UNIVERSITY OF NIŠ
The scientific journal **FACTA UNIVERSITATIS**
Series: **Mechanics, Automatic Control and Robotics** Vol.2, No 10, 2000 pp. 1219 - 1232
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http:// ni.ac.yu/Facta

ON THE FLUID FLOW OVER A COMPLIANT WALL

UDC 532.517(045)

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Abstract. *In this reviewing paper we elucidate two aspects of fluid flow phenomena occuring as a result of strong interaction with flexible walls. The first one is concerned with the possibility of flow control through the supression of Tollmien-Schlichting instabilities, and the reduction of body drag. For that reason the appearance of divergence instability and travelling-wave flutter on the surface of a flexible coating under the effect of fluid flow is discussed. The second aspect is concerned with the flow through collapsible channels, pertinent to blood flow in arteries and pulmonary flow in human airways. Steady, lubrication type flow and boundary layer type flow, interacting with flexible walls which elastic behavior is modeled via von Karman geometrically nonlinear shell theory are treated into some details.*

1. INTRODUCTION

Since the Benjamin's (1960) seminal paper which inagurated the theoretical study of the effects of wall compliance on hydrodynamic stability, much attention has been paid in literature to the problem of interaction between the fluid flow and a flexible boundary. This problem is particularly theoretically attractive and practically important if the interaction is strong in the sense that neither of the two problems involved, fluid flow and wall elasticity behavior can be solved separately. The material of this reviewing paper will be devided into two parts.

In the first part interactions between the fluid flow and the elastic wall will be considered from the point of view of flow control. It will be demonstrated how the presence of a flexible wall on a part of the flow boundary may supress the growth of Tollmien-Schlichting waves, thus postponing laminar-turbulent transition and reducing the drag of the body. It will be also demonstrated how the stress exerted by the fluid on the wall may give rise to wall instabilities in the form of the divergence or the travelling-wave flutter, and how the latter phenomenon may enhance the transition. Since two opposite effects are obviously present in this strong interaction problem, an optimization of wall elastic properties can be performed for best transition delaying achievements by

Received August 15, 2000

using compliant walls made from viscoelastic materials for marine application.

The second part of the paper will be devoted to fluid flow through collapsible channels, such as blood vessels or pulmonary airways. Steady laminar flows will be included only, with a wide range of Reynolds number variations: lubrication type flow, boundary layer type flow (moderately high Reynolds number flow) in which the boundary layer occupies the whole cross section of the channel, and high Reynolds number flow in which the boundary layer is supressed to the immediate neighborhood of the channel walls, with the existence of an effectively inviscid rotational core flow. It is demonstrated how the careful scaling applied in studying these flows imposes the use of von Karman geometrically nonlinear shell theory, and how as a result of the strong fluid structure interaction in all the cases considered, a nonlinear integro-differential equation which describes the wall shape can be readily derived, and numerically integrated.

2. Flow control by compliant coatings

When a fluid flows over a body whose contour is on its part replaced by a flexible coating which may comply with the flow, a complex interaction between instabilities inherent in both fluid flow, known as Tollmien Schlichting (TS) waves, and flexible wall, known as static divergence (SD) and travelling-wave flutter (TWF), may appear, giving rise to either supression or enhancement of TS waves. Supression postpones transition process leading to less drag, while enhancement leads to turbulence augmentation - greater drag and intensified heat exchange between the body and the flow.

Research in this field of fluid mechanics was inspired by spectacular experiments by Kramer (1957), which were motivated by observations that dolphins are able to achieve very high speeds with amazing propulsion efficiency. Kramer designed his coatings in such a way as to match the basic mechanical properties of the dolphins skin, fitted them to the surface of an axisymmetric cylinder and towed the cylinder behind a boat in open sea. Measuring the drag he reported significant drag reductions for certain of the surfaces, and conjectured that the damping in the coatings was responsible for the probable supression of TS waves, thus delaying transition and reducing drag. Early attempts to solve such a problem theoretically from the point of view of hydrodynamic stability theory, made by Benjamin (1960), Landahl (1962) and Kaplan (1964), showed that very modest increase in the critical Reynolds number could be achieved by placing a flexible coating in the flow. Also, several further experiments revealed very controversial results concerning this point, some of them showing an appreciable increase in drag, so that Bushnell et al. (1977) hypothesized that the basic mechanism for reducing drag should be a modulation of the preburst flow in the boundary layer, rather than supression of TS waves. Suitable control of the pressure field in the boundary layer would inhibit burst formation by reducing the number of bursts occuring per unit time, and consequently lowering the skin friction drag.

