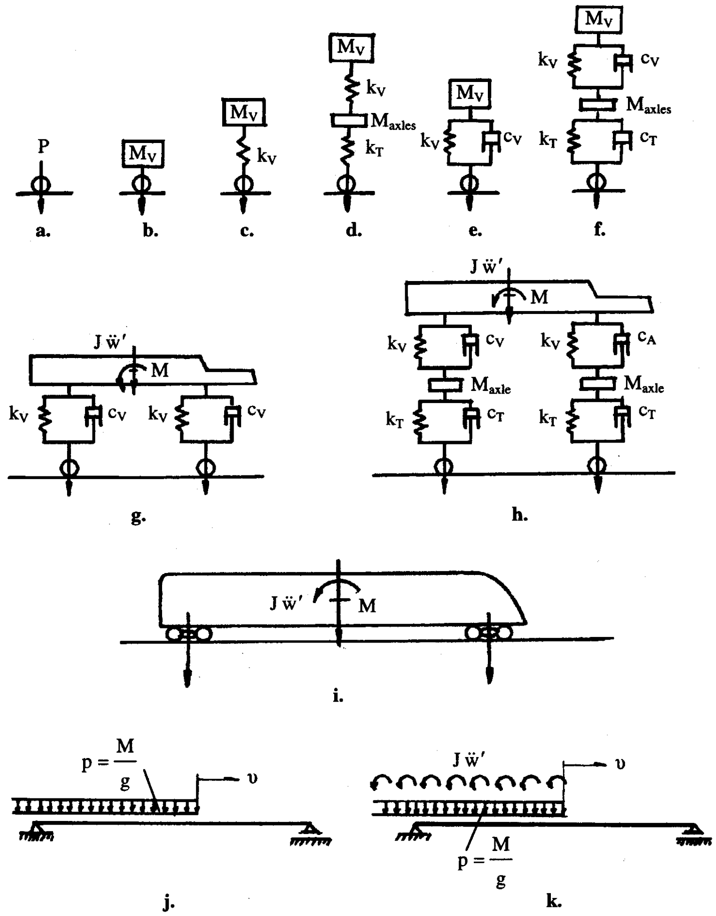

Fig. 2. a. Simple load b. Load-mass c. Load-mass with spring d. As in c, with additional load-mass instead of axles and spring instead of tyres e. Load-mass with spring and damper f. As in e with additional load-mass instead of axles and additional spring and damper instead of tyres g. Simple vehicle with springs and dampers (Drosner model) h. As in g with additional springs and dampers instead of tyres (Veletsos model) i. Locomotive model for railway bridges of short or middle length j. Moving distributed load-mass for long-length bridges k. Moving distributed load-mass with moments caused by mass-rotatory inertia

4. THE UNEVEN DECK

The influence of the uneven deck of a bridge could be significant or not significant.
The used model of the bridge and that of the vehicle are shown in figure 3.
The corresponding equation of motion is:

$$EI\, w'''' + c_b \dot{w} + m\ddot{w} = P \cdot \delta(x - \alpha) \tag{1}$$

with

$$P = M_V \cdot g + k\,[z - w(x) - r(x)] + c_V\,(\dot{z} - \dot{w}) + P_{impact} \tag{2}$$

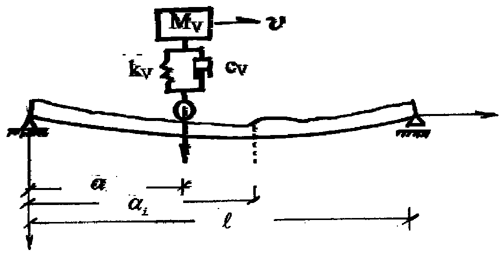

Fig. 3. Hinged single-span bridge with uneven deck

where:

M_V: the mass of the moving load-mass

k: the coefficient of the vehicle spring

c_b: the coefficient of the bridge damping

c_V: the coefficient of the vehicle damping

$r(x)$: the function that gives the uneven surface of the deck

P_{impact}: the arising impact force, which is produced by the impact of tyre-masses on the irregularities (see fig. 4).

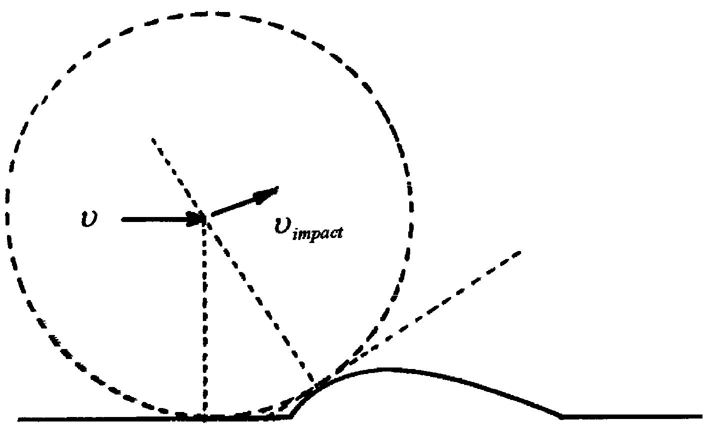

Fig. 4. Impact while the tyre enters an irregularity with speed υ

4.1. The roughness of the deck's surface.

From the above it is clear that roughness can be considered a special case of an uneven deck and therefore can be studied through equations (1) and (2).

