UNIVERSITY OF NIŠ
The scientific journal **FACTA UNIVERSITATIS**
Series: **Mechanics, Automatic Control and Robotics** Vol.2, No 10, 2000 pp. 1415 - 1422
Editor of series: Katica (Stevanovi}) Hedrih, e-mail: katica@masfak.masfak.ni.ac.yu
Address: Univerzitetski trg 2, 18000 Niš, YU, Tel: +381 18 547-095, Fax: +381 18 547-950
http:// ni.ac.yu/Facta

THE MATHEMATICAL DESCRIPTON IN THE VECTOR SPACE OF DYNAMIC BEHAVIOUR FOR INERTIA SYSTEM WITH A DIRECT DIGITAL CONTROL

UDC 531.35:681.513.3:681.523.7:51(045)

Momčilo Milinović[1], Branko Djedović[2]

[1] Faculty of Mechanical Engineering University of Belgrade, 27. marta 80, Yugoslavia
Phone: +(381) 11 33-70-266, Fax: +(381) 11 33-70-364
[2] Yugoslav Army, Brehtova 3, Belgrade, Yugoslavia
Phone: +(381) 11 391-62-69, Fax: +(381) 11 324-47-91, e-mail: gvbg@eunet.yu

Abstract. *At the very beginning the problem of synthesis of digital electro hydraulic servo system as a function of actuators has been presented. Control modeling objects are the masses of large moment inertia, rotating around fixed axis in two normal planes. The power has been realised by the hydrostatic system with damped control components. The functional scheme of the complex hydraulic servo system has been presented. Application of the fluid and other concept of actuators and damping have been compared with distinguishing advantages. It is pointed out universality of application of these types of drives and its generalized applications in a structure of rotating systems. The mathematical description of dynamic system behaviour is given. According to the mass circular distribution, the balance equation has been developed. The digital multivariable control system with microcontrollers as a function of control elements is designed. The law of digital control is accomplished. The feedback of position, velocity and acceleration in rotating system is realised. Direct influence on dynamic performances of control objects is also achieved. Complex system is described by modern mathematical method – state space concept. Dynamic approximation is realised and the mathematical model of the system is represented by linearized differential equations, in the form of state space. Recurrent equations of state with algebraic equations of output are also derived. Computer simulation of the function has been realised. Dynamic analysis enables observation and optimisation of positioning accuracy and speed response, satisfied by criteria of stability, controllability and observability of control system. Relevant parameters of control system quality are represented in real time.*

Key words: *electro hydraulic servo system, rotating masses, digital control, state space, computer simulation, positioning accuracy, speed response.*

Received July 21, 2000

INTRODUCTION

Special objects and high quality control requirements induce the necessity of research in modern theory and practice of digital systems in control systems design. That means development of efficient methods and procedures for analysis and synthesis of control systems realization. Concept of actuators as executive components based on contemporary theory and modern technology of control, in state space, with modern computer modelling and simulation techniques.

1. MATHEMATICAL DESCRIPTON OF CONTROL OBJECT PHYSICAL MODELING

Modelling objects are two rotating masses: the rotating platform of special purpose vehicle that rotates in horizontal plane, and its vertical oscillating functional element jointed in axes on the platform. Range of rotating angles for complete mass in horizontal plane (per azimuth) is $n \times 360^{o}$, and vertical-oscillating mass (per elevation) is from -5^{o} to $+85^{o}$.

Separate actuators subsystems are aimed to provide drive and control of both rotating masses per azimuth and elevation. Its function has been realized by rigid requirements for positioning accuracy and response time delay guided by tracking accuracy for space orientation of functional element.

Control object has coupled motion regime, so their dynamics has been realized in the same manner. Mechanical constraint principle provides description of system motion is described by ordinary differential equations. Rotating mass moves under action of applied torques generated by hydraulic motors.

