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DIGITAL ADAPTIVE CONTROL SYSTEM DESIGN FOR A PARTICULAR CLASS OF HYDRAULIC SYSTEMS

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Abstract. *This paper is devoted to design of adaptive control laws for a particular class of hydraulic systems. Considered hydraulic systems belong to a class of level-systems, which were so far mostly considered with the aim of level control, i.e. keeping the level of a liquid in a reservoir at desired constant value. This control aim could be achieved by linear feedback gain control system. Mentioned hydraulic level-systems are now considered from a different point of view, which demands nonlinear adaptive control system introducing. Namely, if the control aim is to make the level of a liquid in a reservoir change according to a specified function, then the parameters of the system under control are variable due to variable liquid level, which demands adaptive control. The case when the system parameters are unknown is also considered.*
Regarding adaptive control design, different approaches were applied. The paper considers gradient approach (MIT rule for parameter adaptation), reference model adaptive systems based on hyperstability theory and self-tuning regulators. Design procedures were considered in discrete form which enables their implementation as a part of a digital system controled by computer.
For the purpose of adaptive control design, differential equation models of considered systems were converted into appropriate discrete-time models and afterwards represented in polinomial form, which is the basis for adaptive control design procedure. As a result discrete control laws, which provide accomplishing of the control aim, were obtained.
The verification of designed control laws was accomplished by the numerical experiment method, i.e. by digital computer simulation, and obtained simulation results were presented in the work. Simulations were performed for different types of reference input and compared. On the basis of simulation results appropriate conclusions were made. For the purpose of the simulation completion, original programs, which represent software implementation of designed control laws, were written.

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1. INTRODUCTION

Adaptive control represents a special type of control with nonlinear feedback in which the states of the process could be divided into two categories: parameters which change slowly and state variables which change faster. Many processes are characterized by the fact that their parameters are variable or unknown, which demands adaptive systems for their control. Adaptive control systems are nonlinear even if the system under control represents a linear system or a system which could be described by a linearized mathematical model. The structure of adaptive systems and the mechanisms used for the adaptation of unknown or variable parameters impose the nonlinearity of the control system.

The paper considers several types of adaptive control systems (gradient approach - MIT rule for parameter adaptation, reference model adaptive systems based on hyperstability theory and self-tuning regulators) and their application to certain hydraulic systems. Known hydraulic level-systems are now considered from a different point of view. Standard hydraulic level control systems provided keeping the level of a liquid in a reservoir at desired constant value. That could be achieved by linear feedback gain control system. If the aim of control is to make the level of a liquid in a reservoir change according to a specified function, then the adaptive control system is needed, because the parameters of the system under control are variable due to variable liquid level. The case when the system parameters are unknown is also considered.

2. GRADIENT METHOD (MIT RULE) FOR HYDRAULIC LEVEL-SYSTEM

Gradient method is the basic approach in reference model adaptive theory. This method is based on minimization of the quadratic performance index:

$$J(kT,\theta)=\frac{1}{2}e^{2}(kT,\theta)\,, \tag{1}$$

where $e(kT,\theta)=y(kT,\theta)-y_m(kT)$ is error between the system output and the reference model output. In order to minimize the performance index, the parameter vector θ of the adjustable controller has to change in the opposite direction from gradient $\partial J/\partial\theta$. Thus, the adaptation law in discrete version is:

$$\theta(kT+T)=\theta(kT)-\gamma\frac{\partial J(kT,\theta)}{\partial\theta}=\theta(kT)-\gamma e(kT,\theta)\frac{\partial e(kT,\theta)}{\partial\theta}\,. \tag{2}$$

Partial derivative $\partial e(kT,\theta)/\partial\theta$ is called the sensitivity derivative.

For the single input single output system of the first order described by difference equation:

$$y(kT+T)=-ay(kT)+bu(kT), \tag{3}$$

desired closed-loop system is described by the equation:

$$y_m(kT+T)=-a_m y_m(kT)+b_m\omega(kT), \tag{4}$$

where $\omega(kT)$ is bounded reference input and $y_m(kT)$ is the output of the reference model.

