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INTEGRAL METRICS WITH WEIGHT FUNCTIONS TO REGULARIZE SINGULARITIES NUMERICALLY

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Abstract. *The equation of satellite oscillations about its center of masses, which simultaneously moves on given elliptical orbit is under consideration. The solution is calculated on interval of time conforming to one revolution on orbit. The evolution of the Cauchy problem solution with the fixed initial data is studied depending on parameter e of an eccentricity of orbit. The initial conditions are set in a point of an apofocus. The point of pericentre corresponds to the left and right ends of a range where the solution is defined. The eccentric anomaly is used as independent variable. Parameter of an eccentricity can vary in range e н [0, 1]. The conditions of continuous transition to a limit in a space of solutions, when e = 1, are studied. Thus orbit in a limit becomes a line segment and the satellite has collision with center of gravity in pericentre. The equation of satellite oscillations in a point of pericentre has a singularity. When e = 1, the right side of differential equation has a non-uniform limit. The conditions of transition to the limit continuous on parameter of an eccentricity in space of solutions are studied. The computational implementation thus does not demand increase of simulation time when approaching to a limit case e = 1. To achieve the purpose cited at first a reduction of a system of the following differential equations is carried out. Here e is the parameter of a problem such, that e = 1 - e 2. Above ODE system is reduced to an integral equation of a kind in the space of phase variables derivatives. Here x0 is the vector of the initial data. The solution is prolonged on parameter of a problem e from some regular value e 0 № 0, regarding as non-perturbed. The reduction described through the formula preserves uniform topology in a phase space. Usage of the integral metrics for derivatives simultaneously allows construct regular solution prolongation algorithms on parameter even when the regularity in right sides of equations of motion is violated. With this purpose instead of L2([0,2p],R2) it is necessary to use the weight spaces with corresponding weight functions vanishing in singular points. The regularization of algorithms is reached due to topology exhaustion of solutions space in these points. Thus like it is in an implicit function theorem it is possible to construct an iterative process of approximating of the precise solution concurrently in any semi-norm of Frechet space automatically*

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ensuring a uniform convergence on any subinterval of a kind [d, 2p - d] π [0, 2p]. To prolong the solution on parameter the Newton's method in the weight spaces is used. In numerical implementations the algorithm of prolongation on parameter is performed for finite-dimensional Galerkin's systems, received from a system of precise equations.

1. Preliminary remarks

Cauchy problem for an ODE system of a form

$$\dot{\mathbf{x}} = \mathbf{X}(t, \mathbf{x}, \varepsilon), \quad t \in [t_0, t_1], \ \ \mathbf{x} \in D \subset \mathbf{R}^n, \tag{1.1}$$

in an obvious way is reduced to a nonlinear equation of a form

$$\mathbf{y}(t) = \mathbf{X}\left(t, \mathbf{x}_0 + \int_{t_0}^{t} \mathbf{y}(\tau) d\tau, \varepsilon \right), \quad \mathbf{y} \in L_2([t_0, t_1], \mathbf{R}^n) \,. \tag{1.2}$$

in the space of phase variables derivatives. Here ε be the parameter (scalar or vector) of a problem, D be the domain in $\mathbf{R}^n$, $\mathbf{x}_0$ be the vector of initial conditions of source Cauchy problem. Also if D is the domain in $\mathbf{R}^n$ then Ω be the domain in $L_2([t_0,t_1],\mathbf{R}^n)$. The right hand side $\mathbf{X}(t,\mathbf{x},\varepsilon)$ of the equation (1.1) can satisfy, for instance to Caratheodory conditions [1]. The solution is prolonged on the parameter ε from some value ε_0, regarding as unperturbed. The initial conditions one can also include into the set of parameters.

The reduction described is defined by the affine mapping

$$D^{-1}: L_2([t_0,t_1],\mathbf{R}^n) \to C([t_0,t_1],\mathbf{R}^n)$$

according to the formula for an antiderivative

$$\mathbf{x}(t) = \mathbf{x}_0 + \int_{t_0}^{t} \mathbf{y}(\tau) d\tau \,.$$

It is clear use of the integral metric in space of derivatives preserves the uniform topology in a phase space. From the computational point of view such reduction allows to construct regular algorithms to prolong the solution on parameter also when the regularity in right hand sides of ODE system is violated. In such cases it is necessary to use the weight spaces with weight functions vanishing in singular points instead of $L_2([t_0,t_1],\mathbf{R}^n)$. Regularization of algorithms is reached due to the use of a topology of solutions space in equation (1.2) weaker than above one. As a result of such approach a uniform convergence on phase variables in vicinities of singular point is violated. But in Frechet space with the topology defined by the system of seminorms of supremum type on any compact without singularity points, the convergence is ensured.

