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## PROBLEM OF ELASTICITY OF A SHRINK FIT BETWEEN AN ECCENTRIC CIRCULAR ANNULUS AND A SHAFT

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**Abstract.** *The paper treats the thermoelastic problem of a shrink fit between an eccentric circular annulus and a shaft. At ambient temperature, prior to heating and assembly an eccentric annulus has an outer radius  $R$ , inner radius  $r_1$  but the shaft's radius is by a value  $\delta_1$  greater than radius  $r_1$ . An eccentric annulus is homogeneously heated for a certain value of temperature  $\Delta T$  at which the eccentric annulus expands, and the inner radius becomes greater than the radius of the shaft. At this moment the eccentric annulus and the shaft are assembled. After cooling down to the ambient temperature, this assembled system represents a shrink fit. The stresses and displacements in the eccentric annulus and the shaft are determined according to Sherman's theory using complex functions. The results of solving some particular cases are presented in graphs.*

### 1. INTRODUCTION

In 1898 E. Goursat [1] showed that the general solution of the biharmonic differential equation can be expressed by two analytical functions  $\phi(z)$  and  $\psi(z)$  in the original complex plane ( $z$ ),  $z = x + iy$ . In 1909 Kolosov [2] proved that the element of the plane stress tensor  $\sigma_x$ ,  $\sigma_y$  and  $\tau_{xy}$  can be expressed by two Goursat's functions  $\phi(z)$  and  $\psi(z)$ . However, Kolosov did not determine the relationship between the functions  $\phi(z)$  and  $\psi(z)$  and the boundary conditions so they had to be determined by guessing. The relationship between the functions  $\phi(z)$  and  $\psi(z)$  and the boundary conditions was later determined by Muskhelishvili [3], in 1933, who implemented the Plemelj - Sochozki functions, [4], [5].

The complex functions  $\phi(z)$  and  $\psi(z)$  were first used for solving a shrink fit problem by D. I. Sherman [6] in 1938. The boundary conditions between the bodies which represent a shrink fit system were in this case fulfilled by using the first boundary problem. The objective of the present study is to determine the plane stress state tensor

and the displacement vector in an eccentric circular annulus and a shaft.

## 2. ASSUMPTIONS

To simplify the analytical treatment of a shrink fit between an eccentric circular annulus and a shaft, the process of achieving the shrink fit is divided into four phases, as follows:

### a) The initial state

The shaft radius is  $r_1 + \delta'_1$  and the eccentric circular annulus has an inner radius  $r_1$ , outer radius  $R$  and eccentricity  $b_1$ . Both, the shaft and the eccentric circular annulus have an initial temperature  $T_0$  which is constant.

### b) The intermediate phase

The eccentric circular annulus is heated by a temperature  $\Delta T$ , and the temperature field is kept uniform. Due to the uniform temperature field the initial shape of the annulus remains the same after the heating. The result of the heating is an increase of the inner radius of the eccentric circular annulus for  $\alpha \Delta T r_1$ , and the outer one for  $\alpha \Delta T R$ .

In the numerical simulation the temperature difference  $\Delta T$  is chosen at which the expression  $\alpha \Delta T r_1$  is equal to  $\delta'_1$ . At this temperature  $T_0 + \Delta T$  the eccentric circular annulus is simply assembled with the shaft without any interfacial stresses.

### c) The cooling phase

The cooling state depends on the intermediate phase, too. The second part of the intermediate phase which presents a nonstationary cooling process is not the object of study in this paper.

### d) Final phase: achieving the shrink fit

Finally, the shrink fit between the eccentric circular annulus and the shaft is achieved by cooling down to the initial temperature  $T_0$ .

## 3. THE STRESS STATES OF BODIES ASSEMBLED BY THE SHRINK FIT IN THE ISOTROPIC DOMAINS

Numerical evaluation of the stress states in the isotropic domains assembled by the shrink fit is based on the theory developed by Sherman, [6]. In the present paper it is assumed that all assembled bodies have the same elastical properties in the isotropic domains.