The influence of an uneven deck is expressed by two terms (see equation 2). The first term, which contains $r(x)$, has very little influence (in the case of roughness) on the dynamic behaviour of a bridge. The second one, which contains the impact forces has greater influence and depends on the flat part "b" of the tyre (and then on the angle φ of figure 5), which is in contact with the deck (see also fig.5). This last term is expressed by:

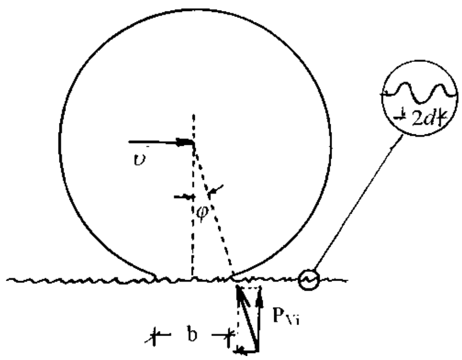

Fig. 5. Impact forces created by roughness

$$P_{impact} = P_{iV} \cdot \sum_{i=1}^{\infty} [\delta(t - t_i)] \quad (3)$$

$$t_i = t_{i-1} + \frac{2d}{\upsilon} \quad (4)$$

where δ is the Dirac's function $2d$ is the distance between two irregularities of the asphalt surface of the deck expressing the surface quality and t_i is the time that the wheel is on the point i which is at a distance α_i from the left end of the bridge. Finally the influence of the roughness is very small amounting to a smallest ratio of the final deformation of the bridge and therefore it is negligible. (The roughness has a significant influence on the car's vibrations and the so caused seasickness to the passengers).

4.2. The surface deck irregularities

By irregularity we mean the change of the surface of the deck deviating from the given form, each time, by function $r(x)$ of equation (1). We suppose that the examined irregularities have a shape that can be expressed in mathematical form. Two types of irregularities are examined:

The first is shown in figure 1a, where the vehicle enters and exits the irregularity abnormally, because of the non-horizontal tangents on the points A and B. This irregularity, named Type I, has a form given by:

$$w_{01}(x) = \frac{f}{\ell^2} x^2 - \frac{2f}{\ell} x \quad (5)$$

The second, in fig. 1b, is the one which has the tangents on the points A and B horizontal. This irregularity, named Type II, has a form given by:

$$w_{02}(x) = -\frac{f}{\ell^4}(x-\ell)^4 + \frac{2f}{\ell^2}(x-\ell)^2 - f \quad (6)$$

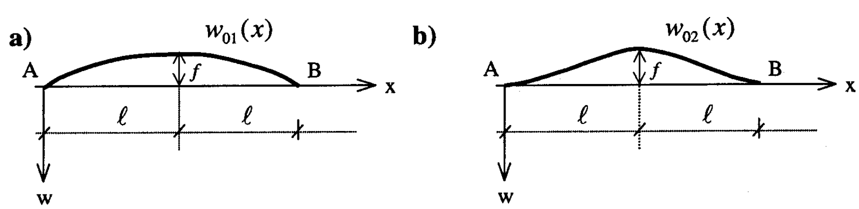

Fig. 6. Types of irregularities: a) Type I, b) Type II

The impact forces are produced at the instant that the mass m_T (mass of tyre) strikes with speed υ against the irregularity (which is assumed to have a mass equal to ∞ and speed equal to zero) and also at the instant that the mass exits the irregularity. The mass m_T, after its impact, bounces in the direction $\overline{k_1 w}$ (entrance) or $\overline{k_2 w}$ (exit), (see fig.7).

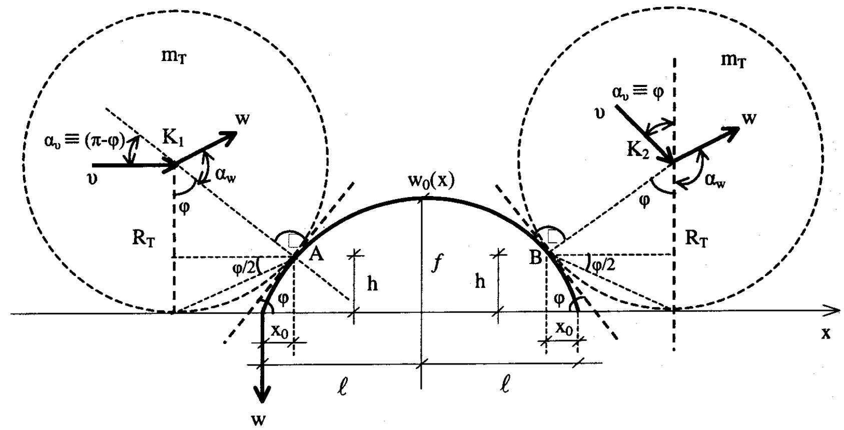

Fig. 7. Impact forces while a mass enters and exits an irregularity

The produced impact forces are (see and figure 7):

$$\left.\begin{aligned} &a)\quad \textit{At the tyre's entrance:} && P_{impact}^{in} = \varepsilon \cdot m_T \cdot \upsilon \cdot \frac{\sin\varphi \cdot \cos(\alpha_w + \varphi)}{\cos\alpha_w} \\ &b)\quad \textit{At the tyre's exit:} && P_{impact}^{out} = \varepsilon \cdot m_T \cdot \upsilon \cdot \sin\varphi \end{aligned}\right\} \tag{7}$$

$$\left.\begin{aligned} &where:\ \sin\frac{\varphi}{2} = \sqrt{\frac{h}{2R_T}}, \quad \tan\alpha_w = -\frac{\cot\varphi}{\varepsilon}, \quad \tan\varphi = w_0'(x_0), \quad h = w_0(x_0) \\ &\quad \varepsilon = \textit{coefficient, which expresses the quantity of the remaining} \\ &\qquad \textit{energy (after impact).} \\ &\quad x_0 \textit{ is the positive solution of the equation:} \\ &\quad w_0'(x_0)[R_T - w_0(x_0)] - \sqrt{w_0(x_0) - [2\cdot R_T - w_0(x_0)]} = 0 \end{aligned}\right\} \tag{8}$$