Differential equation of rotating mass rotation in horizontal plane (per azimuth) is:

$$M_{m1} = I \frac{d^2\theta}{dt^2} + f_{tk}\frac{d\theta}{dt} + M_{vk}$$

where: θ - mass rotating angle per azimuth;
$\dot{\theta}$ - mass angular velocity per azimuth,
$\ddot{\theta}$ - angular acceleration per azimuth,
$I = I(\varphi)$ - the mass inertia moment,
$M_{m1}(t)$ - the torques of hydraulic motors per azimuth,
φ - mass rotating angle per elevation,
f_{tk} - viscous damping coefficient,
M_{vk} - parasite torques per azimuth.

In the same manner, by analogy, motion of rotating mass per elevation could be described.

The platform with functional element rotates under action of torque $M_{m1}(t)$ as control variable for horizontal plane (azimuth) control subsystem. Motor shaft balance equation gives:

$$M_{m1} - M_{vtm1} - M_{\varepsilon m1} - M_{fm1} = 0,$$

otherwise

$$M_{m1} = I_m \frac{d^2\theta_m}{dt^2} + f_{tm1}\frac{d\theta_m}{dt} + \frac{M_{vtm1}}{\eta_{r1} i_1}$$

where: I_{m} - the mass inertia moment reduced on motor shaft, per azimuth,
θ_m - rotating angle of motor shaft per azimuth,
f_{tm1} - viscous damping coefficient of hydraulic motor per azimuth,
η_{r1} - useful effect coefficient of reductors per azimuth,
i_1 - transfer ratio of reductor per azimuth,
M_{vtm1} - the torques of outside loads reduced on motor shaft per azimuth,
M_{fm1} - the torque of viscous damping hydraulic motor per azimuth,
$M_{\varepsilon m1}$ - the torque of hydraulic motor per azimuth for acceleration load.

Functional element rotates by torque $M_{m2}(t)$ as control variable of vertical (elevation) subsystem. Motor shaft balance equation gives:

$$M_{m2} - M_{vtm2} - M_{\varepsilon m2} - M_{fm2} = 0,$$

otherwise

$$M_{m2} = J_m \frac{d^2\varphi_m}{dt^2} + f_{tm2}\frac{d\varphi_m}{dt} + \frac{M_{vtm2}}{\eta_{r2} i_2}$$

where: J_m - the mass inertia moment reduced on motor shaft, per elevation,
φ_m - rotating angle of motor shaft per elevation,
f_{tm2} - viscous damping coefficient of hydraulic motor per elevation,
η_{r2} - useful effect coefficient of reductor per elevation,
i_2 - transfer ratio of reductor per elevation,
M_{vtm2} - the torque of hydraulic motor per elevation for acceleration load,
M_{fm2} - the torque of viscous damping hydraulic motor, per elevation,
$M_{\varepsilon m2}$ - the torque of hydraulic motor per elevation for acceleration load,

Differential equation of motion for known parameters gives solution in time that describes motion of control objects.

The paper deduces idealization of dynamic and mathematical model. The problem of motion is solved in several different ways.

2. PRINCIPLE SOLUTION OF ACTUATORS CONTROL SYSTEM

Control quality requirements are determined by properties of actuator's control function. It is concepted complex digital servo system, with microprocessor as a regulator, which is composed from three subsystems: control, azimuth control subsystem and elevation control subsystem.

Structure of actuators on the basis of electro hydraulic servo systems with damping control (principle "brain" and "brawn") is accepted. Concept of control system approves its complexity in three mutual joints – couple of control system for rotating mass per azimuth and per elevation. Combination of mutual influence is realized in: control subsystem by microcontroller, generator of hydraulic energy subsystem and control object. Electro hydraulic servo system (EHSS) is given in variant of complex hydraulic system with two hydro motors controlled by electro hydraulic servo valves, supplied by hydraulic energy from a servo pump with pressure stabilizer. Two encoders (for azimuth and elevation) give information about parameters of control object motion. Functional hydraulic scheme of complex EHSS is shown on Fig. 1.