Let the isotropic domain  $S$  present a finite two-fold connected domain with an outer boundary  $L_0$  and an inner boundary  $L_0^*$ , Fig. 1.

The domain is assembled of  $p+2$  domains. It consists of multi-fold connected domain  $S_0$  with boundaries  $L = L_0 + L_1 + L_2 + \dots + L_{p+1}$ , of one-fold connected domains  $S_j$ ,  $j = 1, 2, \dots, p$ , that present shafts with radii  $r_j$  and their centers of circles  $b_j$  and of a two-fold connected domain  $S_{p+1}$  that presents an annular with an outer circular boundary with radius  $r_{p+1}$  and inner boundary  $L_0^*$ . The domains  $S_j$ ,  $j = 1, 2, \dots, p+1$  with overmeasures  $\delta'_j$

are inserted into domain  $S_0$  by the shrink fit. It is assumed that the vector of outer loads is zero on the boundaries  $L_0$  and  $L_0^*$ .

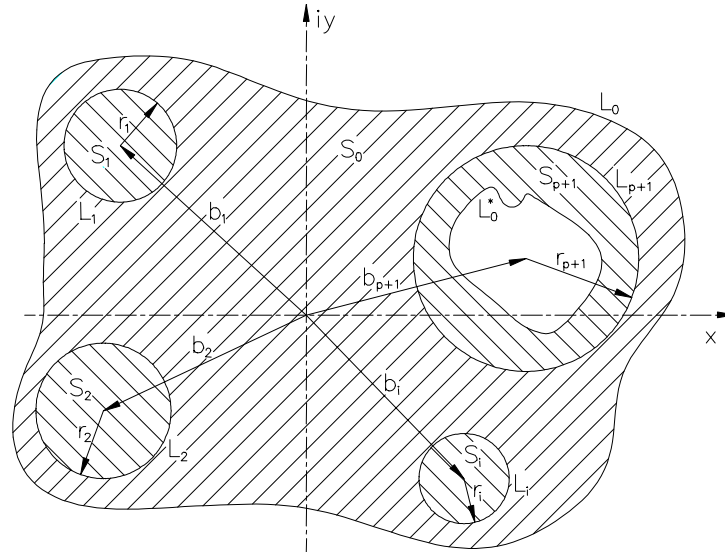


Fig. 1. Multi-fold connected domain  $S_0$  with shrink fits

According to Sherman's theory, the problem of determining the stress state in the shrink fit can be reduced to a determination of the functions  $\varphi_j(z)$  and  $\psi_j(z)$ ,  $j = 0, 1, 2, \dots, p+1$ , which are holomorphic functions in domains  $S_j$ . These functions can be determined by using the boundary conditions as follows:

On the boundary  $L_0$  of the multi-fold connected domain  $S_0$  and on the boundary  $L_0^*$  of the two-fold connected domain  $S_{p+1}$  the vectors of outer load are equal to zero. So, the equations of boundary conditions are, [6]:

$$\varphi_0(t) + t\overline{\varphi_0'(t)} + \overline{\psi_0(t)} = C_0 \quad \text{on } L_0 \quad (1)$$

$$\varphi_{p+1}(t) + t\overline{\varphi_{p+1}'(t)} + \overline{\psi_{p+1}(t)} = C_0^* \quad \text{on } L_0^* \quad (2)$$

where  $t$  is a point on the boundaries  $L_0, L_1, \dots, L_{p+1}$  and  $C_0, C_0^*$  are constants.