Tracking of the reference model can be achieved with the controller which outputs the

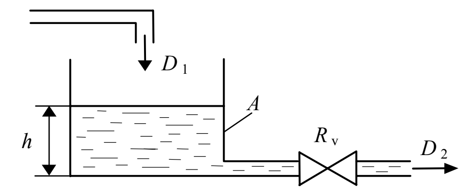

Fig. 1. Hydraulic level-system described by the single input single output linear model of the first order

control law:

$$u(kT) = -f(kT)y(kT) + g(kT)\omega(kT) \quad (5)$$

where the controller parameters f and g are determined through the following adaptation mechanism (MIT rule) obtained using gradient approach [1], [9], [10]:

$$g(kT+T) = g(kT) - \gamma e(kT)\frac{\partial e(kT)}{\partial g} = g(kT) - \gamma \frac{q^{-1}}{1+a_m q^{-1}}\omega(kT)e(kT) \quad (6)$$

$$f(kT+T) = f(kT) - \gamma e(kT)\frac{\partial e(kT)}{\partial f} = f(kT) + \gamma \frac{q^{-1}}{1+a_m q^{-1}}y(kT)e(kT) \quad (7)$$

γ is adaptation gain and sensitivity derivatives were determined using appropriate approximations based on the reference model following in case of known model parameters [1], [9], [10].

Gradient approach was applied to adaptive control design for the hydraulic level-system shown in Fig. 1. Basic regulation problem represents the problem of keeping the liquid level in the reservoir at desired nominal value. It can be solved easily by using some of the known methods for regulator design. In case when the control aim is to provide the level of the liquid change as an appropriate function of time (determined by the reference model output), regulator design is not enough for solving the problem. Variable liquid level represents variable system parameter and an adaptive control is required to accomplish the control aim.

The plant [9] is described by difference equation (8) where the plant output y is the level of the liquid in the reservoir and control u is the flow D_1.

$$\dot{y} = -0.625y + u \quad (8)$$

The control aim is that the plant output changes according to the output of the reference model y_m.

$$\dot{y}_m = -2y_m + 2\omega, \quad (9)$$

where ω is the reference model input.

Discretizing the plant model (8) and the reference model (9) with sampling interval 0.1min, discrete-time plant model and reference model were obtained in the form (3) and (4) respectively, where $a = -0.9394$, $b = 0.0969$, $a_m = -0.8187$, $b_m = 0.1813$.

According to the procedure for adaptive control system design based on gradient approach and quoted equations, adaptive control law was obtained in the form (5). Block diagram for this adaptive system based on MIT rule is shown in Figure 2.

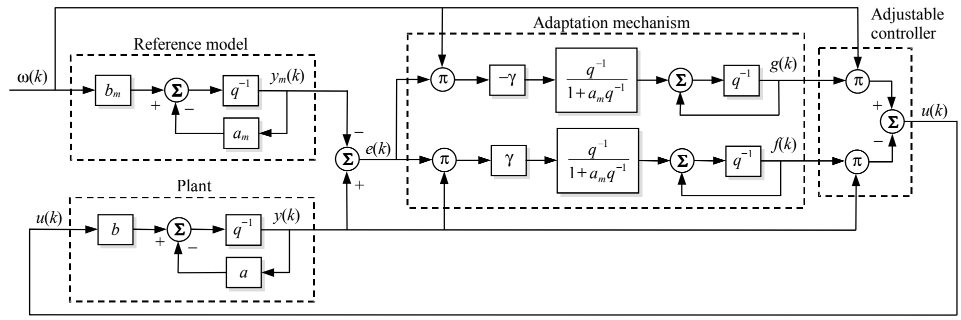

Fig. 2. Block diagram of discrete reference model adaptive system based on gradient approach for the hydraulic level-system of the first order

Simulations were performed for different types of reference input. Simulation results obtained for rectangular and sine reference input are shown in Figure 3.