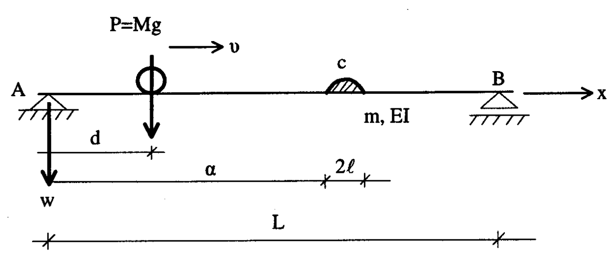

Fig 8. Single-span bridge with an irregularity at α and moving mass at d

The equation of motion of the bridge of figure 8 has the form:

$$\left.\begin{array}{ll} EIw''''(x,t)+m\ddot{w}(x,t)=P\delta(x-d) & for: 0\le t\le \dfrac{\alpha}{\upsilon} \\ EIw''''(x,t)+m\ddot{w}(x,t)=(P+P_i)\delta(x-d)+P_{impact}^{in}\delta(x-\alpha)\delta(t-\dfrac{\alpha}{\upsilon}) & for: \dfrac{\alpha}{\upsilon}\le t\le \dfrac{\alpha+2\ell}{\upsilon} \\ \left.\begin{array}{r} EIw''''(x,t)+m\ddot{w}(x,t)=(P+P_i)\delta(x-d)+P_{impact}^{in}\delta(x-\alpha)\delta(t-\dfrac{\alpha}{\upsilon})+ \\ +P_{inpact}^{out}\delta(x-\alpha-2\ell)\delta(t-\dfrac{\alpha+2\ell}{\upsilon}) \end{array}\right\} & for: t\ge \dfrac{\alpha+2\ell}{\upsilon} \end{array}\right\} \quad (9)$$

From the solution of the above equation (9), and after some manipulations, one can find dimensionless forms and study the influence of the shape of an irregularity, its length or its position on the dynamic deflections of a bridge in connection to the weight and the length of the moving vehicle. The results of such an investigation [20] are shown in plots of figures 9 and 10. The study of diagrams 1 to 20 of figures 9 and 10 leads to the following general results:

1. *Regarding the models of vehicles*:

The use of the exact biaxial model is necessary in light vehicles, where the difference goes up to 44% in relation to the one-axis model, when this difference reaches 85% in relation to the simple moving load (without irregularity, spring, e.t.c.).

For short bridges, the deviation from the true values is about 25% and 62% respectively. For big lorries the exact model vehicle gives results less than 40% (for short bridges) to 2-5% (for long bridges). But if we take into account that a significant factor in the design of a long bridge is economy, then it is necessary to use the exact biaxial model.

2. *Regarding the type of irregularity*:

The one of type I, gives an increment of the dynamic deflection of a bridge from 9 to 52% for light vehicles, but from 0 to 5% for the big lorries. The irregularities of type II give minor rates of the increment on the dynamic deflection of a bridge from 2 to 12% for light vehicles, but from 0 to 2% for the big lorries.

3. *Regarding the position of the irregularity*:

The existence of an irregularity at the beginning of a bridge gives more unfavorable results, with significant difference (it doubles the influence), while the existence of an irregularity at positions 2L/8 to 4L/8 is, also, quite dangerous.

Therefore, the construction (for the reduction of the speed of a vehicle) or the accidental existence of an irregularity on the bridge, especially at its beginning, is the worst design (or case) of all.

Finally, we would like to point out that the short irregularities up to 1 m, have the most unfavorable influence. Therefore, we propose that the whole length of the irregularity be from 2 to 4 m. A longer length has the effect of two irregularities instead of one because the beginning and the end of the long irregularity act as two independent irregularities, especially for that of type I (where we have the impact phenomenon).

We, finally, note that the optimum length of an irregularity is connected with the particular characteristics of a bridge and, especially, with the specter of the eigenfrequencies.