In order to resolve existing controversies Gaster and Willis (s. Willis, 1986) undertook very careful and sofisticated laboratory experiments with the point on studying the effect of a flexible coating on the evolution of TS waves in a flat plate boundary layer. Some of their results are shown in Fig. 1 in which the amplification factor versus the free stream velocity is given for a range of modal frequencies. Dramatic decrease of instabilities in the case of a flexible coating in comparison with the rigid wall case is sound.

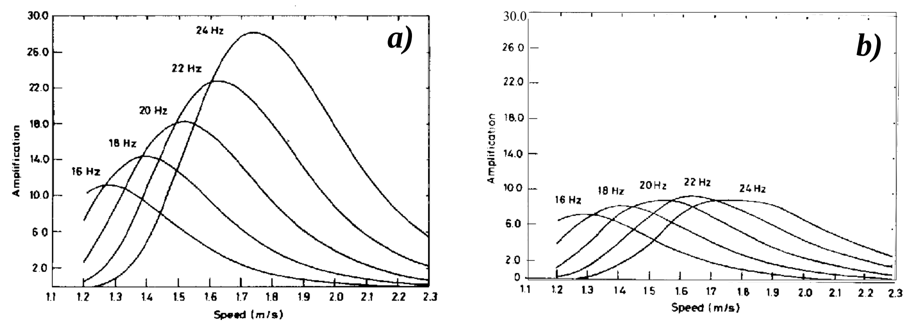

Fig. 1. The amplification factor versus the speed for various modal frequences, for (a) rigid wall and (b) flexible coating. Reproduced from Riley et al. (1988).

Progress made in development of efficient numerical techniques for the solution of stiff differential equations, and the recent advent of high speed computers has made the analytic approach to this problem much more feasible than before. The model problem extensively studied in the literature is depicted in Fig. 2. At that, like in Carpenter and Garard (1985), elastic properties of the coating are modeled by the spring backed elastic membrane or plate, which is a relatively simple model, but contains characteristics representative of a broad range of surfaces.

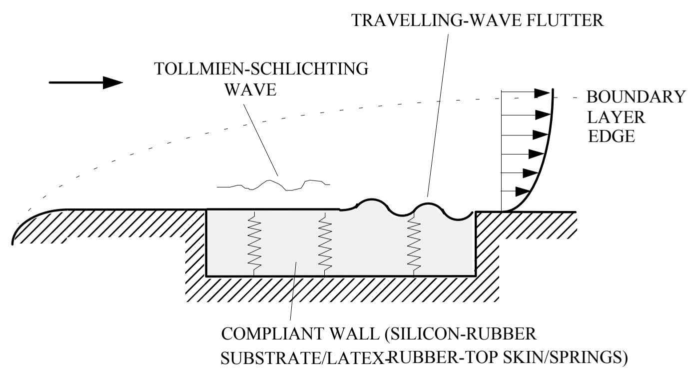

Fig. 2. Schematic of the considered problem.

If coordinate axis are chosen with the x axis lying along the undisturbed free surface and the y axis normal to this surface, then the equation for the y component of the momentum of the compliant coating is:

$$\frac{\partial^2\eta}{\partial t^2}+d\frac{\partial\eta}{\partial t}-\frac{T}{m}\frac{\partial^2\eta}{\partial x^2}+\frac{B}{m}\frac{\partial^4\eta}{\partial x^4}+\frac{k}{m}\eta=f \quad (1)$$

Here $\eta(x,t)$ is the y-displacement of the surface from its equilibrium state at time t and position x, T is the longitudinal tension and B the flexural rigidity of the plate, m is the mass per unit area, d is the damping coefficient, k is spring constant,and f is an external forcing term. This equation is coupled to the fluid flow through the forcing term f and the fluid boundary conditions at the wall. The forcing term is assumed to depend upon the fluid pressure $p(x,y,t)$ only, the effect of the shear stress being usually neglected, i.e. $f = (1/m)p(x,\eta, t)$ while the no slip boundary conditions read:

$$u(x,\eta,t) = 0\,,\ \ v(x,\eta,t) = \frac{\partial \eta}{\partial t}$$

where (u,v) are the x and y velocity components of the fluid flow, respectively, which together with the pressure satisfy the Navier-Stokes equations. The linear stability properties of this coupled system are studied by presenting the fluid velocity components in the form:

$$u(x,\eta,t) = U(y) + F'(y)\,e^{i\alpha(x-ct)}$$
$$v(x,\eta,t) = -i\alpha F(y)\,e^{i\alpha(x-ct)}\ .$$