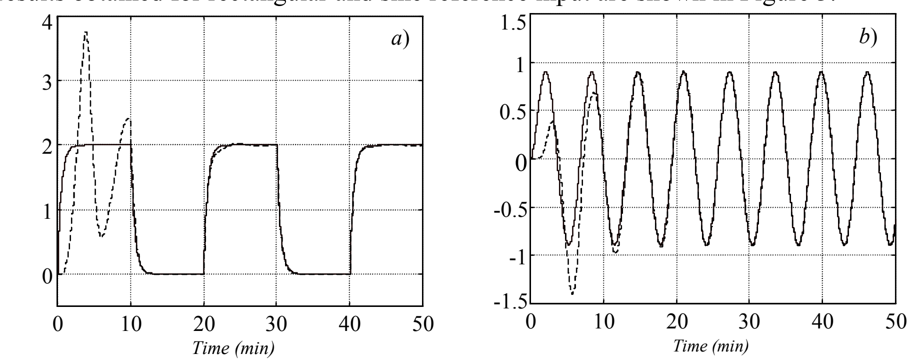

Fig. 3. Simulations of the output of reference model adaptive system based on MIT rule for hydraulic level-system of the first order (system output – dashed line; output of the reference model – solid line). a) Reference input: rectangular wave with height 2, width 10 and period 20 min. Adaptation gain: γ=0.005. b) Reference input: sine wave, period 2π min, adaptation gain: γ=0.02.

Simulation results in Figure 3 show that quite good tracking of the reference model output is accomplished after the first cycle, approximately. Simulations performed during the investigation also showed that designed adaptive system could also be used with step reference input, i.e. for regulation. Depending on the adaptation gain, different step responses were obtained and they varied from extremely oscillating to very slow responses. Adaptive system with adaptation gain $\gamma = 0.001$ resulted in acceptable step response.

3. REFERENCE MODEL ADAPTIVE SYSTEM BASED ON HYPERSTABILITY APPROACH

For the same hydraulic level-system shown in Figure 1 model reference adaptive system based on hyperstability approach, which tracks the output of the reference model was designed. In this case plant model (10) and reference model (13) are represented in polynomial form.

$$A(q^{-1})y(k+d) = B(q^{-1})u(k) \;\text{ or }\; A(q^{-1})y(k) = q^{-d}B(q^{-1})u(k), \quad y(0) \neq 0, \tag{10}$$

$$A(q^{-1}) = 1 + a_1 q^{-1} + ... + a_{n_A} q^{-n_A} = 1 + q^{-1}A^*(q^{-1}), \tag{11}$$

$$B(q^{-1}) = b_0 + b_1 q^{-1} + ... + b_{n_B} q^{-n_B} = b_0 + q^{-1}B^*(q^{-1}), \quad b_0 \neq 0. \tag{12}$$

$$A_m(q^{-1})y(k) = q^{-d}B_m(q^{-1})\omega(k) \tag{13}$$

$$A_m(q^{-1}) = 1 + a_1^m q^{-1} + ... + a_{n_{Am}}^m q^{-n_{Am}}, \tag{14}$$

$$B_m(q^{-1}) = b_0^m + b_1^m q^{-1} + ... + b_{n_{Bm}}^m q^{-n_{Bm}}. \tag{15}$$

According to discrete-time model of the plant, after its transformation to discrete transfer function in backward shift operator q^{-1} and afterwards to polynomial form, the coefficients of the appropriate polynomials were obtained: $a_1 = -0.9394$, $b_0=0.0969$, $a_m=0.8187$, $b_m=0.1813$. As in [9] and [10], asymptotic stable polynomial $\Gamma(q^{-1}) = 1 + \gamma_1 q^{-1} = 1 + 0.5q^{-1}$ was introduced to describe dynamics of the system during regulation. Hyperstability approach [10] considers a special form of Diophantine equation [2], [6]:

$$\Gamma(q^{-1}) = A(q^{-1})S(q^{-1}) + q^{-d}R(q^{-1}), \tag{16}$$

and the conditions under which the solution of (16), i.e. the polynomials $R(q^{-1})$ and $S(q^{-1})$ are unique. Discussion leads to polynomials $S(q^{-1}) = 1$, $R(q^{-1}) = r_0 = 1.4394$ of the controller, which in case of known plant parameters produces control law:

$$u(k) = \frac{\Gamma(q^{-1})y_m(k+d) + R(q^{-1})y(k)}{b_0} = \frac{y_m(k+1) + \gamma_1 y_m(k) - r_0 y(k)}{b_0}. \tag{17}$$

In case of unknown or variable plant parameters, they are estimated through adaptation mechanism. According to the certainty equivalence principle, the control law (18) is of the same form as in case of known parameters, but the parameters are now substituted with their estimates obtained through adaptation algorithm.

$$u(k) = \frac{y_m(k+1) + \gamma_1 y_m(k) - \hat{r}_0(k)y(k)}{\hat{b}_0(k)} \tag{18}$$