On the contact boundaries  $L_j$  between domain  $S_0$  and domains  $S_j$ ,  $j = 1, 2, \dots, p+1$ , the equilibrium law has to be fulfilled, [6]:

$$\varphi_0(t) + t\overline{\varphi_0'(t)} + \overline{\psi_0(t)} = \varphi_j(t) + t\overline{\varphi_j'(t)} + \overline{\psi_j(t)} \quad \text{on } L_j \quad (j = 1, 2, \dots, p+1) \quad (3)$$

On the contact boundaries  $L_j$ ,  $j = 1, 2, \dots, p+1$  the difference between the elements of displacement vectors of domains  $S_0$  and  $S_j$ ,  $j = 1, 2, \dots, p+1$  has to be equal to overmeasures  $\delta'_j$ ,  $j = 1, 2, \dots, p+1$ , [6]:

$$(u_o + iv_o) - (u_j + iv_j) = \delta'_j e^{i\varphi} \quad \text{on } L_j \quad (j = 1, 2, \dots, p+1) \quad (4)$$

or expressed by the complex functions  $\varphi_j(z)$  and  $\psi_j(z)$ ,  $j = 1, 2, \dots, p+1$ :

$$\chi\varphi_0(t) - \overline{t\varphi_0'(t)} - \overline{\psi_0(t)} - \chi\varphi_j(t) + \overline{t\varphi_j'(t)} + \overline{\psi_j(t)} = \frac{2\mu\delta_j'}{r_j}(t-b_j) \text{ on } L_j \ (j = 1, 2, \dots, p+1) \quad (5)$$

$$\text{where } \chi = \frac{3-\nu}{1+\nu}, \quad \mu = \frac{E}{2(1+\nu)}$$

and  $E$  is Young's modulus and  $\nu$  is Poisson's ratio

Equations (1-3) and Eq. (5) represent the basic equations from which can be determined the functions  $\varphi_j(z)$  and  $\psi_j(z)$ ,  $j = 1, 2, \dots, p+1$ . D. I. Sherman solved the problem of the shrink fit with the transformation to the first boundary problem. Using Eqs. (3) and (5) he obtained:

$$\varphi_0(t) = \varphi_j(t) + \frac{\delta_j}{1+\chi}(t-b_j) \text{ on } L_j \ (j = 1, 2, \dots, p+1) \quad (6)$$

$$\psi_0(t) = \psi_j(t) - \frac{\delta_j}{1+\chi} \left( \frac{2r_j^2}{t-b_j} + \overline{b_j} \right) \text{ on } L_j \ (j = 1, 2, \dots, p+1) \quad (7)$$

$$\text{where } \delta_j = \frac{2\mu\delta_j'}{r_j}$$

Introducing two holomorphic functions into domain  $S_0$ :

$$\varphi_*(z) = \varphi_0(z), \psi_*(z) = \psi_0(z) + \sum_{j=1}^{p+1} \frac{2\delta_j r_j^2}{1+\chi} \cdot \frac{1}{z-b_j} \quad (8)$$

and applying the Eqs. (6) and (7), these two functions can be expressed on boundaries  $L_j$ ,  $j = 1, 2, \dots, p+1$ :

$$\varphi_*(t) = \varphi_j(t) + \frac{\delta_j}{1+\chi}(t-b_j) \text{ on } L_j \ (j = 1, 2, \dots, p+1) \quad (9)$$

$$\psi_*(t) = \psi_j(t) - \frac{2\delta_j r_j^2}{1+\chi} \cdot \frac{1}{t-b_j} - \frac{\delta_j \overline{b_j}}{1+\chi} + \sum_{j=1}^{p+1} \frac{2\delta_j r_j^2}{1+\chi} \cdot \frac{1}{t-b_j} \text{ on } L_j \ (j = 1, 2, \dots, p+1) \quad (10)$$

Functions  $\varphi_*(t)$  and  $\psi_*(t)$  are uniformly continuous functions on the boundaries  $L_j$ ,  $j = 1, 2, \dots, p+1$ , and they present boundary functions of the holomorphic functions  $\varphi_*(z)$  and  $\psi_*(z)$  in domains  $S_j$ ,  $j = 1, 2, \dots, p+1$ . It is obvious that the complex functions  $\varphi_*(z)$  and  $\psi_*(z)$  are the holomorphic functions on the complete domain  $S$ .