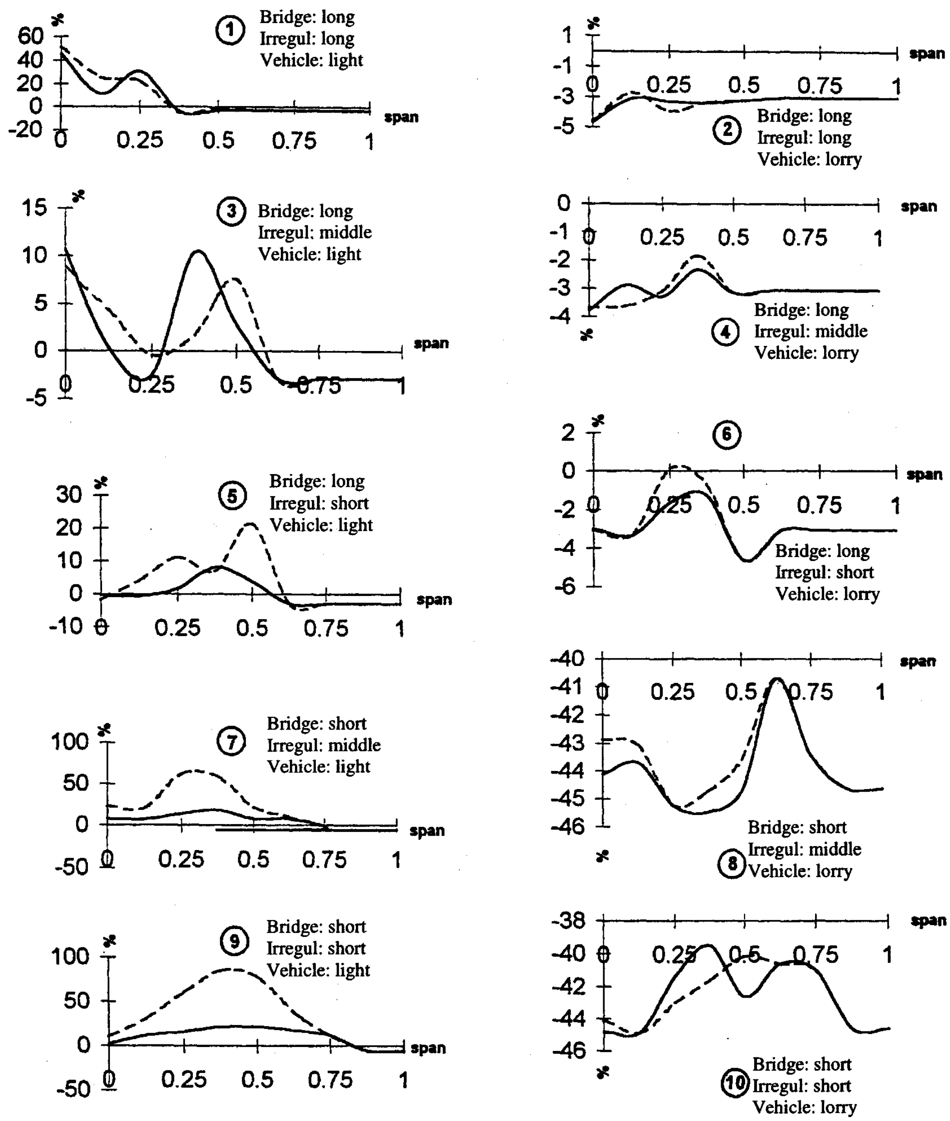

Fig. 9. Percentile comparison of the change of the displacement of the middle of the bridge for irregularity of type I.
______ $(W_{oo}-W_o) / W_o$ % biaxial model to one axis model
- - - - - $(W_{oo}-W) / W$ % biaxial model to without mass moving force

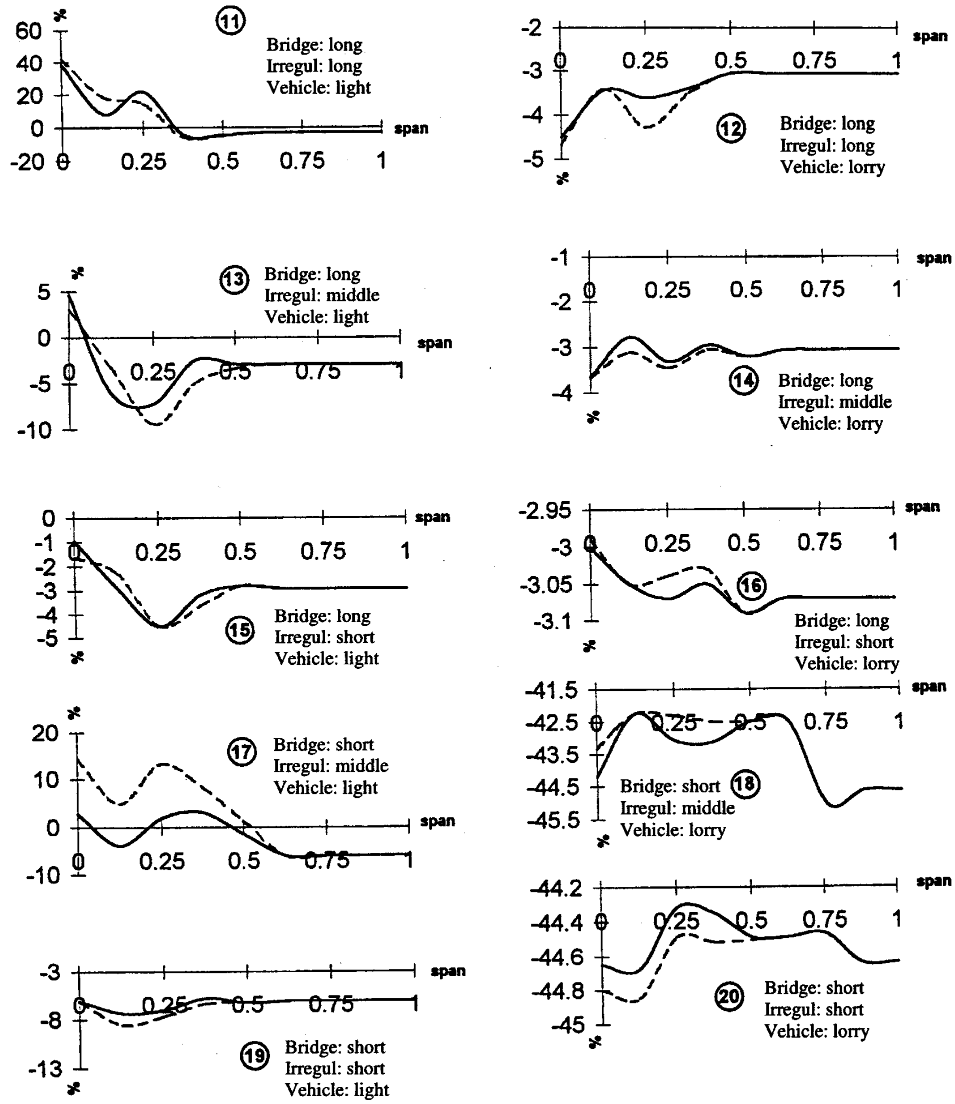

Fig. 10. Percentile comparison of the change of the displacement of the middle of the bridge for irregularity of type II.
______ $(W_{oo}-W_o) / W_o$ % biaxial model to one axis model
- - - - - $(W_{oo}-W) / W$ % biaxial model to without mass moving force