The plant parameter vector is estimated at each sampling interval and it is:

$$\hat{\theta}^{\mathrm{T}}(k) = [\hat{b}_0(k) \quad \hat{r}_0(k)] = [\hat{\theta}_1(k) \quad \hat{\theta}_2(k)]. \tag{19}$$

It is estimated through the following adaptation algorithm.

$$\mathbf{F}(0) = \frac{1}{10^{-3}}\mathbf{I}_2$$

$$\phi^{\mathrm{T}}(k) = [u(k) \quad y(k)] \qquad \text{for} \quad k = 0, 1, 2, ...$$

$$\begin{aligned}
&\tilde{\varepsilon}(k) = y(k) + \gamma_1 y(k-1) - \hat{\theta}^{\mathrm{T}}(k-1)\phi(k-1) && \text{for} \quad k = 0, 1, 2, \ldots \\
&\varepsilon(k) = \frac{\tilde{\varepsilon}(k)}{1 + \phi^{\mathrm{T}}(k-d)\mathbf{F}(k)\phi(k-d)} && \text{for} \quad k = 0, 1, 2, \ldots \qquad (20) \\
&\hat{\theta}(k) = \hat{\theta}(k-1) + \mathbf{F}(k)\phi(k-d)\varepsilon(k) && \text{for} \quad k = 0, 1, 2, \ldots \\
&\mathbf{F}^{-1}(k+1) = 0.98(k)\mathbf{F}^{-1}(k) + \phi\,(k-d)\,)\phi^{\mathrm{T}}(k-d) && \text{for} \quad k = 0, 1, 2, \ldots
\end{aligned}$$

Simulation diagrams shown in Figure 4 were obtained on the basis of the software [9] for simulation of reference model adaptive control system. Initial parameter vector is $\hat{\theta}^{\mathrm{T}}(0) = [1 \quad 1]$ and initial value of the output is $y(0)=1$. Step, sine and rectangular reference inputs were considered. In each case the same diagrams of estimated parameters were obtained (Figure 4 d). Simulations show that exact tracking of the reference model output is accomplished after approximately 1min, which is faster than in the case of gradient approach to adaptive control of the same level-system.

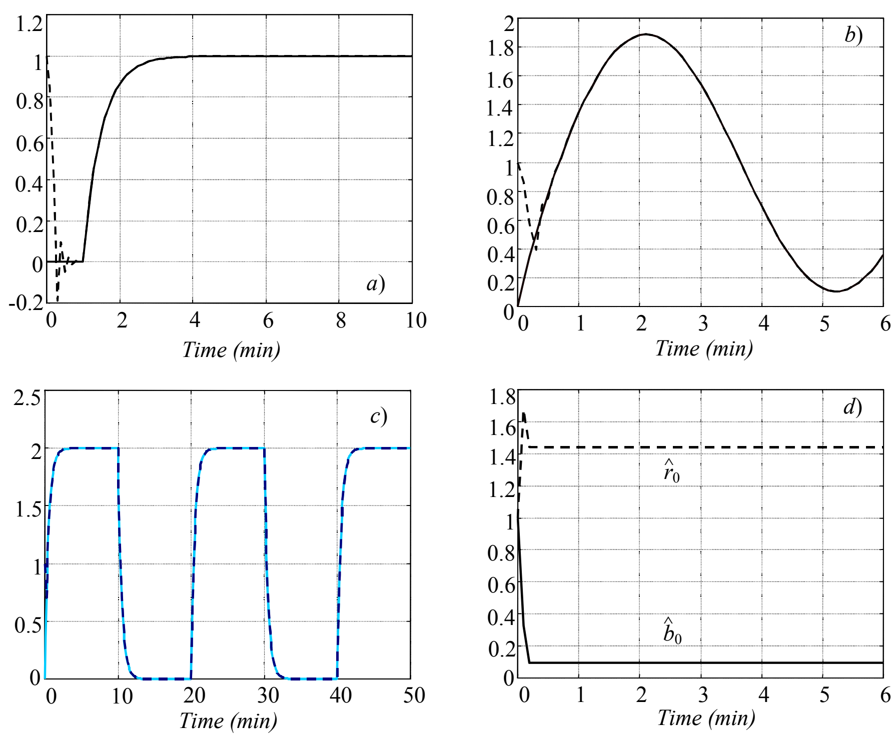

Fig. 4. a) Reference input is unit step function with delay 1. b) Reference input is sine wave with period 2π. c) Reference input is rectangular wave, period 20, height 2, width 10. For a), b) and c): output of the reference model – solid line, system output – dashed line. d) Estimated parameters.