The holomorphic functions  $\varphi_*(z)$  and  $\psi_*(z)$  have to fulfill the boundary conditions Eqs. (1) and (2) on the boundaries  $L_0$  and  $L_0^*$ , as follows:

$$\varphi_*(t) + \overline{t\varphi_*'(t)} + \overline{\psi_*(t)} = \sum_{j=1}^{p+1} \frac{2\delta_j r_j^2}{1+\chi} \cdot \frac{1}{\overline{t-b_j}} + C_0 \text{ on } L_0 \quad (11)$$

$$\varphi_*(t) + \overline{t\varphi_*'(t)} + \overline{\psi_*(t)} = \frac{2\delta_{p+1}}{1+\chi}(t-b_{p+1}) + \sum_{j=1}^p \frac{2\delta_j r_j^2}{1+\chi} \cdot \frac{1}{\overline{t-b_j}} + C_0^* \text{ on } L_0^* \quad (12)$$

The problem of the shrink fit is solved when the holomorphic functions  $\varphi_*(z)$  and

$\psi_*(z)$  are known. Using Eqs. (8-10), we get:

$$\varphi_0(z) = \varphi_*(z), \quad \psi_0(z) = \psi_*(z) - \sum_{j=1}^{p+1} \frac{2\delta_j r_j^2}{1+\chi} \cdot \frac{1}{z-b_j} \quad (13)$$

$$\varphi_j(z) = \varphi_0(z) - \frac{\delta_j}{1+\chi} (z-b_j) \quad (j = 1, 2, \dots, p+1) \quad (14)$$

$$\psi_j(z) = \psi_0(z) + \frac{2\delta_j r_j^2}{1+\chi} \cdot \frac{1}{z-b_j} + \frac{\delta_j \bar{b}_j}{1+\chi} \quad (j = 1, 2, \dots, p+1) \quad (15)$$

and after applying the functions  $\varphi_j(z)$  and  $\psi_j(z)$ ,  $j=0, 1, 2, \dots, p+1$  in the Kolosov's expressions [6], the stress states and elements of displacement vectors are determined:

$$\sigma_x + \sigma_y = 4 \cdot \text{Re}[\varphi_j'(z)] \quad (16)$$

$$\sigma_y - \sigma_x + 2i\tau_{xy} = 2[\bar{z}\varphi_j''(z) + \psi_j'(z)] \quad (17)$$

$$2\mu(u - iv) = \overline{\chi\varphi_j(z)} - \bar{z}\varphi_j'(z) - \psi_j(z) \quad (18)$$

#### 4. STRESS AND DISPLACEMENT STATE IN THE SHRINK FIT BETWEEN AN ECCENTRIC CIRCULAR ANNULUS AND A SHAFT

The shrink fit between an eccentric circular annulus and a shaft represents a one-fold connected domain. So, there is no boundary condition on the boundary  $L_0^*$ , Fig. 2.

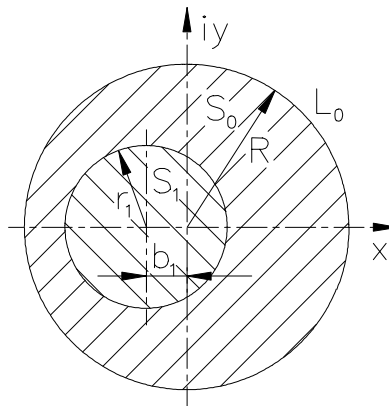


Fig. 2. A shrink fit between an eccentric circular annulus and a shaft  
Applying Eq. (11), the boundary condition on the boundary  $L_0$  can be written:

$$\varphi_*(t) + \overline{t\varphi_*(t)} + \overline{\psi_*(t)} = \frac{2\delta_1 r_1^2}{1+\chi} \cdot \frac{1}{\bar{t}-b_1} + C_0 \quad (19)$$

where constants  $\varphi_*(0)$  and  $\psi_*(0)$  must be equal to zero.