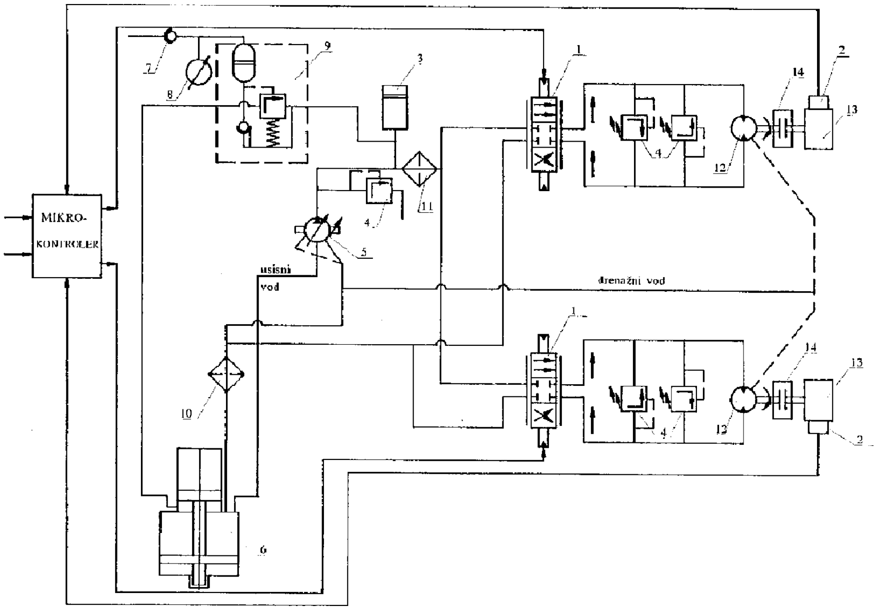

Fig. 1. Functional hydraulic scheme of complex EHSS
1. Electro hydraulic valves of azimuth and elevation, 2 encoders of azimuth and elevation, 3. hydraulic accumulator, 4. safety valve, 5. hydraulic pump, 6. autopressurisation reservoir, 7. charging valve, 8. manometer, 9. pressure maintenance valve, 10. rough filter, 11. fine filter, 12. hydro motors of azimuth and elevation, 13. object of control, 14. reductors.

3. STATE SPACE SYNTHESIS OF A CONTROL SISTEM

Subsystems for azimuth and elevation control must satisfy corresponding design requirements.

Providing that the state of system motion is determined by coordinates of point position and coordinates of momentum, inertial masses' motion is observed in terms of generalized coordinates and generalized momentums, which represents phase space or state space. The system with concentrate parameters is considered.

Modern control theory based on the concept of state of system, using mathematical terms: state variable, state vector and state space, enables precise definition of stability, controlability, observability, susceptibility and adaptibility of itself. The use of computers and modern control theory enables efficient analysis of dynamic characteristics. This method provides analysis of wide class stationary nonlinear, nonstationary (linear and nonlinear) systems, and also enables synthesis of optimal and adaptive complex multi variable control systems.

Extrapolator of zero order realizes D/A conversion. As D/A converter has a transfer function $G_{h0} = (1 - e^{-ts})/s$, then the overall transfer function $G(s)$ of continual part of system is:

$$G(s) = \frac{1-e^{-Ts}}{s} G_p(s) \text{ ,}$$

where $G_p(s)$ is a process transfer function, given by relation:

$$G_p(s) = \frac{Y(s)}{U(s)} = \frac{K}{s(T_1 s+1)(T_2 s^2 + 2\xi T_2 s + 1)(T_3 s^2 + 2\xi T_3 s + 1)} \text{ .}$$

The analysis and synthesis of the control system is accomplished by computer simulation. Characteristic equations are formed, and transfer functions of azimuth subsystem (G_1) and elevation subsystem (G_2) are represented in the form of vectoral recurrent equation:

$$x[(k+1)T] = Ax(kT) + Bu(kT) \text{ .}$$

Vectoral algebraic equation of output of (linear steady continual dynamic) the system is:

$$\mathbf{y}(kT) = \mathbf{C}\mathbf{x}(kT) + \mathbf{D}\mathbf{u}(kT) \text{ ,}$$

where $\mathbf{x}(kT)$, $\mathbf{u}(kT)$ and $\mathbf{y}(kT)$ are state vectors of input and output, whence $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{D}$ are constant matrices with appropriate dimensions. On the basis of matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{D}$, and disposition of nulls and poles, which are specified for both subsystems and for both cases (with or without regulator), conclusion about dynamic characteristics of system has been reached.