4. SELF-TUNING REGULATOR BASED ON EXPLICIT POLE PLACEMENT METHOD

Pole placement method for self-tuning regulator design represents general self-tuning regulator design method. It can be applied in the case when the plant is not minimum-phase system. Explicit and implicit adaptive schemes can be formulated. In this case explicit pole placement method is considered, which means that the plant parameters are estimated explicitly through the adaptation algorithm and the controller is designed afterwards on the basis of the plant parameter estimates at each sampling interval.

The plant model should be represented in polynomial form (10) for self-tuning regulator design, where the plant parameter polynomials $A(q^{-1})$ and $B(q^{-1})$ are (11) and (12), respectively. The polynomial $A(q^{-1})$ is monic, and $A(q^{-1})$ and $B(q^{-1})$ are relatively prime. The control problem is to determine such a controller which should provide that the relation between the reference input $\omega(k)$ and the system output is (13). This is accomplished by R–S–T controller:

$$R(q^{-1})u(k) = T(q^{-1})\omega(k) - S(q^{-1})y(k). \tag{21}$$

The polynomial $B(q^{-1})$ is factorized as $B(q^{-1}) = B^{+}(q^{-1})B^{-}(q^{-1})$, where $B^{+}(q^{-1})$ contains stabile (well damped) plant zeros which could be canceled and $B^{-}(q^{-1})$ contains unstable and poorly damped zeros that should not be canceled. According to discussion performed in [9] and [10] the polynomial $B_m(q^{-1})$ is factorized as: $B_m(q^{-1}) = B^{-}(q^{-1})B_m^{+}(q^{-1})$, where $B_m^{+}(q^{-1})$ can be chosen freely. It should also be: $T(q^{-1}) = A_0(q^{-1})B_m^{+}(q^{-1})$, where the observer polynomial $A_0(q^{-1})$ is chosen to have well damped roots.

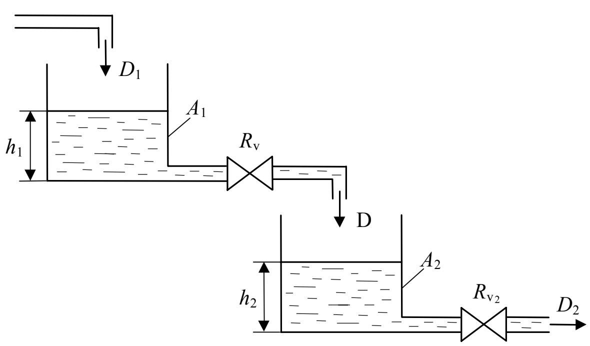

Fig. 5. Hydraulic level-system described by the second order model

Design procedure is performed for hydraulic level-system represented in Fig. 5. According to [4], [9], [11] continuous state space model of the plant is:

$$\dot{\mathbf{x}} = \begin{bmatrix} -0.625 & 0 \\ 0.5 & -1 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}u, \quad y = [0 \quad 1]\,\mathbf{x} \tag{22}$$

Discretizing the previous state space model with sampling time 0.1min and representing the model in the form of discrete transfer function in backwards shift operator q^{-1} and afterwards in polynomial form the coefficients of the polynomials $A(q^{-1})$ and $B(q^{-1})$ were determined: $a_1 = -1.8443$, $a_2 = 0.85$, $b_0 = 0.0024$, $b_1 = 0.0022$. It is required that desired closed loop behavior is described by (13), where the polynomials $A_m(q^{-1})$ and $B_m(q^{-1})$ are:

$$A_m(q^{-1}) = 1 + a_1^m q^{-1} + a_2^m q^{-2} = 1 - q^{-1} + 0.25q^{-2} \tag{23}$$

$$B_m(q^{-1}) = B^{-}(q^{-1})B_m^{+}(q^{-1}) = b_0^m + b_1^m q^{-1} = 0.0355 + 0.0118q^{-1} \tag{24}$$