The holomorphic functions  $\varphi_*(z)$  and  $\psi_*(z)$  are determined by using Muskhelishvili's method, [3]. These functions are expressed with sums:

$$\varphi_*(z) = \sum_{k=1}^{\infty} a_k z^k \quad \psi_*(z) = \sum_{k=1}^{\infty} b_k z^k \quad (20)$$

The boundary condition Eq. (19) can be written also in the form:

$$\overline{\varphi_*(t) + t\varphi_*(t) + \psi_*(t)} = \frac{2\delta_1 r_1^2}{1+\chi} \cdot \frac{t}{R^2 - b_1 t} + C_0 \quad (21)$$

Let us now insert Eqs. (20) into Eq. (19). Multiplying Eq. (21) by the expression:

$$\frac{1}{2\pi i} \frac{dt}{t-z} \quad (22)$$

and integration Eq. (21) on the boundary  $L_0$  for the domain  $|z| < R$ , we get:

$$\begin{aligned} \frac{1}{2\pi i} \left[ \int_{L_0} \frac{\sum_{k=1}^{\infty} a_k t^k dt}{t-z} + \int_{L_0} \frac{\sum_{k=1}^{\infty} k \bar{a}_k R^{2(k-1)} t^{2-k} dt}{t-z} + \int_{L_0} \frac{\sum_{k=1}^{\infty} \bar{b}_k R^{2k} t^{-k} dt}{t-z} \right] = \\ = \frac{1}{2\pi i} \int_{L_0} \frac{2\delta_1 r_1^2}{1+\chi} \cdot \frac{tdt}{(R^2 - b_1 t)(t-z)} + \frac{C_0}{2\pi i} \int_{L_0} \frac{dt}{t-z} \end{aligned} \quad (23)$$

Using the rules for complex functions that are continuous functions on the boundary  $L_0$  and holomorphic functions in the domain  $|z| < R$ , Eq. (23) becomes:

$$\sum_{k=1}^{\infty} a_k z^k + \bar{a}_1 z + 2\bar{a}_2 R^2 = \frac{2\delta_1 r_1^2}{1+\chi} \cdot \frac{z}{R^2 - b_1 z} + C_0 \quad (24)$$

Multiplying the Eq. (21) by Eq. (22) and integrating it on the boundary  $L_0$  for the domain  $|z| > R$ , and applying the rules for complex functions that are continuous functions on the boundary  $L_0$  and holomorphic functions in the domain  $|z| > R$ , we get:

$$-\sum_{k=1}^{\infty} k \bar{a}_k R^{2(k-1)} z^{2-k} + \bar{a}_1 z + 2\bar{a}_2 R^2 - \sum_{k=1}^{\infty} \bar{b}_k R^{2k} z^{-k} = 0 \quad (25)$$

Equations (24) and (25) can be rearranged as:

$$\varphi_*(z) = -\bar{a}_1 z - 2\bar{a}_2 R^2 + \frac{2\delta_1 r_1^2}{1+\chi} \cdot \frac{z}{R^2 - b_1 z} + C_0 \quad (26)$$

$$\psi_*(z) = -\bar{z}\varphi'_*(z) + a_1 \bar{z} + 2a_2 R^2 \quad (27)$$

In Eq. (26), from which we get holomorphic function  $\varphi_*(z)$ , the expression  $z/(R^2 - b_1 z)$  has to be written in the form of a sum:

$$\frac{z}{R^2 - b_1 z} = \frac{1}{R^2} \left( z + \frac{b_1}{R^2} z^2 + \frac{b_1^2}{R^4} z^3 + \frac{b_1^3}{R^6} z^4 + \dots \right) \quad (28)$$