The controlability of motion is property to move an object by a given program forced by generalized forces. The program comprises trajectories and velocities subprograms. Rectilinear orthonorm system of coordinates is used for realization of the motion program. The system of bodies with constant mass should be constrained to move in a prescribed manner.

Using the mechanical principle of constrain, differential equations of motion can be established. Transfer function G_1 for azimuth subsystem is given by relation:

$$G_1 = \frac{0.000044^{*}560236^{*}0.004545^{*}6241^{*}9025/0.002273}{(s+1/0.002273)s(s+71-34^{*}i)(s+71+34^{*}i)(s+0{,}76-95^{*}i)(s+0.76+95^{*}i)} \text{ .}$$

Transfer function of azimuth control is determined and controlability requirements are defined. Matrix of controlability gets a form:

$$\mathbf{E}_1 = [\,\mathbf{B}_1 \mid \mathbf{A}_1\mathbf{B}_1 \mid \mathbf{A}_1{}^2\mathbf{B}_1 \mid \mathbf{A}_1{}^3\mathbf{B}_1 \mid \mathbf{A}_1{}^4\mathbf{B}_1 \mid \mathbf{A}_1{}^5\mathbf{B}_1\,] \text{ .}$$

Rank of matrix $\mathbf{E}_1$ is six. The system is completely controlable. The matrix of observability is determined as:

$$\mathbf{F}_1 = [\,\mathbf{C}_1{}^T \mid \mathbf{A}_1{}^T\mathbf{C}_1{}^T \mid (\mathbf{A}_1{}^T)^2\mathbf{C}_1{}^T \mid (\mathbf{A}_1{}^T)^3\mathbf{C}_1{}^T \mid (\mathbf{A}_1{}^T)^4\mathbf{C}_1{}^T \mid (\mathbf{A}_1{}^T)^5\mathbf{C}_1{}^T\,] \text{ .}$$

The rank of matrix $\mathbf{F}_1$ is also six. The system is also completely observable.

Transfer function G_2 for elevation control subsystem is given by relation:

$$G_2 = \frac{0.000044^{*}560236^{*}0.004545^{*}19044^{*}29929/0.002273}{(s+1/0.002273)s(s+110-83^{*}i)(s+110+83^{*}i)(s+1.6-173^{*}i)(s+1.6+173^{*}i)} \text{ .}$$

Transfer function of elevation control is determined and controlability requirements are also defined. The matrix of controlability gets a form:

$$\mathbf{E}_2 = [\,\mathbf{B}_2 \mid \mathbf{A}_2\mathbf{B}_2 \mid \mathbf{A}_2^{2}\mathbf{B}_2 \mid \mathbf{A}_2^{3}\mathbf{B}_2 \mid \mathbf{A}_2^{4}\mathbf{B}_2 \mid \mathbf{A}_2^{5}\mathbf{B}_2\,].$$

and rank of matrix $\mathbf{E}_2$ is six. The system is completely controlable. The matrix of observability is determined as:

$$\mathbf{F}_2 = [\,\mathbf{C}^T{}_2 \mid \mathbf{A}_2^{T}\mathbf{C}_2^{T} \mid (\mathbf{A}_2^{T})^2\mathbf{C}_2^{T} \mid (\mathbf{A}_2^{T})^3\mathbf{C}_2^{T} \mid (\mathbf{A}_2^{T})^4\mathbf{C}_2^{T} \mid (\mathbf{A}_2^{T})^5\mathbf{C}_2^{T}\,].$$

with rank of matrix $\mathbf{F}_2$ as a six. The system is completely observable.