According to conditions [9], [10] regarding the order of the polynomials in the control law, which should be satisfied in order to determine unique and causal control law, after solving Diophantine equation and determining the controller parameters, the following control law was obtained in the case of known plant parameters:

$$u(k)=\frac{15+5q^{-1}}{1+0.9473q^{-1}}\omega(k)-\frac{356.4208-267.411q^{-1}}{1+0.9473q^{-1}}y(k) \tag{25}$$

In the case of unknown plant parameters, they are estimated at each sampling interval. The plant model can be written in the form: $(1 + a_1q^{-1} + a_2q^{-2})\, y(k) = (b_0 + b_1q^{-1})u(k-1)$. It follows that:

$$y(k)=-a_1y(k-1)-a_2y(k-2)+b_0u(k-1)+b_1u(k-2) \quad \text{or} \quad y(k)=\boldsymbol{\theta}^{\mathrm{T}}(k)\boldsymbol{\phi}(k) \tag{26}$$

where the plant parameter vector is

$$\theta^{\mathrm{T}}=[a_1 \quad a_2 \quad b_0 \quad b_1], \tag{27}$$

and the regression vector:

$$\phi^{\mathrm{T}}(k)=[-y(k-1) \quad -y(k-2) \quad u(k-1) \quad u(k-2)]. \tag{28}$$

Unknown parameters are estimated at each sampling interval according to the following algorithm.

$$\begin{aligned}
&\mathbf{F}(0)=\mathbf{I}_4/10^{-3}\\
&\mathbf{F}(k+1)=\frac{1}{0.99}\left(\mathbf{F}(k)-\frac{\mathbf{F}(k)\phi(k)\phi^{\mathrm{T}}(k)\mathbf{F}(k)}{1+\phi^{\mathrm{T}}(k)\mathbf{F}(k)\phi(k)}\right)\\
&\varepsilon(k)=y(k)-\phi^{\mathrm{T}}(k)\hat{\theta}(k-1)\\
&\hat{\theta}(k)=\hat{\theta}(k-1)+\mathbf{F}(k)\phi(k)\varepsilon(k)\\
&\hat{\theta}^{\mathrm{T}}(k)=[\hat{a}_1(k) \quad \hat{a}_2(k) \quad \hat{b}_0(k) \quad \hat{b}_1(k)]
\end{aligned} \tag{29}$$

The controller is designed on the basis of estimated plant parameters at each sampling interval by solving Diophantine equation and using the procedure described in [9]. The control law is calculated and applied at each sampling interval in the form:

$$u(k)=\frac{T(q^{-1})}{\hat{R}(q^{-1})}\omega(k)-\frac{\hat{S}(q^{-1})}{\hat{R}(q^{-1})}y(k). \tag{30}$$

Presented design algorithm was implemented through the software program designed in [9]. The program was used to obtain simulation results for the system output, input and estimated parameters. Simulation results for rectangular reference input are shown in Figure 6.

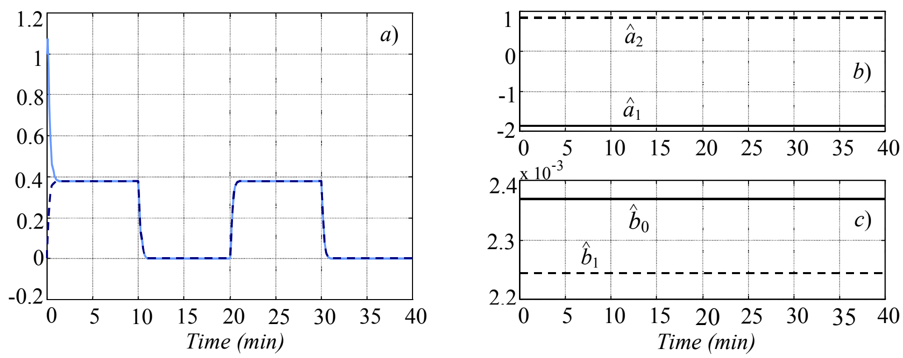

Fig. 6. Simulation results for rectangular wave reference input, period 2π, height 2, width 10. a) Output of the reference model – dashed line, system output – solid line. b) and c) Estimated plant parameters.