Having inserted Eqs. (20) and (28) into Eq. (26) we get the equation:

$$(a_1 z + a_2 z^2 + a_3 z^3 + \dots) + \bar{a}_1 z + 2\bar{a}_2 R^2 - C_0 = \frac{2\delta_1 r_1^2}{(1+\chi)R^2} \left( z + \frac{b_1}{R^2} z^2 + \frac{b_1^2}{R^4} z^3 + \dots \right) \quad (29)$$

By equalizing the coefficients at the same powers of complex variable  $z^k$ , a linear nonhomogeneous system of equations is obtained:

$$k = 0: \quad 2\bar{a}_2 R^2 - C_0 = 0$$

$$k = 1: \quad a_1 + \bar{a}_1 = \frac{2\delta_1 r_1^2}{(1+\chi)R^2}$$

$$k = 2: \quad a_2 = \frac{2\delta_1 r_1^2 b_1}{(1+\chi)R^4}$$

etc.

If in solving the system of equations it is considered that domain  $S$  is symmetrical to the axis  $x$ , ( $a_k = \bar{a}_k$ ), the holomorphic function  $\varphi_*(z)$  from the Eq. (20) becomes:

$$\varphi_*(z) = \frac{\delta_1 r_1^2}{1+\chi} \left[ \frac{2z}{R^2 - b_1 z} - \frac{z}{R^2} \right] \quad (30)$$

Applying the Eq. (30) in the Eq. (27), the holomorphic function  $\psi_*(z)$  is obtained:

$$\psi_*(z) = \frac{2\delta_1 r_1^2}{1+\chi} \left[ \frac{2b_1}{R^2} + \frac{1}{z} - \frac{R^4}{z(R^2 - b_1 z)^2} \right] \quad (31)$$

and with some rearrangments:

$$\psi_*(z) = -\frac{2\delta_1 r_1^2}{1+\chi} \left[ -\frac{2b_1}{R^2} + \frac{b_1}{R^2 - b_1 z} + \frac{R^2 b_1}{(R^2 - b_1 z)^2} \right] \quad (32)$$

Equations (30) and (32) lead us to the conclusion that functions  $\varphi_*(z)$  and  $\psi_*(z)$  are holomorphic functions in the domain  $S$ . To determine the stress state and vector of displacement of an eccentric circular annulus, applying Eqs. (13-15), it is possible to write:

$$\varphi_0(z) = \frac{\delta_1 r_1^2}{1+\chi} \left[ \frac{2z}{R^2 - b_1 z} - \frac{z}{R^2} \right] \quad (33)$$

$$\psi_0(z) = -\frac{2\delta_1 r_1^2}{1+\chi} \left[ -\frac{2b_1}{R^2} + \frac{b_1}{R^2 - b_1 z} + \frac{R^2 b_1}{(R^2 - b_1 z)^2} + \frac{1}{z - b_1} \right] \quad (34)$$

and for a shaft:

$$\varphi_1(z) = \frac{\delta_1 r_1^2}{1+\chi} \left[ \frac{2z}{R^2 - b_1 z} - \frac{z}{R^2} - \frac{z - b_1}{r_1^2} \right] \quad (35)$$

$$\psi_1(z) = -\frac{2\delta_1 r_1^2}{1+\chi} \left[ -\frac{2b_1}{R^2} + \frac{b_1}{R^2 - b_1 z} + \frac{R^2 b_1}{(R^2 - b_1 z)^2} - \frac{b_1}{2r_1^2} \right] \quad (36)$$