Here $U(y)$ is the mean velocity, $F(y)$ is the stream function amplitude, α is the horizontal wave number, and c is the complex phase speed. Navier-Stokes equations after eliminating the pressure then yield the well known Orr-Sommerfeld equation:

$$F^{(IV)} - [2\alpha^2 - i\alpha R(U-c)]F'' + [\alpha^4 + i\alpha^3 R(U-c) + i\alpha R U'']F = 0\,, \tag{2}$$

where R is the Reynolds number. The appropriate boundary conditions are:

$$y \to \infty\ \ :\ \ F, F' \to 0\,,\ \text{and} \tag{3}$$

$$[U'(0)/c]F(0) + F'(0) = 0\,, \tag{4}$$

which are nothing but the linearized boundary condition derived from matching tangential velocities at the wall, and

$$-\frac{ic}{\alpha m R}F'''(0) + \frac{i\alpha c}{mR}F'(0) + \left(\frac{T}{m}\alpha^2 + \frac{B}{m}\alpha^4 - \alpha^2 c^2 - i\alpha dc + \frac{k}{m}\right)F(0) = 0\,. \tag{5}$$

The latter equation is derived from the linearized x momentum equation by assuming that, at the wall, the fluid pressure and the normal velocity component match their counterparts on the surface of the coating.

Equation (2) with homogeneous boundary conditions (3)–(5) represents an eigenvalue problem which can be now numerically readily solved, yielding the phase speeds and growth rates of modal solutions as functions of the governing parameters. It is found that it might be possible to use compliant coatings to reduce the drag by both increasing the critical Reynolds number and decreasing the growth rates of instabilities. For a coating to be effective in reducing drag, it should have a density similar to fluid density, be highly flexible and have low damping. Optimistic predictions concerned with the value of the critical Reynolds number show that this value can be duplicated with respect to its counterpart for a rigid wall.

However, the compliant wall itself is a wave bearing medium capable of supporting wall based waves which can develop into already mentioned two classes of hydroelastic instabilities: SD and TWF instabilities. SD is a slowly progressive instability, with a very low phase speed and zero group velocity. Thus, it is an absolute instability developing in both the upstream and downstream directions from the point of initiation. Among several other authors, it was thoroughly investigated in experiments conducted by Gad-el-Hak et al. (1984). In Fig. 3 large eddy structures in a turbulent boundary layer are presented for a rigid wall (a) and a flexible viscoelastic wall (b), made of plastisol gel. SD instability waves consisting of quite sharp peaks and broad and shallow valleys are clearly seen in Fig. 3*b*. Whenever they appear the height of the peaks was of the same order of magnitude as the depth of the gel, so that from this point of view SD instabilities seem to be highly influenced by nonlinear effects. Eddy structures above the flexible wall are obviously more intense than those above the rigid wall, because existing peaks act as roughness elements on the surface, with the consequence that SD waves lead to an increase of drag.

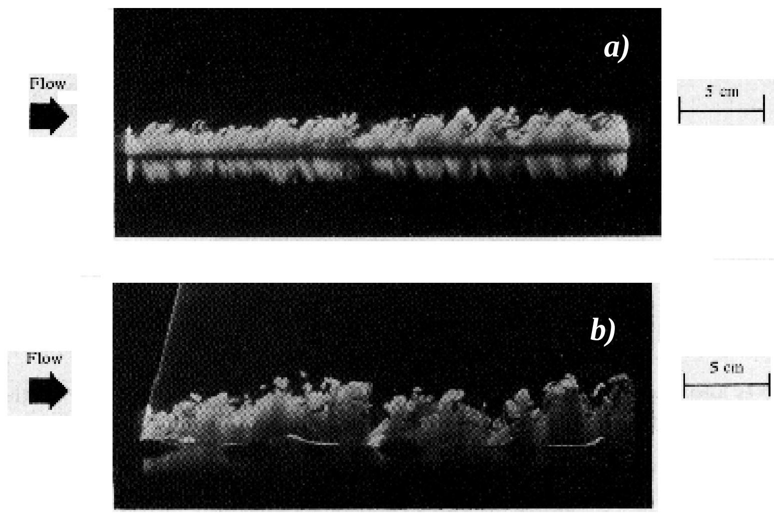

Fig. 3*a*, 3*b*. Large-eddy structures in a turbulent boundary layer over (a) rigid wall and (b) compliant wall with pronounced SD instabilities.
Reproduced from Gad-el-Hak et al. (1984).

TWF instabilities are characterized by relatively high phase speeds and group velocities, so that they are a convective instability, like TS waves. It is a very dangereous instability, because, under certain conditions, it may interact with TS waves and cause a very sudden onset of transition, observed in several experiments. Thus, both types of hydroelastic instabilities must be avoided in order to retain transition delaying capabilities of flexible coatings. In conclusion we may say that the flexible coating technique for delaying the onset of turbulence in the boundary layer flow is a promising mean for future applications in naval technology. Also, recent results indicate that the study of more complex surfaces, e.g. multi-layer and/or anisotropic ones, as well as the study of nonlinear and/or three dimensional effects may lead to new advances in flow stabilization.