5. THE INFLUENCE OF THE MOVING LOAD'S MASS.

The problem of the forced motion of a beam, subjected to a moving load, is associated with serious difficulties when the effect of the mass of the load is taken into account. The problem becomes, mathematically, non-linear and therefore we are unable to give an exact and closed solution form.

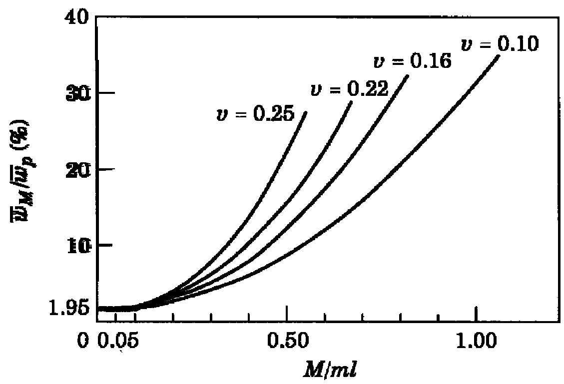

Fig. 11. The influence of the ratio $M/m\ell$ (for various velocities) on the ratio $\overline{w_M}/\overline{w_P}$ (%) of the dynamic middle-span deflections, where $\overline{w_M}$ includes the load-mass M while $\overline{w_P}$ does not.

In spite of the above difficulties there are some recent publications dealing with this problem. We can easily estimate this influence regarding the plot of figure 11 (where the effects of Centripetal, Coriolis forces and of the mass rotatory inertia are neglected [18]).

The solution technique that is developed and applied in the laboratory of Dynamics of NTUA, consists in the use of a solution (as a first approximation) of the form which is produced by the movement of a simple load neglecting its mass influence. The convergence, as is proved in [23], is extremely rapid and then a second step of approach is not necessary.

5.1. The concentrated load-mass

We consider a beam that is traversed by a constant load P having mass M moving at constant velocity υ. Hence, its position α, from the left end of the beam, at any time t is equal to $\upsilon t(=a)$, where t is measured from the instant the load enters the span. Before that instant, the deflection throughout the length of the beam is assumed to be zero. Continuing we adopt the assumptions of the theory of small vibrations, Navier's hypothesis and Saint-Venant's principle.

Under these assumptions one can establish the governing differential equation of motion for a slender beam, after neglecting the effects of longitudinal motion and damping [21]:

$$EIw''''(x,t) + m\ddot{w}(x,t) = \{Mg - M[\ddot{w}(x,t) + \upsilon^2 w''(x,t) + 2\upsilon \dot{w}'(x,t)]\}\delta(x-\alpha) \qquad (10)$$

where $0 \le t \le \ell/\upsilon$.

From the solution of the above equation (10), and after some manipulations, one can find dimensionless forms and study the individual and coupling effects of Centripetal and Coriolis forces, combined with various parameters such as load-mass and velocity, on the dynamic response of a single-span bridge. The results of such an investigation [21] are shown in plots of figure 12.

On the basis of the model chosen, one may draw the following conclusions:

1. The effect of Centripetal and Coriolis forces on the dimensionless dynamic mid-span deflection increases with the increase of the dimensionless mass $\overline{M}(> 0.2)$ of the moving load. Such an increase which becomes more appreciable for $\overline{M} > 0.5$ and for small dimensionless velocities $\overline{\upsilon} < 0.30$ may reach 60%.

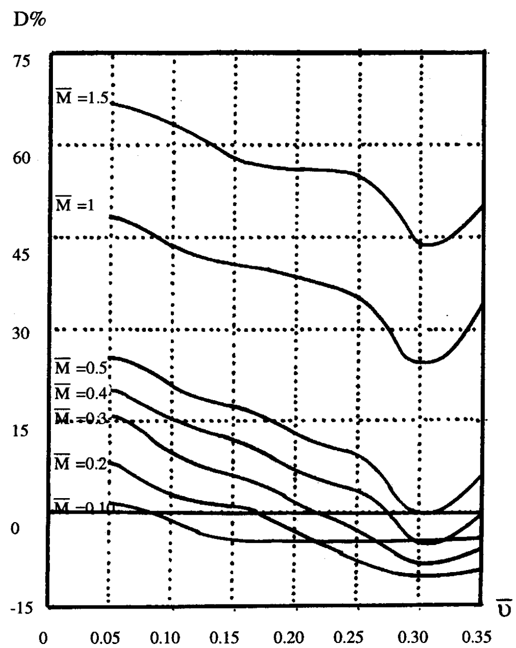

Fig. 12. Relationship between the ratio D = $(w_c - w_o) / w_o$ % and the dimensionless velocity $\overline{\upsilon}$ for various values of the dimensionless load-mass $\overline{M}$.