To prolong the solution numerically on parameter ε it is possible to use Newton's method in weight space. When implementing the corresponding algorithms it is necessary to construct a finite-dimensional Galerkin's system of equations, derived from a precise equation (1.2).

2. Example of a satellite on elliptic orbit

Consider oscillations of an asymmetrical satellite in a plane of elliptical orbit [2]. The orbital motion is considered as predefined. According to [3] satellite rotational motion equation about the center of mass one can present in a form

$$(1-e\cos t)\ddot{\delta}-(e\sin t)\dot{\delta}+\mu\sin(\delta-2\nu(t))=0\,. \qquad (2.1)$$

Here e be an eccentricity of satellite center of mass Keplerian orbit. As independent variable t the eccentric anomaly ($t \in [0,2\pi]$) is selected. The role of dependent variable play the one $\delta = 2\theta$ with θ as an angle between the central principal axis and the direction of orbit pericenter. True anomaly $\nu(t)$ be known function of independent variable, and μ be the numeric parameter describing satellite dynamic asymmetry.

Denote $x_1 = \delta$, $x_2 = \dot{\delta}$, and consider deformations of the Cauchy problem solution of the following system of ODE

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{(e\sin t)x_2+\mu\sin 2\nu(t)\cos x_1-\mu\cos 2\nu(t)\sin x_1}{1-e\cos t} \end{aligned} \qquad (2.2)$$

derived in a natural way from the equation (2.1). Initial conditions given in a midpoint of a segment where the solution is defined: $x_i(t_0) = x_{i0}$ $(i = 1,2)$, with $t_0 = \pi$. These deformations are constructed depending on parameter $e \in [0,1]$, which is the eccentricity of orbit. Parameter μ is considered thus as fixed. Instead of e the solutions will be regarded further depending on parameter $\varepsilon \in [-1,1]$ such, that $e = 1 - \varepsilon^2$.

When $\varepsilon = 0$ the system (2.2) has singularities at $t = 0,\pi$. This case corresponds to the satellite limit orbit, which has a shape of closed segment and allows encounters with center of a gravity at $t = 0,\pi$. Besides it is easy to check, that when $\varepsilon \to 0$ right hand sides of a system (2.2) have a non-uniform limit on an interval $(0,2\pi)$.

Indeed, one can see, that at any fixed $t \in (0,2\pi)$, when parameter $e \to 1$, functions

$$\cos\nu(t) \to -1, \qquad \sin\nu(t) \to 0.$$

Also by the same manner one can see, that this convergence is non-uniform on an interval $t \in (0,2\pi)$. On the other hand at $t = t_e = \arccos e$ for all $e \in [0,1)$ the following equalities are fulfilled

$$\cos\nu(t_e) = 0, \sin\nu(t_e) = 1.$$

Therefore at any fixed $t \in (0,2\pi)$, but also non-uniformly on all interval $(0,2\pi)$ one has

$$\cos 2\nu(t) \to 1, \qquad \sin 2\nu(t) \to 0.$$

In accordance with the general approach described above, the solutions space topology is defined by the integral metrics for phase variables derivatives. However singularity at $\varepsilon = 0$ is overcome due to the integral metrics with weight functions. For functions $y_1 = \dot{x}_1$ weight space $L_2^{\omega_1}[0,2\pi]$ with the weight $\omega_1(t) = 1 - \cos t$ is used. Correspondingly for $y_2 = \dot{x}_2$ one uses the weight space $L_2^{\omega_2}[0,2\pi]$ with the weight function $\omega_2(t) = (1 - \cos t)^2$.