3. FLUID FLOW THROUGH COLLAPSIBLE CHANNELS

Another aspect of fluid flow over compliant walls is referred to flow through collapsible channels which are of immense practical importance in medicine and bioengineering, because they are inevitably involved in the description of fluid flows in humans, like the blood flow in arteries and veins, the flow of urine in the urethra, and the flow of air in lung airways. An excellent review of the large body of these and other problems pertinent to so called physiological fluid mechanics can be found in Grotberg (1994) and in Ku (1997). A representative example of such a flow is sketched in Fig. 4 in which a part of a channel with rigid walls is replaced on its length l' by thin, flexible walls which may comply with the flow. If the channel is, at least at its downstream part, exposed to a positive transmural pressure (defined here as external minus internal pressure), being large enough, the channel may experience very large reduction of area followed by the correspondigly large transverse displacements of the channel walls, and if the channel is made up of an initially axisymmetric tube, the displacement in such a case may result in a nonaxisymmetric tube deformation, which is commonly identified as the tube collapse (for a brief review on the problems of flow in collapsible tubes, s. Kamm and Pedley, 1989). Depending on a realistic practical situation, the flow in a collapsible channel and the elastic behavior of its walls can be modeled in many different ways. The flow can be steady, and one of low Reynolds number flow, with neglected inertia, which may experience slow variations in the direction of flow (lubrication type flow), or the variations in the direction of flow can be arbitrary (Stokes flow). If the channel collapses, the inclination of the channel walls toward the channel axis can be so large that the Stokes flow should be given clear prevalence over the lubrication approximation. For high Reynolds numbers, inertia should be included into consideration so that, in general, full Navier-Stokes equations have to be employed, which makes the problem extremely complex and practically amenable to possible numerical analyses only. Elastic behavior of the channel walls is usually modelled either according to the membrane theory, with neglected bending stiffness, or according to the thin shell theory.

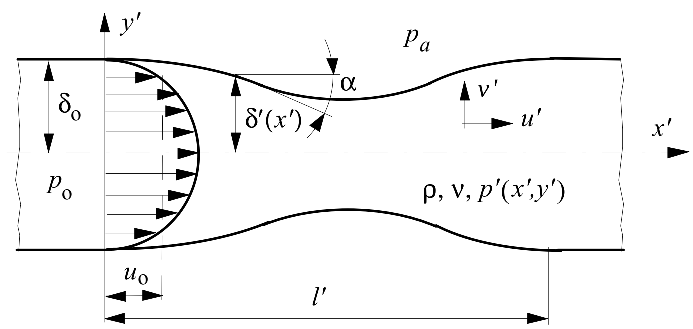

Fig. 4. Steady, viscous flow in a two-dimensional channel.

As clearly shown in several experiments (s. for example Conrad (1969)) steady flow in collapsible channels may break down under certain conditions. The breakdown is associated with two different reasons. One of them is the phenomenon of choking: steady flow simply ceases to exist for some values of control parameters, and another reason is that steady flow exists, but is unstable. Instabilities usually generate self-excited oscillations, and they are held responsible for the oscillations of the artery wall under an externally applied pressure cuff, heard during sphygmomanometry (Korotkoff sound), and for the wheezing experienced by some subjects during forced expiration.

Problems stated above have been until now the subject of study in numerous published papers, of which we will mention just a few. Shapiro (1977) treated steady, inviscid flow in collapsible tubes by a simple 1 - D model of flow. At that the elastic properties of the tube wall were modeled by an one-to-one empirical relation between cross-sectional area and transmural pressure, which was referred to in the literature as the "tube law" model. The results obtained are strongly based on the analogy between this flow and (a) the compressible gas flow in rigid, uniform tubes, or (b) the free surface liquid flow in uniform, horizontal channels. In Shapiro's theory the "tube law" serves as an "equation of state" that closes the system of governing equations. In Pedley (1992) lubrication theory and membrane theory were used for studying 2-D flow in a collapsible channel. Particular attention was paid to the effect that the longitudinal tension in the channel wall, which decreases downstream due to the viscous shear strees exerted by the fluid, and may fall to zero before the end of the channel, might have on the breakdown of stready flow. A hypothesis was set that the breakdown of steady flow was directly attributed to the drop to zero of the longitudinal tension. This hypothesis was abandoned later in Luo and Pedley (1996) who treated numerically unsteady flow in collapsible channels by full Navier-Stokes equations. They find that the breakdown of steday flow appears as a consequence of its instability, and that instabilities set up when the tension falls below a certain value which decreases with the Reynolds number. Also, instabilities give rise to self-exciting oscillations which become increasingly complicated as the tension is decreased from its critical value. Purely steady flow in collapsible channels was investigated by using membrane theory in Lowe and Pedley (1995), combined with Stokes equations to model the fluid flow, and in Luo and Pedley (1995), combined with full Navier-Stokes equations. In both papers the approach was entirely numerical, with finite element method applied. In the first of them the effect of control parameters, in particular of volume flow rate and upstream transmural pressure, on the wall configuration was analyzed, and it was found that the wall of the channel could be: inflated everywhere for negative and some low positive transmural pressures inflated upstream and collapsed downstream for some intermediate positive transmural pressures and collapsed everywhere for sufficiently large positive values of transmural pressure. In the second one the analysis was enriched by including inertia, so that even separated flows in collapsible channels and their effect on the wall slope could be encountered.