2. The increase of the dimensionless velocity $\overline{\upsilon}$ decreases the effect of the aforementioned forces (on the dimensionless mid-span deflection) up to $\overline{\upsilon} = 0.30$ while this effect increases for $\overline{\upsilon} > 0.30$. Such a decrease varies from 6% (for $\overline{M} = 0.1$) up to $\approx 25\%$ (for $\overline{M} = 1.5$).
3. All the curves D vs. $\overline{\upsilon}$ regardless of the value of $\overline{M}$ exhibit a minimum in the vicinity of $\overline{\upsilon} = 0.30$ (Fig.12).

5.2. The moving vehicle

Using the model of a locomotive like the one in figure 2i one can write the equation of motion of a single-span bridge, that is traversed by the above vehicle as follows:

$$\begin{aligned} EIw''''(x,t) + m\ddot{w}(x,t) &= \frac{M}{2}[\,g - \ddot{w}(x,t) - \upsilon^2 w''(x,t) - 2\upsilon\dot{w}'(x,t)]\,\delta(x-\alpha) \\ &\quad + \frac{M}{2}[\,g - \ddot{w}(x,t) - \upsilon^2 w''(x,t) - 2\upsilon\dot{w}'(x,t)]\,\delta(x-\alpha+2\,d) \\ &\quad + J\,\ddot{w}'(x,t)\,\delta'(x-\alpha+d) \end{aligned} \tag{11}$$

From the solution of the above equation (11), through the described technique, one can find dimensionless forms and study the individual and coupling effect of Centripetal and Coriolis forces and of the mass rotatory inertia of the vehicle, combined with various parameters.

The above problem is dealt in [22], in which the individual and coupling effects on the dynamic response of Centripetal and Coriolis forces combined with various parameters, such as vehicle-mass, distance of wheel axles (wheelbase) and velocity of the moving vehicle are discussed in detail.

The mathematical model discussed in [22] is related to a real bridge with a span of ~100 m, weight per unit length of ~40 KN/m and moment of inertia $I = \sim 1.50$ m^4, which is traversed by a moving load with velocities $\upsilon = 50$, 100, 150, and 200 km/h or in dimensionless form velocities $\overline{\upsilon} =$ 0.10, 0.20, 0.30, 0.40 respectively. The velocities are combined with the dimensionless values of vehicle-masses $\overline{M} =$ 0.10, 0.20, 0.30, 0.40, and 0.50 and the dimensionless wheelbases $2\overline{d} = 0.10$, 0.20, 0.30 and 0.40.

From the plots of Fig. 13 to 16 one can see graphically the relationship between the dimensionless wheelbases $2\overline{d}$ and the deflection ratios $D_1 = (WC-WCM)/WCM$ %, $D_2 = (WP-WCM)/WCM$ %, $D_3 = (W-WCM)/WCM$ %, in connection with the dimensionless vehicle-masses $\overline{M}$, where:

W: are the dimensionless deflections of the middle of the bridge, including the effect of Centripetal and Coriolis forces, for a moving load-mass $\overline{M}$.

WP: are the dimensionless deflections of the middle of the bridge for a vehicle-model with dimensionless mass $\overline{M}$ and dimensionless wheelbase $2\overline{d}$, in which the effects of Centripetal and Coriolis forces are neglected.

WC: are the dimensionless deflections of the above vehicle-model, in which the effects of Centripetal and Coriolis forces are included.

WCM: are the dimensionless deflections of the above vehicle-model, in which except for the effect of Centripetal and Coriolis forces the effect of the vehicle's mass rotatory inertia is also included. This last model is considered (for the present paper) as the real vehicle's model.

The above deflections concern the middle of the bridge (at $\xi = 0.5$).

From the above plots, we can easily get the following results:

c. The model of the single mass-load is accurate for values up to $\overline{M} = 0.20$. For higher values of $\overline{M}$ (even if the speeds are low) the differences from the real model are higher of 10% and we often meet values for the differences which amount from 25 to 40% (see ratio D_3).
b. The model of a vehicle with two axes and mass $\overline{M}$ is more accurate than the above model of one mass-load $\overline{M}$. In this two-axis model the influences of Centripetal and Coriolis forces are neglected. The differences from the exact model are high for low

speeds (independently of the value of wheelbase) amounting from about –8 to 15%, but they are low for higher speeds and masses amounting from about –4 to 5%.

c. The third model which neglects only the vehicle's mass rotatory inertia is the most accurate of the others for low values of wheelbase (up to $2\overline{d} = 0.20$) independent of speed and magnitude of mass. For wheelbases with values higher than $2\overline{d} = 0.20$, the influence of the neglected parameter J becomes very significant and then the differences from the deflections of the real model are from about 10 to 15%.

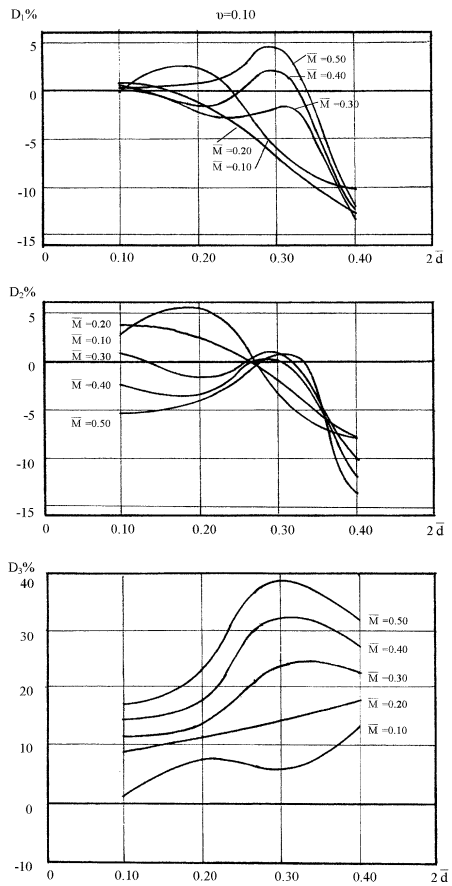

Fig. 13. Relationship between the ratios D_1, D_2, D_3 and the dimensionless wheelbase $2\overline{d}$ for $\overline{\upsilon} = 0.10$.