Using the program MATLAB, simulative block chart is formed and analyzed response of system for a prescribed trajectory of motion. The response of the complex control system (by azimuth and elevation) for real function – trajectory of motion, is given on Fig. 2. and 3.

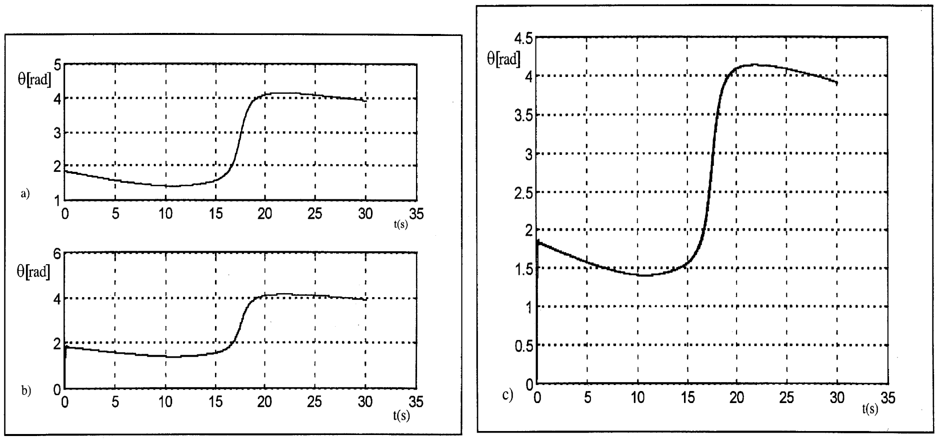

Fig. 2. Response of digital azimuth control system for a real trajectory of motion
a) programed trajectory b) response of control system
c) programed trajectory and response of control system

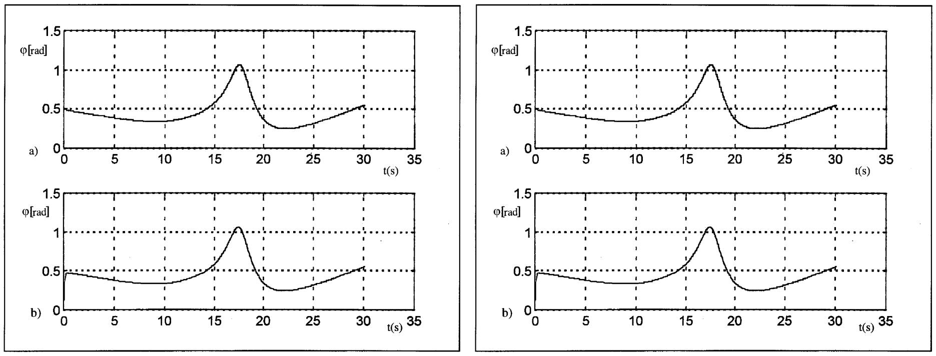

Fig. 3. Response of digital elevation control system for a real trajectory of motion
a) programed trajectory b) response of control system
c) programed trajectory and response of control system

Control law is realized by using regulator of I effect and regulator of PD effect in a local feedback loop. For a direct digital control, the law is given by relation:

$$u(k) = K_p \left\{ -y(k) + \frac{T}{T_i} \sum_{i=0}^{k} [r(k) - yk)] - \frac{T_d}{T} [y(k) - y(k-1)] \right\},$$

where T_i and T_d are time constants of *I* and *D* effect, and K_p is amplifing factor.

Requested solution gets in time domain. Results of actuator response simulation for a real control system input function approves accomplishing of requirement of positioning and tracking accuracy. Overall positioning error in steady state conditions inclines to zero, and stabilization time is a few tenths of a second. Dynamic parameters of motion indicate that the proposed solution of actuators' control structure is valid, whereby it is satisfied required quality of dynamic characteristics of control system.