A more sophisticated modelling of the wall elasticity behavior was performed by Heil and Pedley (1995) and Heil (1997), where the collapsible tube was modelled according to the geometrically nonlinear shell theory. In both papers the finite element method was used for studying this large-displacement fluid-structure interaction problem. In the first paper fluid flow was modeled using lubrication theory, and only axisymmetric deformations were investigated, while in the second one 3-D Stokes flow model was used, combined with non-axisymmetric tube deformations. Numerical results obtained in the second paper were supported by experiments. It was also shown a little bit surprisingly that the results obtained by lubrication theory and by full Stokes equations differed very slightly, which spoke in favor of the computationally much less expensive lubrication theory in treating low Reynolds number flow through collapsible tubes.

Here, we wish to elucidate a couple of characteristic cases of steady flow in collapsible channels, in particular the low Reynolds number lubrication type flow, and moderately high Reynolds number boundary layer type flow. In the latter one the boundary layer spreads over entire cross section of the channel so that its centerline plays the role of the free stream boundary. Equations governing the flow sketched in Fig. 4 are Navier-Stokes equations in x' and y' directions, and continuity equation. They will be written in non dimensional form by using δ_o, u_o and p_o as the scales for all lengths, velocities and pressure, respectively. For a steady, 2-D flow they read:

$$
\begin{aligned}
u\frac{\partial u}{\partial x}+\upsilon\frac{\partial u}{\partial y}&=\mathrm{Eu}\frac{\partial p}{\partial x}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)\\
u\frac{\partial v}{\partial x}+\upsilon\frac{\partial v}{\partial y}&=\mathrm{Eu}\frac{\partial p}{\partial y}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 \upsilon}{\partial x^2}+\frac{\partial^2 \upsilon}{\partial y^2}\right)\\
\frac{\partial u}{\partial x}+\frac{\partial \upsilon}{\partial y}&=0\,.
\end{aligned}
\tag{6}
$$

In here, $x = x'/\delta_o$, $u = u'/u_o$, etc., $\mathrm{Eu} = \frac{p_o}{\rho u_0^{\,2}}$ is the Euler number, $\mathrm{Re} = \frac{u_o\delta_o}{\nu}$ is the Reynolds number and p is the nondimensional transmural pressure defined as: $p(x,y) = (p_a - p'(x',y'))/p_o$. In order to simplify equ. (1) we will now make some assumptions which will be crucial for validity of the theory to follow. We will assume that the collapsible segment of the channel is relatively long, i.e. $\delta_o/l' \ll 1$, and that $\alpha_{\max} \ll 1$, both of these two small quantities being of the same order of magnitude, so that the channel changes its width slowly. It is to be expected in this case that all physical quantities related to fluid flow will also experience slow variations in the direction of flow and that transverse velocity component will be much smaller than longitudinal one throughout. In order to make these slow variations explicit we will introduce a slow coordinate $\xi = \varepsilon x$ and write the transverse velocity component in the form: $\upsilon = \varepsilon V(\xi,y)$, where $V = O(1)$. In here, $0 < \varepsilon \ll 1$ is a small parameter to be defined precisely later. It suffices to say for the moment that $\delta_o/l' = O(\varepsilon)$ and $\alpha_{\max} = O(\varepsilon)$, and that $\delta = \delta(\xi)$. Also we will assume that the channel walls are characterized by relatively small values of the elasticity modulus (say, they are made of rubber), and that consequently only the values of the transmural pressure much smaller than the reference pressure p_0, which is supposedly of the order of 10^5 Pa, can be transmitted through the wall. In order to make this statement explicit we will write: $p = \varepsilon^n P(\xi,y)$, $P = O(1)$, where the parameter $n > 0$ will be also specified later.