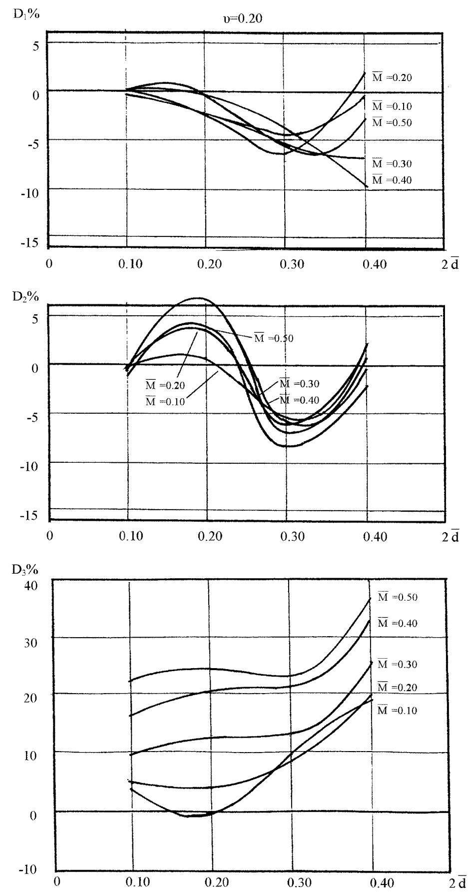

Fig. 14. Relationship between the ratios D_1, D_2, D_3 and the dimensionless wheelbase $2\overline{d}$ for $\overline{\upsilon} = 0.20$.

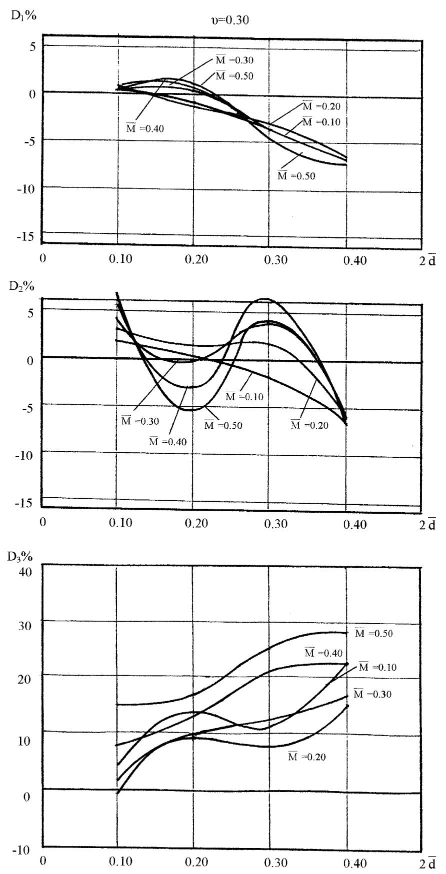

Fig. 15. Relationship between the ratios D_1, D_2, D_3 and the dimensionless wheelbase $2\bar{d}$ for $\bar{\upsilon} = 0.30$.

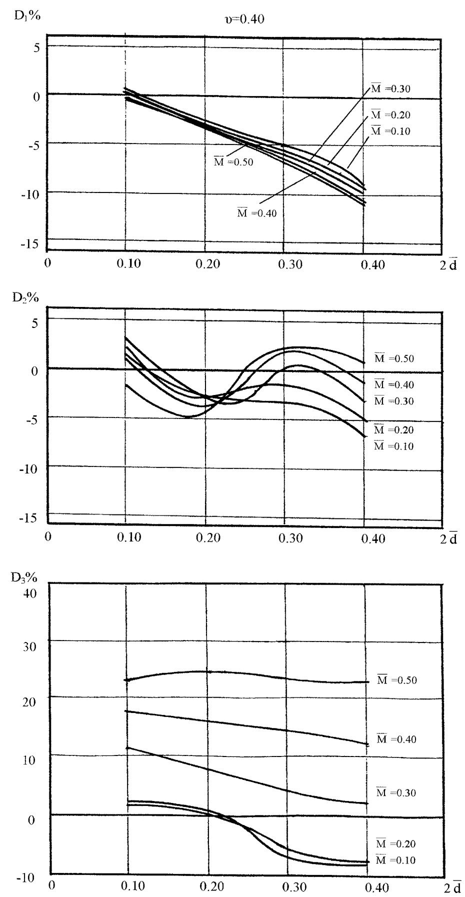

Fig. 16. Relationship between the ratios D_1, D_2, D_3 and the dimensionless wheelbase $2\bar{d}$ for $\bar{\upsilon} = 0.40$.