One can transform source ODE system (2.2) to the following form

$$y_i = F_i(y_1, y_2,\varepsilon), \quad (i = 1,2),$$

or in vector notation

$$\mathbf{y} = \mathbf{F}(\mathbf{y},\varepsilon) \tag{2.3}$$

with the non-linear operator on a right hand side, defined according to formulae

$$\begin{aligned}[F_1(\mathbf{y},\varepsilon)](t) &= x_2[y_2](t),\\ [F_2(\mathbf{y},\varepsilon)](t) &= \frac{(e\sin t)x_2[y_2](t) + \mu\sin 2\nu(t)\cos x_1[y_1](t) - \mu\cos 2\nu(t)\sin x_1[y_1](t)}{1-e\cos t},\end{aligned}$$

Here functions $x_i[y_i](t)$ $(i = 1,2)$ are calculated on a segment $[0,2\pi]$ from functions $y_i(t)$ through the operator D^{-1}, which computes corresponding antiderivative

$$x_i[y_i](t) = [(D^{-1})y_i](t) = x_{i0} + \int_{t_0}^{t} y_i(\tau)d\tau .$$

The solutions space for an equation (2.3) is under construction by the way

$$Y = L_2^{\omega_1}[0,2\pi]\times L_2^{\omega_2}[0,2\pi] .$$

Through Hardy generalized inequality [4] of a kind

$$\left[\int_0^\infty \left| w(x)\int_0^x f(t)dt \right|^q dx\right]^{1/q} \le \text{const}\left[\int_0^\infty | v(x) f(x) |^p \, dx\right]^{1/p}$$

one can prove the following

Theorem 2.1.

The space Y is invariant when the operator of equation (2.3) right hand side acts

$$\mathbf{F}(\cdot,\varepsilon) : Y \to Y$$

for any fixed $\varepsilon \in [-1,1]$.

Below the solutions will be viewed to be in the metric of space Y. The algorithms of solutions prolongation on the parameter specified for equations of a kind (2.3) require the feasibility of some regularity conditions imposed on the operator $\mathbf{F}(\mathbf{y}, \varepsilon)$. The properties of interest are the regularity of this operator when $\varepsilon = 0$.

So, consider properties of mapping $\mathbf{F} : Y\times[-1,1] \to Y$ constructed above. Except the non-linear operator $\mathbf{F}(\mathbf{y},\varepsilon)$ itself at fixed $\varepsilon \in [-1,1]$ one must also consider its Frechet derivative on a variable $\mathbf{y}$. This one $d_\mathbf{y}\mathbf{F}(\cdot,\varepsilon) : Y \to \mathrm{L}(Y,Y)$ is an operator valued function with values in algebra $\mathrm{L}(Y,Y)$ of continuous linear operators, mapping the space Y in itself. Prove, that a non-linear function $\mathbf{F}(\mathbf{y},\varepsilon)$ defined on set $Y\times[-1,1]$ is continuous, and on the first argument is differentiated in sense of Frechet.

Through a technique similar above one, the following proposal is established.

Proposition 2.1.

In any point of the set $Y\times[-1,1]$ the operator $\mathbf{F} : Y\times[-1,1] \to Y$ is continuous and on the first variable has a Frechet derivative.

The continuity of a Frechet derivative one can establish in the topology weaker, than one in Y. For this purpose it is necessary to consider Frechet space Z, consisting from measurable vector-valued functions of a form $(y_1(t),y_2(t))$ with $t \in [0,2\pi]$. The topology in

Z is defined through variety of seminorms of a kind $\|\cdot\|_Y^\delta$, producing from the corresponding integral norm $\|\cdot\|_Y$ in the space Y, by restricting of integrating on subsegment of a kind $[\delta, 2\pi - \delta] \subset (0, 2\pi)$. From properties of weight functions it follows, that the topology in Z, described by the set of seminorms $\{\|\cdot\|_Y^\delta\}$, is equivalent to one given by the norms of the spaces of a kind $L_2[\delta, 2\pi - \delta] \times L_2[\delta, 2\pi - \delta]$. It is clear also, that the solutions space $Y \subset Z$ is continuously included in Z.

Regarding at fixed δ the solution on smaller segment $[\delta, 2\pi - \delta]$ after component-wise estimations for addends of a Frechet derivative expression , one can conclude

Proposition 2.2.

Frechet derivative $d_{\mathbf{y}}\mathbf{F} : Y \times [-1,1] \to \mathrm{L}(Y,Y)$ is the map, continuous on any seminorm $\|\cdot\|_Y^\delta$ with $0 < \delta < \pi$.