CONCLUSION

The study of determined problem enables generalization of premises given in the form of the following conclusions:

1. Characteristics of object, process and control system are determined and adequately presented in the form that enables their efficient application.
2. Used concept of actuators' configuration on the basis of electro hydraulic servo system with damping digital control, provides high control quality requirements for special purposes objects.
3. Synthesis of control system by using state space concept, represents a good mathematical tool, which gives complete description of system and comprehensive analysis of important performances that determinate the quality of the system. All advantages of this concept are presented in the given example of the system.

REFERENCES

1. Bangert, H., *Trends der Elektronik in der Hidraulik*, "Olhydraulik und Pneumatik", 38, Nr. 11-12, 1994
2. Backe, W., *Technische Trends der Fluidtechnik*, "Olhydraulik und Pneumatik" 38, Nr. 11-12, 1994
3. Debeljkovic, D.LJ., S.A. Milinkovic, L.L. Pezo, S.D. Šarboh, V.M. Milentijevic, *Dinamika procesa sa raspodeljenim parametrima*, Beograd, 2000.
4. Merritt, H. E., *Hidraulic control systems*, J. Wilev, New York, 1967
5. Meyer, U.A., *Modern control principles and applications.* New York. San Francisko. Toronto. London. Sydney, 1972
6. Sekulic, M.R., *Osnovi teorije automatskog upravljanja - servomehanizmi*, Beograd 1982
7. Stojic, M.R., *Digitalni sistemi upravljanja*, Beograd, 1989
8. Vujicic, V.A., *Preprincipi mehanike*, Beograd, 1998

MATEMATIČKI PRIKAZ U PROSTORU STANJA DINAMIČKOG PONAŠANJA INERCIONOG DIREKTNO UPRAVLJANOG DIGITALIZOVANOG SISTEMA

Momčilo Milinović, Branko Djedović

Na početku rada postavlja se problem sinteze digitalnog elektrohidrauličkog servosistema u funkciji izvršnih organa upravljanja. Objekti upravljanja - mase velikog momenta inercije izvode obrtna kretanja oko nepokretnih osa u dve uzajamno normalne ravni. Prenos energije je realizovan hidrostatičkim putem sa prigušnim upravljanjem. Daje se funkcionalna hidraulička shema složenog servosistema. Ukazuje se na prednosti primene tehnike fluida i prigušnog upravljanja u odnosu na druga koncepcijska rešenja izvršnih organa. Ističe se univerzalnost primene i generalizuju stavovi o implementaciji ovih pogona u koncepcijskoj strukturi pogona rotirajućih sistema. Daje se matematički prikaz dinamičkog ponašanja sistema. Razvija se postavljena bilansna jednačina shodno odgovarajućim tokovima mase. Projektuje se multivarijabilni digitalni zatvoreni sistem upravljanja u kojem upravljačku funkciju realizuje mikrokontroler. Izvodi se zakon upravljanja. Ostvaruje se povratna sprega po poziciji, brzini i ubrzanju obrtnog kretanja, čime se direktno utiče na dinamičke karakteristike objekata upravljanja. Složeni sistem se opisuje primenom savremenih matematičkih metoda konceptom prostora stanja. Izvedena je dinamička idealizacija i matematički model sistema je predstavljen linearizovanim diferencijalnim jednačinama u vidu vektorske diferencijalne jednačine stanja i vektorske diferencne jednačine stanja sa vektorskim algebarskim jednačinama izlaza. Izvodi se simulacija diskretnog sistema upravljanja na računaru. Pri dinamičkoj analizi posebno se posmatraju i optimiziraju tačnost pozicioniranja i brzina reagovanja uz obezbeđenje zahteva stabilnosti, kontrolabilnosti i opservabilnosti podsistema složenog sistema upravljanja. Relevantni pokazatelji kvaliteta ponašanja sistema upravljanja prikazuju se u realnom vremenskom domenu.

Ključne reči: *elektrohidraulički servosistem, obrtne mase, digitalno upravljanje, prostor stanja, računarska simulacija, tačnost pozicioniranja brzina reagovanja.*