According to Eqs. (16-18), the elements of stress tensor and displacements vector in an eccentric circular annulus are:

$$\sigma_x + \sigma_y = \frac{4\delta_1 r_1^2}{1+\chi} \operatorname{Re} \left[ \frac{2R^2}{(R^2 - b_1 z)^2} - \frac{1}{R^2} \right] \quad (37)$$

$$\sigma_y - \sigma_x + 2i\tau_{xy} = \frac{4\delta_1 r_1^2}{1+\chi} \left[ \frac{2R^2 b_1}{(R^2 - b_1 z)^3} (\bar{z} - b_1) - \frac{b_1^2}{(R^2 - b_1 z)^2} + \frac{1}{(z - b_1)^2} \right] \quad (38)$$

$$u - iv = \frac{\delta_1 r_1}{1+\chi} \left[ \chi \left( \frac{2\bar{z}}{R^2 - b_1 \bar{z}} - \frac{\bar{z}}{R^2} \right) + \frac{2R^2}{(R^2 - b_1 z)^2} (b_1 - \bar{z}) + \frac{\bar{z}}{R^2} + \frac{2b_1}{R^2 - b_1 z} + \frac{2}{z - b_1} - \frac{4b_1}{R^2} \right] \quad (39)$$

and in the shaft:

$$\sigma_x + \sigma_y = \frac{4\delta_1 r_1^2}{1+\chi} \operatorname{Re} \left[ \frac{2R^2}{(R^2 - b_1 z)^2} - \frac{1}{R^2} - \frac{1}{r_1^2} \right] \quad (40)$$

$$\sigma_y - \sigma_x + 2i\tau_{xy} = \frac{4\delta_1 r_1^2 b_1}{(1+\chi)(R^2 - b_1 z)^2} \left[ \frac{2R^2(\bar{z} - b_1)}{R^2 - b_1 z} - b_1 \right] \quad (41)$$

$$u - iv = \frac{\delta_1 r_1}{1+\chi} \left[ \chi \left( \frac{2\bar{z}}{R^2 - b_1 \bar{z}} + \left( \frac{1}{\chi} - 1 \right) \left( \frac{\bar{z}}{R^2} + \frac{\bar{z} - b_1}{r_1^2} \right) \right) + \frac{2R^2}{(R^2 - b_1 z)^2} (b_1 - \bar{z}) + \frac{2b_1}{R^2 - b_1 z} - \frac{4b_1}{R^2} \right] \quad (42)$$

If in the Eqs. (37-42) it is chosen that the constant  $b_1 = 0$ , the equations for a shrink fit between the centric circular annulus and a shaft are obtained.

## 5. NUMERICAL RESULTS

In the continuation a numerical example is presented. Elements of the stress tensor are determined for an eccentric circular annulus with an outer radius  $R = 60$  mm, inner radius  $r_1 = 15$  mm and  $b_1 = -30$  mm. The overmeasure of the shaft is  $\delta_1 = 0.01$  mm. The



annulus and the shaft are made of steel with the Young's modulus  $E = 2,1 \cdot 10^5$  MPa and the Poisson's ratio  $\nu = 0.3$ . The results of the elements of the stress tensor in some points of domain of the eccentric annulus and the shaft are shown in Figs. 3-5.