Below to construct Newton's iterative process when prolonging the solution on parameter one need establish property of continuous invertibility for a Frechet derivative. Let a point $(\mathbf{y}^0, \varepsilon^0) = (\mathbf{y}^0, 0) \in Y \times [-1,1]$ be fixed. As it was established above, the linear continuous operator $I - d_{\mathbf{y}}\mathbf{F}(\mathbf{y}^0, 0) : Y \to Y$ of Frechet derivative is defined in tangent space $T_{\mathbf{y}^0}Y = Y$. Here the problem of continuous invertibility of this operator in the same point $(\mathbf{y}^0, 0)$ is regarded. For this purpose it is necessary to establish, at first, unique solvability of an equation

$$(I - d_{\mathbf{y}}\mathbf{F}(\mathbf{y}^0, 0))\mathbf{h} = \mathbf{g}$$

for an arbitrary $\mathbf{g} \in Y$ and, secondly, continuity of corresponding inverse operator. One can put some sufficient conditions of continuous.

Proposition 2.3.

When $\mu > 9/8$ and for all initial conditions x_{10}, x_{20}, excluding points of some curves in a phase space Frechet derivative $I - d\mathbf{F}_{\mathbf{y}}(\mathbf{y}^0, 0)$ of the left-hand side operator of an equation

$$\mathbf{y} - \mathbf{F}(\mathbf{y}, \varepsilon) = \mathbf{0}$$

on a variable $\mathbf{y} \in Y$ in a point $(\mathbf{y}^0, 0) \in Y \times [-1,1]$ is continuously invertible in tangent space Y.

In the proof the results of work [5] on asymptotic properties of the equation (2.2) solutions in a limit ($\varepsilon = 0$) case have been used.

3. Computational Consequences

Now consider a procedure of an equation (2.2) solution prolongation on parameter ε. Using for this purpose Newton's method one can rewrite an equation (2.2) by the way

$$\mathrm{H}(\mathrm{y}, \varepsilon) = 0 \qquad (3.1)$$

with the left-hand side operator calculated according to the formula

$$\mathbf{H}(\mathbf{y}, \varepsilon) = \mathbf{y} - \mathbf{F}(\mathbf{y}, \varepsilon)$$

The solution is constructed in the form of implicit functions $\mathbf{y}(\varepsilon)$, depending upon the parameter ε, in vicinity of some value $\varepsilon = \varepsilon^0$. When $\varepsilon^0 \neq 0$ the regular case regarded earlier elsewhere takes place. The case of interest is the singular one, when $\varepsilon^0 = 0$.

In section 2 was established, that besides exceptional cases Frechet derivative of the operator $\mathbf{F}(\cdot,0)$ when $\varepsilon = 0$ is the continuously invertible. The same concerns the operator $\mathbf{H}(\cdot,0)$ on the solution $\mathbf{y}^0$ of an equation (3.1), and therefore

$$A = d_{\mathbf{y}} \mathbf{H}(\mathbf{y},\varepsilon)\Big|_{\mathbf{y}=\mathbf{y}^0,\varepsilon=0} \in \mathrm{L}(Y,Y).$$

Further for definiteness one will guess as fulfilled sufficient conditions of continuous invertibility from the proposition 2.3.

By virtue of work [5] results when $\varepsilon = 0$ equation (3.1) has the solution $\mathbf{y}^0 \in Y$. The uniqueness of this solution follows from the known ODE theorem. To solve the equation (3.1) in neighborhood of a point $\varepsilon = \varepsilon^0 = 0$ let replace it on equivalent one

$$\mathbf{y} = \mathbf{Q}(\mathbf{y},\varepsilon)$$

with

$$\mathbf{Q}(\mathbf{y},\varepsilon) = \mathbf{y} - A^{-1}\mathbf{H}(\mathbf{y},\varepsilon).$$

According to previous results operator $\mathbf{F}(\cdot,\varepsilon) : Y \to Y$ is correctly defined and is continuous. Moreover, for any $\varepsilon \in [-1,1]$ this operator is differentiated in sense of Frechet. Unfortunately continuity of the derivative itself one can establish only in a topology of space Z.