After introducing ξ, V and P into equ.(1) we get:

$$
\begin{aligned}
\varepsilon\left(u\frac{\partial u}{\partial \xi}+V\frac{\partial u}{\partial y}\right)&=\varepsilon^{n+1}\mathrm{Eu}\frac{\partial P}{\partial \xi}+\frac{1}{\mathrm{Re}}\left(\varepsilon^2\frac{\partial^2 u}{\partial \xi^2}+\frac{\partial^2 u}{\partial y^2}\right)\\
\varepsilon^2\left(u\frac{\partial V}{\partial \xi}+V\frac{\partial V}{\partial y}\right)&=\varepsilon^{n}\mathrm{Eu}\frac{\partial P}{\partial y}+\frac{\varepsilon}{\mathrm{Re}}\left(\varepsilon^2\frac{\partial^2 V}{\partial \xi^2}+\frac{\partial^2 V}{\partial y^2}\right)\\
\frac{\partial u}{\partial \xi}+\frac{\partial V}{\partial y}&=0.
\end{aligned}
\tag{7}
$$

Two characteristic cases of the dominant balance can now be extracted from (2). The first one, to be referred to in what follows by Case A, in which:

$$\frac{1}{\mathrm{Eu}} = \varepsilon^{n+1}, \quad \text{and} \quad \frac{1}{\mathrm{Re}} = \lambda = O(1), \tag{8}$$

and the second one, to be referred to in what follows by Case B, in which:

$$\frac{1}{\mathrm{Eu}} = \varepsilon^{n}, \quad \text{and} \quad \frac{1}{\mathrm{Re}} = \lambda\varepsilon = O(1)\,. \tag{9}$$

In Case A equations (7) become:

$$\begin{gathered} \varepsilon\left(u\frac{\partial u}{\partial \xi} + V\frac{\partial u}{\partial y}\right) = \frac{\partial P}{\partial \xi} + \lambda\frac{\partial^2 u}{\partial y^2} + O(\varepsilon^2) \\ \frac{\partial P}{\partial y} = O(\varepsilon^2) \\ \frac{\partial u}{\partial \xi} + \frac{\partial V}{\partial y} = 0, \end{gathered} \tag{10}$$

while in Case B they attain the form:

$$\begin{gathered} u\frac{\partial u}{\partial \xi} + V\frac{\partial u}{\partial y} = \frac{\partial P}{\partial \xi} + \lambda\frac{\partial^2 u}{\partial y^2} + O(\varepsilon^2) \\ \frac{\partial P}{\partial y} = O\,(\varepsilon^2) \\ \frac{\partial u}{\partial \xi} + \frac{\partial V}{\partial y} = 0\cdot \end{gathered} \tag{11}$$

In Case A Reynolds number is low: $\mathrm{Re} = O(1)$, and the first order equations which can be derived from (10) are typical equations of the lubrication type flow.

In Case B Reynolds number attains moderately high values, and the first order equations which can be derived from (6) are typical equations of the boundary layer type flow. In this case the boundary layer occupies the whole cross-section of the channel and, consequently, the effect of viscosity is spread over all the points in the flow field, centerline of the channel playing the role of the free-stream boundary. In both cases the boundary and the symmetry conditions to be satisfied by the flow field are:

$$\begin{gathered} y = 0: \quad \frac{\partial u}{\partial y} = \frac{\partial^3 u}{\partial y^3} = \ldots\ldots = 0\,, \quad V = 0 \\ y = \delta(\xi): \quad u = 0\,, V = 0\,. \end{gathered} \tag{12}$$

In the lubrication type flow the solutions of the first order equations can be readily found to be:

$$u = \frac{3}{2\delta}\left(1 - \frac{y^2}{\delta^2}\right), \quad P = P_o + 3\lambda\int_o^{\xi}\frac{d\xi}{\delta^3(\xi)}, \tag{13}$$

where P_o is the nondimensional transmural pressure at the entrance of the channel. Of course, $\delta(\xi)$ is not known beforehand, so that question concerned with the form of the channel cannot be answered before the equations describing elastic behavior of the channel walls are involved into the analysis.

In Case B, derived from (11), first order equations are:

$$u\frac{\partial u}{\partial \xi}+V\frac{\partial u}{\partial y}=\frac{dP}{d\xi}+\lambda\frac{\partial^2 u}{\partial y^2}$$
$$\frac{\partial u}{\partial \xi}+\frac{\partial V}{\partial y}=0 \qquad (14)$$

and represent typical boundary layer type equations. In comparison with the classical boundary layer theory, (s. Schlichting, 1979), however, the problem considered herein is characterized by an important difference. Since, as said already, centerline of the channel plays now the role of a free stream boundary, the boundary (symmetry) conditions at the centerline are not satisfied asymptotically, but on a finite distance $\delta(\xi)$ from the wall. This implies the application of an approximate method based mainly on the Karman Pohlhausen method, known from the classical boundary layer theory which contributed much to its early development (Schlichting, 1979). Following the procedure outlined in Djordjevic and Vukobratovic (2000), in which the velocity profile in the channel is approximated with a sixth order polynomial, one can readily derive the following expression for the pressure distribution inside the channel:

$$P=P_o+\frac{\lambda}{5}\int_0^{\xi}\frac{80+32g-105f}{f\,\delta^3}d\xi\,, \qquad (15)$$

where $f=1/u_e\delta$ and $g=\frac{\delta^2}{6\lambda}\frac{du_e}{d\xi}$ are parameters which can be determined from the momentum integral equation. As before, P_o is the nondimensional transmural pressure at the entrance of the channel and $u_e(\xi)$is the centerline velocity.

