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Address: Univerzitetski trg 2, 18000 Niš, YU, Tel: +381 18 547-095, Fax: +381 18 547-950
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ON MODELLING OF TRANSITION LAYERS IN MATRIX COMPOSITES REINFORCED BY COMPLEX INCLUSIONS

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Ludmila Kudrjavceva

Faculty of Mechanical Engineering, Kragujevac, Yugoslavia
E-mail: mmicun@knez.uis.kg.ac.yu

Abstract. *In the paper matrix composites reinforced by multilayered inclusions possessing transition layers bounding diverse phases are considered. For description of effective viscoelastic properties as well as of thermal stresses the model of self consistens field is applied.*

Key words: *self consistens field method, vibrodamping effect*

1 Introduction

Structure of matrix composites is characterized by a presence of a continuous phase (matrix polymer) as well as a disperse phase (filler) being a manifold of inclusions of diverse geometries. Examples of such systems are: ASA copolymer (a shock resistant system), numerous vibro damping compositions based on polymer mixtures, polyethylene mixtures and polypropylene reinforced by a mineral filler.

Features of mechanical behavior as well as whole spectrum of their physical properties is determined by the corresponding characteristics of their components as well as of topology of geometric microstructure of the considered composite. Component properties could be also considerably different from their properties in virgin state. Such a difference essentially appears by means of transition layers generated on interboundaries among diverse phases during synthesis of a composition (cf. [1, 2]).

Influence of the layer existence on behavior of the composition is practically negligible and could be ignored if

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- thickness of a transition layer is much smaller from average dimensions of particles as well as from their mutual distances and
- layer properties are inside the interval of component properties.

If anyone of them above conditions is not fulfilled, then influence of transition layers must not be neglected. In such a case, its size and features have essential influence on deformation behavior of the whole composition.

2 Analysis

Let us briefly outline the proposed way of modelling of effective viscoelastic characteristics of the composite reinforced by a manifold of quasispherical multilayered inclusions.

Suppose that the medium be isotropic with Lame parameters λ and μ which are piecewise smooth functions of r - the distance from the centre of the inclusion having spherical shape. If application of an external field to the medium is described by a constant strain tensor ε_0,then due to linearity of the field equations strain tensor in the medium is represented in the form:

$$\varepsilon_{\alpha\beta}(r,n) = \varepsilon_{0\alpha\beta}(r,n) + \varepsilon_{1\alpha\beta}(r,n), \quad \varepsilon_{1\alpha\beta}(r,n) = \mathbf{A}^{\lambda\mu}_{\alpha\beta}(r,n)\varepsilon_{0\alpha\beta}(r,n) \qquad (1)$$

Here r and n are spherical coordinates of a point x when origin is in the centre of the inhomogeneity having spherical form, $r = |x|$, $n = x/|x|$ is a vector on the unit sphere and the fourth rank tensor $\mathbf{A}(r,n)$ vanishes at the limit when $r \to \infty$. In the paper [3] it has been shown that this tensor has the following form:

$$\begin{aligned} \mathbf{A}(r,n) = {} & [E_1 + E_5(n)D]\,(5+D)\alpha_1(r) + [E_2 + E_4(n)D]\,\alpha_2(r) + \\ & + [E_2 + 2E_1 + (E_3(n) + E_4(n) + 4E_5(n))\,D + E_6(n)D(D-2)]\,(\alpha_3(r) - \alpha_1(r)). \end{aligned} \qquad (2)$$