REFERENCES

1. Willis R. et al.: *Preliminary essay to the appendix B.: Experiments for determining the effects produced by causing weights to travel over bars with different velocities*. In: Grey G. et al.: Report of the Commissioners Appointed to inquire into the application of iron to railway structures. W.Clowes and Sons, London 1849. Reprinted in: Barlow P.: Treatise on the strength of Timber, Cast Iron and Malleable Iron. London 1851.
2. Stokes G.: *Discussion of a differential equation relating to the breaking of railway bridges*. Trans. Cambridge Philosoph. Soc., **8** (1849), Part 5, 707-735. Reprinted in: Mathematical and Physical Papers, **2** (1883), 178 to 220.
3. Zimmermann H.: *Die Schwingungen eines Tragers mit bewegter Lasts*. Centralblatt der Bauverwaltung, **16** (1896), No. 23, 249-251; No. 23, 249-251; No. 23 A, 257-260; No. 24, 264-266; No. 26, 288.
4. Krylov A. N.: *Mathematical Collection of Papers of the Academy of Sciences*, Vol. 61. (Matematischeskii sbornik Akademii Nauk). Peterburg 1905. Kriloff A. N.: Uber die erzwungenen Schwingungen von gleichformigen elastischen Staben. Mathematische Annalen, **61** (1905), 211.

5. Timoshenko S.P.: *Forced Vibration of Prismatic Bars* (in Russian). Izvestiya Kievskogo politekhnicheskogo instituta, 1908. Also in German: Erzwungene Schwingungen prismatischer Stabe. Zeitsch. f. Mathematik u. Physik, 59 (1911), No. 2, 163-203.
6. Lowan A. N.: *On transverse oscillations of beams under the action of moving variable loads*. Philosophical magazine, ser. 7., 19 (1935), No. 127, 708-715.
7. Steuding H.: *Die Schwingungen von tragern bei bewegten lasten*. I, II. Ingenieur-Archiv, **5** (1934), No. 4, 275-305, **6** (1935), No. 4, 265-270.
8. Schallenkamp A.: *Schwingungen von tragern bei bewegten lasten*. Ingenieur-Archiv, **8** (1937), 182-198.
9. Bolotin V.V.: *Problem of bridge vibration under the action of a moving load* (in Russian). Izvestiya AN SSSR, Mekhanika I mashinostroenie, 1961, No. 4, 109-115.
10. Inglis C.E.: *A mathematical treatise on vibration in railway bridges*. The University Press, Cambridge, 1934.
11. Hillerborg A.: *Dynamic influences of smoothly running loads on simply loads on simply supported girders*. Kungl. Tekn. Hogskolan, Stockholm 1951.
12. Biggs J.M., Suer H.S., Louw J.M.: *Vibration on simple-span highway bridges*. Trans. Amer. Soc. Civ. Engrs., 124 (1959), No. 2979, 291-318.
13. Timoshenko S.P.: *History of the strength of materials*. D. van Nostrand Co., New York 1953.
14. Kolousek V.: *Dynamics of civil engineering structures*. Part I-General problems, second eidition; Part II-Continuous beams and frame systems, second edition; Part III-Selected topics (in Czech.) SNTL, Prague 1967, 1956, 1961.
15. Fryba L.: *Vibration of solids and structures under moving loads*. Research Institute of Transport, Prague (1972).
16. Karaolides Ch. and Kounadis A.N. (1983), *Forced motion of a simple frame subjected to a moving force*, J. of Sound and Vibration, 88, pp. 37-45.
17. Sophianopoulos D.S. and Kounadis A.N. (1989), *The Axial Motion Effect on the Dynamic Response of a Laterally Vibrating Frame Subject to a Moving Load*, Acta Mechanica, 79, pp. 277-294.
18. Michaltsos G.T, Sophianopoulos D. and Kounadis A.N, (1996), *The effect of a moving mass and other parameters on the dynamic response of a simply supported beam*, J. of Sound and Vibration, 191, pp. 357-362.
19. Stavridis L.T. and Michaltsos G.T, (1999), *Eigenfrequency Analysis of Thin-Walled girders, curved in plan*, J. of Sound and Vibration, 227(2), pp.383-396
20. Michaltsos G.T. and Konstantakopoulos T.G, (2000), *Dynamic response of a bridge with surface deck irregularities*, J. of Vibration and Control, (in printing).
21. Michaltsos G.T. and Kounadis A.N. *The Effects of Centripetal and Coriolis Forces on the dynamic response of light bridges under moving loads*, submitted for publication.
22. Michaltsos G.T. *The influence of Centripetal and Coriolis forces on the dynamic response of light bridges under moving vehicles*, submitted for publication.
23. Kounadis A. *A very efficient approximate method for solving nonlinear boundary-value problems*, Scientific Papers of NTUA, 9, (3,4), pp.1-10

PARAMETRI KOJI UTIČU NA DINAMIČKI ODZIV LAKIH (ČELIČNIH) MOSTOVA

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U prvom delu dat je pregled istorijskog razvoja oblasti. Zatim su razmotreni najznačajniji parametri kao i teškoće koje se javljaju pri stvarnim ispitivanjima. Najznačajniji parametri su klasifikovani u dve velike kategorije: oni koji su u vezi sa projektovanjem, oblikom mosta i karakteristikama vozila i koji se mogu nazvati fizičkim parametrima i oni koji zavise od prihvatanja i od potrebnih uprošćenja povezanih sa poteškoćama koje se susreću uvek pri formiranju matematičkog modela problema (nazivaju se projektnim parametrima). Obe kategorije parametara su detaljno objašnjene. Karakteristični primeri su prikazani za najnovija istraživanja u laboratoriji Steel Bridge NTUA.