Elastic behavior of the channel walls is modeled according to the geometrically nonlinear Karman thin shell theory. It is shown in Djordjevic and Vukobratovic (2000) that the application of this theory is not a matter of taste. The theory is simply imposed by the properties of fluid flow in both cases considered, A and B. Equations expressing the balance of forces in the transverse and the longitudinal direction in nondimensional form will be:

$$\frac{E\varepsilon^4}{12(1-\sigma^2)}\left(\frac{h}{\delta_o}\right)^3\frac{d^4b}{d\xi^4}-\frac{E\varepsilon^4}{1-\sigma^2}\frac{h}{\delta_o}\left[\frac{dA}{d\xi}+\frac{1}{2}\left(\frac{db}{d\xi}\right)^2\right]+\varepsilon^n p_o P=0$$
$$\frac{E\varepsilon^5}{12(1-\sigma^2)}\left(\frac{h}{\delta_o}\right)^3\frac{d^2b}{d\xi^2}\frac{d^3b}{d\xi^3}+\frac{E\varepsilon^3}{1-\sigma^2}\frac{h}{\delta_o}\left[\frac{dA}{d\xi}+\frac{1}{2}\left(\frac{db}{d\xi}\right)^2\right]+O(\varepsilon^{n+1}p_o)=0\,, \qquad (16)$$

where $b = 1 - \delta$ is the transverse displacement of the shell, A is an order one longitudinal displacement, E is the elasticity modulus, σ is Poisson's ratio, and $h \ll \delta_o$ is the wall thickness. Obviously, the dominant balance in the first of equations (16) takes place between the second and the third term. Thus, we choose:

$$\varepsilon^n p_o = \frac{E\varepsilon^4}{1-\sigma^2}\frac{h}{\delta_o} \ .$$

This relation, together with either (8) in Case A, or (9) in Case B, serves for the final definition of ε and n. Then, the second term will be the dominant term in the second of equation (16), so that its first order solution is:

$$\frac{dA}{d\xi} + \frac{1}{2}\left(\frac{db}{d\xi}\right)^2 = K = const., \tag{17}$$

which means that the longitudinal tension is constant. However, it cannot be arbitrarily chosen, like in Lowe and Pedley (1995) and Luo and Pedley (1966). It is determined from the governing equations by numerically performed iterations, as will be seen later, together with all the other physical quantities. Equation (17) can be formally integrated between $\xi = 0$ and $L = \varepsilon l'/\delta_o$. At that, if the shell is not previously streched out, so that $A(0) = A(L) = 0$, we obtain:

$$\int_0^L \left(\frac{db}{d\xi}\right)^2 d\xi = 2KL \ . \tag{18}$$

Finally, equations (13), and the first of (16) can be combined to yield a single integro-differential equation for δ in Case A:

$$m_o^2\delta^{(IV)} - m\delta'' + m_1\left(m_2 + \int_0^t \frac{dt}{\delta_3(t)}\right) = 0 \ , \tag{19}$$

where $m_o = h/\delta_o$, $m = 12KL^2$, $m_1 = 36\lambda L^5$, $m_2 = \frac{P_o}{3\lambda L}$ and $t = \xi/L$, while the condition (18) is transformed into:

$$6\int_0^1 \delta'^2\, dt = m \ .$$

Boundary conditions are:

$$\delta(0) = \delta(1) = 1 \, , \ \delta'(0) = \delta'(1) = 0 \ .$$

In Case B a similar integro differential equation is supplemented by two ordinary nonlinear differential equations for the parameters f and g, and two extra boundary conditions for a Poiseuille's velocity profile at the entrance of the channel:

$$f(0) = \frac{2}{3} \ \ , \ g(0) = 0 \ .$$

Equation (19) with the condition (20) was integrated by finite differences, and the results are stated in Fig. 5. In Fig. 5*a* and 5*b* the shell is "thin" ($m_o = 0.05$). For L fixed increase of m_1, means increase in λ and decrease in $\dot{V}$ (note that $\lambda = 2\upsilon/\dot{V}$, where $\dot{V} = 2\delta_o u_o$, which follows from (8)). At that for m_2 fixed P_o increases also, which leads to larger displacements of channel walls in Fig. 5*b*, than in Fig. 5*a*. In Fig. 5*c* the shell is "thick" ($m_o = 0.15$), but for the same values of other control parameters displacements of the channel walls are only slightly smaller (cf. Fig. 5*b* and Fig. 5*c*). All curves evaluated in this case are "smooth" in the sense that no wavy displacements of channel walls are detected. Such a wavy behavior of a collapsible tube with lubrication type flow was revealed in Heil and Pedley (1995), but never confirmed experimentally.