In the above expression $D \equiv r\ \ d/dr$ is the differential operator, $\alpha_i(r)$ - scalar functions of r, where $i \in \{1,2,3\}$, $E_k(n)$ - elements of a tensor basis (with $k \in \{1,..,6\}$), necessary and sufficient for representation of fourth rank tensors having the explicit form ($\delta_{\alpha\beta}$ is Cronecker second rank tensor):

$$E_{1\ \alpha\beta\lambda\mu} = \frac{1}{2}(\delta_{\alpha\lambda}\delta_{\beta\mu} + \delta_{\alpha\mu}\delta_{\beta\lambda}), \qquad E_{2\ \alpha\beta\lambda\mu} = \delta_{\alpha\beta}\delta_{\lambda\mu}$$

$$E_{3\ \alpha\beta\lambda\mu} = \delta_{\alpha\beta}n_\lambda n_\mu, \qquad E_{4\ \alpha\beta\lambda\mu} = n_\alpha n_\beta \delta_{\lambda\mu},$$

$$E_{5\ \alpha\beta\lambda\mu} = \frac{1}{4}(\delta_{\alpha\lambda}n_\beta n_\mu + \delta_{\beta\lambda}n_\alpha n_\mu + \delta_{\alpha\mu}n_\beta n_\lambda + \delta_{\beta\mu}n_\alpha n_\lambda), \ E_{6\ \alpha\beta\lambda\mu} = n_\alpha n_\beta n_\lambda n_\mu.$$

The functions α_1, α_2 and α_3 in the relationship (2) fulfils a system of ordinary differential equations of fourth order. Such a system has the simplest form in the special case of the spherical layered inhomogeneity when Lame coefficients of an inclusion $\lambda(r), \mu(r)$ are piecewise constant functions with discontinuities at the points $r = a_i, i = 1, 2, ..., N;\ 0 < a_1 < a_2 < ... < a_N$. If this holds then the mentioned system of equations may be solved analyticially in the intervals $a_{i-1} < r < a_i$ $(i = 1, 2, ..., N+1;\ a_0 = 0;\ a_{N+1} \to \infty)$ such that the functions α_1, α_2 and α_3 take the following simplified form:

$$\begin{aligned} \alpha_1(r) &= Y_1^i + Y_2^i r^2 + Y_3^i r^{-3} + Y_4^i r^{-5}, \quad \alpha_2(r) = \beta(r) - (5 + D)\alpha_3(r) \\ \alpha_3(r) &= Y_5^i + Y_6^i r^2 + Y_7^i r^{-3} + Y_8^i r^{-5}, \quad \beta(r) = Y_9^i + Y_{10}^i r^{-3} \end{aligned} \tag{3}$$

Here Y_j^i are arbitrary constants. In this way, the solution inside each layer is determined with precision up to ten constants. Corresponding algorithm for determining all the constants for all the layers is reduced to solving an algebraic equation of fifth order with multiplication of known matrices (not larger than 10×10) whose number depends on the number of layers dressing the inclusion. It has been shown that such an algorithm is numerically stable and permits analysis of inclusions with unlimited number of layers [3, 5]. Moreover, such obtained solution of the problem of one spherically layered inhomogeneity is subsequently applied to construction of tensor of effective elastic moduli of the considered composite reinforced by random manifold of such inclusions. For such a procedure, the *self-consistent method of effective field* permits taking into account interactions inside manifold of inclusions immersed into a composite. The expression for the *tensor of effective elasticity moduli* of the composite with spherical layers is then obtained in the form:

$$c_* = K_* E_2 + 2\mu_*(E_1 - \frac{1}{3}E_2), \tag{4}$$

where the effective bulk modulus and shear modulus (K_*, μ_*) are given by the following relationships:

$$\begin{aligned} \frac{K_*}{K_0} &= 1 + 3p < q_1 > \left[1 - p < q_1 > \frac{3K_0}{3K_0 + 4\mu_0}\right]^{-1} \\ \frac{\mu_*}{\mu_0} &= 1 + p < q_2 > \left[1 - p < q_2 > \frac{6(K_0 + 2\mu_0)}{5(3K_0 + 4\mu_0)}\right]^{-1}, \end{aligned} \tag{5}$$

where K_0 and μ_0 are, respectively, bulk modulus and shear modulus of the matrix, p is the volume inclusion concentration, $< \cdot >$ is taken to show volume averaging operation, wheras q_1 and q_2 have the form:

$$q_1 = \frac{1}{3}\sum_{i=1}^{N} \frac{K_1^i}{K_0}(1 + Y_9^i)(\bar{a}_i^3 - \bar{a}_{i-1}^3), \quad \bar{a}_i = a_i/a_N,$$

$$q_2 = \sum_{i=1}^{N} \frac{\mu_1^i}{\mu_0}\left[1 + 3Y_1^i + 2Y_5^i)(\bar{a}_i^3 - \bar{a}_{i-1}^3) + \frac{7}{5}(3Y_2^i + 2Y_6^i)(\bar{a}_i^5 - \bar{a}_{i-1}^5)a_N^2\right].$$

Here K_1 and μ_1 are, respectively, bulk modulus and shear modulus of the inclusion while a denotes its radius.

An application of the proposed algorithm for solving of composite properties in the presence of tranisition layers among the matrix and its inlusions has been considered in papers [6, 7].

Let h be thickness of the transition layer, a - inclusion radius, E_h, ν_h and E_a, ν_a are, respectively, Young modulus and Poisson coefficient for layer and inclusion. For soft layers $h/a \ll 1$ and $E_h/E_a \ll 1$ with an allowed precision the finite size layer can be approximated by the corresponding *equivalent singular layer* according to the scheme explained in [7]. Approaching towards singular representation of soft layers inclusion is based on limit process with $h \to 0$ and $E_h \to 0$ with finite and fixed ratio $d = (h/a)(E_a/E_h)$.

It has been shown that solution of such limit problem in the presence of singular transition layer on phase boundaries essentially depends on the dimensionless parameter d. The error inherent to such replacement of finite size layers by means of a singular layer has the the order of magnitude h/a and, therefore, is much smaller than unity.

Effective bulk and shear moduli of a composite with spherical layers and soft transition layers on phase boundaries have again the form (5), but the coefficients q_1 and q_2 are represented now by means of:

$$q_1 = \frac{K_0 - K_a}{K_0}\left(1 + Y_9^i\right) + \frac{d\chi}{\mu_a}(3\lambda_a + 2\mu_a)\left(1 + Y_9^i\right), \quad \chi = \frac{1 - 2\nu_h}{1 - \nu_h},$$

$$q_2 = \frac{\mu_0 - \mu_a}{\mu_0}\left[1 + 3Y_1^i + 2Y_5^i + \frac{7}{5}(3Y_2^i + 2Y_6^i)a^2\right] + \frac{2}{5}d \times \tag{6}$$

$$\times \left\{(3 + 2\chi)(1 + 3Y_1^i + 2Y_5^i) + 18(1 + \chi)Y_2^i a^2 + 2\left[9 + \frac{\chi}{\mu_a}(9\lambda_a + 10\mu_a)\right] Y_6^i a^2\right\}.$$

3 Concluding remarks

At the end let us conclude that the effect when decrease of the composite elasticity modulus in the glass resembling state bellow the smallest elasticity modulus of the composition components is met in praxis and happens quite often for many polymeric systems. Introducing of the "softened" transition layer in the computational scheme makes possible a satisfactory description of dynamic characteristics of the considered composite and produces a "judgement" of the bonding quality on matrix-inclusion boundaries being residual during the process of partial delamination.

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O MODELIRANJU PRELAZNIH SLOJEVA U MATRIČNIM KOMPOZITIMA SA KOMPLEKSNIM UKLJUČCIMA

Ludmila Kudrjavceva

U radu se razmatraju matrični kompoziti pojačani višeslojnim uključcima koji poseduju prenosne slojeve na granicama različitih faza. Za snimanje efektivnih viskoelastičnih slojeva kao i termičkih naprezanja primenjen je model samousaglašavajućeg polja koji su razvili Kanaun i Levin. Predloženi metod dinamičke spektroskopije je primenjen na sintezu vibroprigušujućih kompozicija kao i na pojedinačne slojeve.