REFERENCES

1. Åström K., Wittenmark B., *Adaptive Control*, Addison-Wesley Publishing Company, New York, 1989.
2. Åström K., Wittenmark B., *Computer Controlled Systems*, Prentice-Hall, Inc., Englewood Cliffs, 1984.
3. Åström K., Wittenmark B., On Self-tuning Regulators, *Automatica*, Vol. 9, pp. 185-199, 1973.
4. Debeljković D., Dinamika objekata i procesa – matematički modeli objekata i procesa u sistemima automatskog upravljanja, Mašinski fakultet, Beograd, 1989.
5. Friedland B., *Advanced Control System Design*, Prentice Hall, New Jersey, 1996.
6. Kučera V., Diophantine Equations in Control – A survey, *Automatica*, Vol. 29, No. 6, pp. 1361-1375, 1993.
7. Merrit H. E., *Hydraulic Control Systems*, John Wiley&Sons, New York, 1967.
8. Nestorović T., Nikolić V., Digital Tracking System Design for the Hydraulic Positioning System of the Fourth Order, *Theoretical and Applied Mechanics*, Jugoslovensko društvo za mehaniku, (to be issued).
9. Nestorović T., Application of Digital Control Laws to some groups of Hydraulic Systems and Possibilities for their Microprocessor Implementation, master's thesis, Faculty of Mechanical Engineering, University of Niš, April 2000.
10. Paraskevopulos P. N., *Digital Control Systems*, Prentice Hall, New Jersey, 1996.
11. Ротач В. Я. Теория автоматического управления тепло-энергетическими процессами, Энергоатомиздат, Москва, 1985.
12. Vaccaro R. J., *Digital Control – A State-Space Approach*, McGraw-Hill Inc., Singapore, 1995.
13. Walters R. B., *Hydraulic and Electro-Hydraulic Control Systems*, Elsevier Applied Science, Elsevier Science Publishers Ltd., London and New York, 1991.

PROJEKTOVANJE ADAPTIVNOG UPRAVLJANJA ZA ODREĐENU KLASU HIDRAULIČKIH SISTEMA

Tamara Nestorović

Rad je posvećen projektovanju adaptivnih upravljačkih zakona u cilju upravljanja određenim hidrauličkim sistemima. Razmatrani hidraulički sistemi pripadaju klasi nivo sistema, koji su do sada razmatrani sa stavnovišta održavanja nivoa tečnosti u rezervoaru na željenoj konstantnoj vrednosti. Ovakav upravljački cilj mogao se ostvariti linearnim upravljačkim sistemom sa povratnim spregama po promenljivim stanja. Pomenuti hidraulički nivo sistemi sada se razmatraju

sa drugačije tačke gledišta, što zahteva uvođenje nelinearnog adaptivnog upravljačkog sistema. Naime, ukoliko je cilj upravljanja da se nivo tečnosti u rezervoaru menja po unapred zadatoj funkciji, onda su parametri objekta upravljanja promenljivi zbog promenljivog nivoa tečnosti u rezevoaru, zbog čega je potrebno uvesti adaptivno upravljanje. Ovde se razmatra i slučaj kada su parametri objekta upravljanja nepoznati.

Pri projektovanju adaptivnog upravljanja korišćeni su različiti pristupi. U ovom radu razmotren je gradijentni pristup (MIT pravilo), adaptivni sistemi sa referentnim modelom zasnovani na teoriji hiperstabilnosti i samopodešavajući regulatori. Postupci projektovanja razmatrani su u diskretnom obliku, što omogućava njihovu primenu u sastavu digitalnog sistema kojim upravlja računar.

U cilju projektovanja adaptivnog upravljanja, modeli razmatranih sistema predstavljeni diferencijalnim jednačinama, prevedeni su u odgovarajuće diskretne modele, a zatim su predstavljeni u polinomnom obliku koji predstavlja osnovu za projektovanje adaptivnog upravljanja. Kao rezultat projektovanja dobijeni su diskretni upravljački zakoni kojima se ostvaruje cilj upravljanja.

Verifikacija projektovanih upravljačkih zakona ostvarena je metodom numeričkog eksperimenta, odnosno simulacijom na digitalnom računaru, a dobijeni rezultati prikazani su u radu. Simulacije su izvršene za različite tipove refrentnih signala i izvršeno je njihovo upoređivanje. Na osnovu rezultata simulacije izvedeni su odgovarajući zaključci. U cilju izvršenja simulacija napisani su i određeni originalni programi koji predstavljaju softversku implementaciju projektovanih upravljačkih zakona.