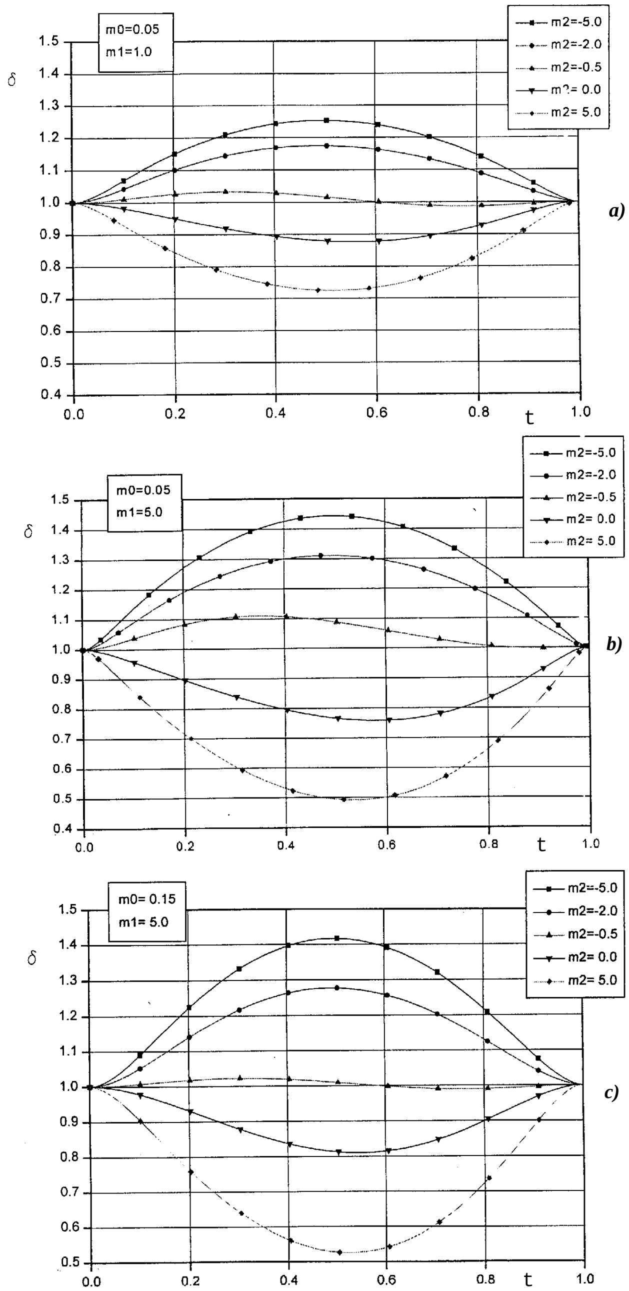

Fig. 5. Wall configuration in the lubrication type flow for different values of control parameters.

In flows with higher values of the Reynolds number the position of the boundary layer will be restricted to the immediate neighborhood of channel walls with an effectively inviscid, but rotational core. It is our opinion that this problem is amenable to same analytic methods, and from this point of view it is very chalenging for a possible future work. Also, instabilities in the form of self exciting oscillations, studied purely numerically by full Navier-Stokes equations in Luo and Pedley (1995) seem to be tractable, at least to a certain degree, analytically in terms of normal modes, by classical methods of hydrodynamic and hydroelastic stability theory (s. Davies and Carpenter, 1997).

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O STRUJANJU FLUIDA PREKO PODATLJIVIH POVRŠINA

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U ovome preglednom radu tretiraju se dva aspekta onih strujanja fluida koja nastaju kao rezultat jake interakcije sa fleksibilnim zidom. Prvi aspect se odnosi na mogućnosti upravljanja strujanjem fluida preko potiskivanja Tolmin-Šlihtingovih nestabilnosti i smanjivanja otpora tela. U tu svrhu diskutuju se pojava divergencije i flatera u vidu putujećeg talasa, pod uticajem strujanja fluida. Drugi aspect se odnosi na strujanje fluida kroz kolapsibilne kanale, a u vezi sa strujanjem krvi kroz arterije i strujanjem vazduha u plućima čoveka. Detaljno se diskutuju stacionarna strujanja tipa podmazivanja i tipa graničnog sloja u interactiji sa fleksibilnim zidovima čije se elastično ponašanje modelira pomoću Karmanove geometrijski nelinearne teorije ljuski.