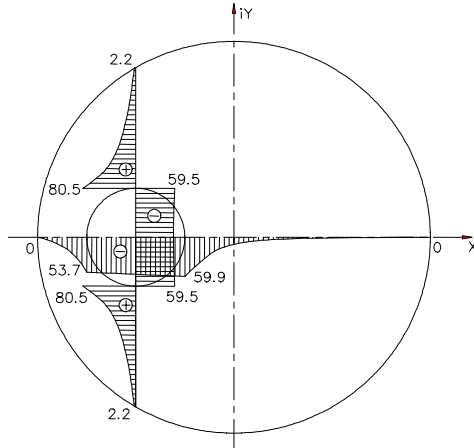


Fig. 3. Normal stress  $\sigma_x$  [MPa] in the shrink fit

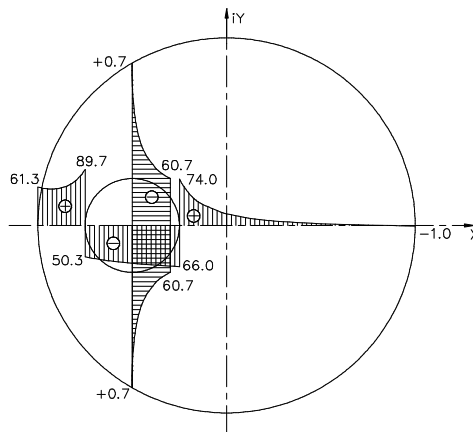


Fig. 4. Normal stress  $\sigma_y$  [MPa] in the shrink fit

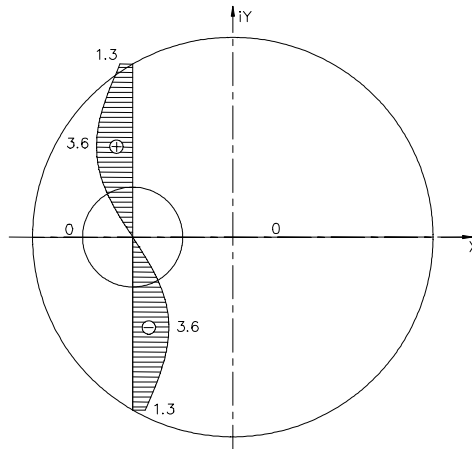


Fig. 5. Shear stress  $\tau_{xy}$  [MPa] in the shrink fit

#### REFERENCES

1. Goursat, E., *Sur l'équation  $\Delta\Delta u=0$* , Bull. Soc. Math., France, V. 26, 236, (1898).
2. Timoshenko, S., Goodier, J. N., *Theory of Elasticity*, p. 144. McGraw-Hill Book Company, Inc., New York, Toronto, London 1951.
3. Mushelišvili, N. I., *Nekotarie osnovnie zadači matematičeskoj teorii uprugosti*, 5th ed., p. 294. Izdatel'stvo "Nauka", Moskva 1966.

4. Plemelj, J., *Ein Ergänzungssatz zur Chauchyschen Integraldarstellung analytischer Funktionen, Randwerte betreffend.*, Monatsch., Math. Phys., XIX. Jahrgang, p. 205-210 (1908).
5. Plemelj, J., *Potentialtheoretische Untersuchungen*, p. 20-23. Leipzig 1911.
6. Ugodčikov, A. G., Dugač, M. I., and Stepanov, A. E., *Rešenje kraevih zadač ploskoj teorii uprugosti na cifrovih i analogovih mašinah*, p. 14-16., p. 344-351. Visšaja škola, Moskva 1970..

## PROBLEM ELASTIČNOSTI TESNOG SKLOPA EKSCENTRIČNOG KRUŽNOG OTVORA I VRATILA

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*Članak tretira problem termoelastičnosti tesnog sklopa između ekscentričnog kružnog otvora i vratila. U temperaturnom polju, prvo zagrevanja, a onda montaže jednog ekscentričnog otvora koji ima spoljašnji prečnik  $R$  i unutrašnji radijus  $r_1$ , ali radijus vratila je za veličinu  $\delta_1'$  veći od radijusa  $r_1$ .*

*Ekscentrični otvor je homogeno zagrevan za neku vrednost priraštaja temperature  $\Delta T$  pri kojoj se ekscentrični otvor širi i unutrašnji radijus postaje veći od radijusa vratila. U tom momentu ekscentrični otvor i vratilo su namontirani. Posle hlađenja, na nižoj temperaturi, namontirani sistem predstavlja tesni sklop. Naponi i pomeranja u ekscentričnom otvoru i vratilu su određeni u saglasnosti sa Sherman-ovom teorijom korišćenjem funkcija kompleksne promenljive.*

*Rezultati rešavanja nekih posebnih slučajeva su predstavljani grafički.*