Summarize the results earlier obtained. Consider sequence of elements $\{\mathbf{y}^n\}_{n=1}^{\infty} \subset Y$, constructed by the iterative formula

$$\mathbf{y}^{n+1} = \mathbf{Q}(\mathbf{y}^n,\varepsilon). \tag{3.2}$$

Observe the convergence of this sequence. For this purpose construct in space Y the set of seminorms $\{\|\cdot\|_Y^\delta\}$, defined by the formula

$$\|\mathbf{y}\|_Y^\delta = [(\| y_1 \|_2^{\omega_1,\delta})^2 + (\| y_2 \|_2^{\omega_2,\delta})^2]^{1/2},$$

where each seminorm, included in a right hand side looks like ($i = 1,2$)

$$\|y_i\|_2^{\omega_i,\delta} = \left[\int_\delta^{2\pi-\delta} \omega_i(t)\,|\,y_i(t)\,|^2\,dt\right]^{1/2}.$$

One can easy to see any seminorm $\|\cdot\|_2^{\omega_i,\delta}$ is equivalent to the norm $\|\cdot\|_2^\delta$ in a topology of space $L_2[\delta,2\pi - \delta]$. Together, at every possible δ, these seminorms define a topology of Frechet space Z, mentioned in the previous section.

Proposition 3.1.

For any seminorm $\|\cdot\|_Y^\delta$ there exist such positive Λ, Δ, that for any $\varepsilon \in [-\Lambda,\Lambda]$ the operator $\mathbf{Q}(\cdot,\varepsilon) : Y \to Y$ is compression in $\|\cdot\|_Y^\delta$ in neighborhood $U_\Delta^\delta(\mathbf{y}^0)$, where

$$U_\Delta^\delta(\mathbf{y}^0) = \{\mathbf{y} \in Y : \| \mathbf{y} - \mathbf{y}^0 \|_Y^\delta < \Delta\} .$$

Fix below positive $q < 1$. Then find such a condition, which ensures the invariance of closed ball $[U_\Delta^\delta(\mathbf{y}^0)]$ in seminorm $\|\cdot\|_Y^\delta$ when map $\mathbf{Q}(\cdot,\varepsilon) : Y \to Y$ acts. Return temporarily to the metric of space Y. One has the estimation

$$\|\mathbf{Q}(\mathbf{y}^0,\varepsilon) - \mathbf{y}^0\|_Y = \|A^{-1}\mathbf{H}(\mathbf{y}^0,\varepsilon)\|_Y \leq \|A^{-1}\|_Y \|\mathbf{H}(\mathbf{y}^0,\varepsilon)\|_Y.$$

By virtue of proposition 2.1 the operator $\mathbf{H} : (\mathbf{y},\varepsilon) \mapsto \mathbf{H}(\mathbf{y},\varepsilon)$ is continuous on all its arguments. Therefore $\mathbf{H}(\mathbf{y},\varepsilon) \to \mathbf{H}(\mathbf{y},0)$ when $\varepsilon \to 0$. Now select Λ such, that for all $\varepsilon \in [-\Lambda,\Lambda]$ the following inequality takes place

$$\|A^{-1}\|_Y \|\mathbf{H}(\mathbf{y}^0,\varepsilon)\|_Y \leq (1 - q)\Delta$$

Thus, when $\mathbf{y} \in U_\Delta^\delta(\mathbf{y}^0)$ one can derive

$$\left\|\mathbf{Q}(\mathbf{y},\varepsilon) - \mathbf{y}^0\right\|_Y^\delta \leq \left\|\mathbf{Q}(\mathbf{y},\varepsilon) - \mathbf{Q}(\mathbf{y}^0,\varepsilon)\right\|_Y^\delta + \left\|\mathbf{Q}(\mathbf{y}^0,\varepsilon) - \mathbf{y}^0\right\|_Y^\delta \leq q\left\|\mathbf{y} - \mathbf{y}^0\right\|_Y^\delta + \left\|A^{-1}\right\|_Y \left\|\mathbf{H}(\mathbf{y}^0,\varepsilon)\right\|_Y \leq$$
$$\leq q\Delta + (1-q)\Delta = \Delta,$$

That provides an invariance of a ball $[U_\Delta^\delta(\mathbf{y}^0)]$ under $\mathbf{Q}(\cdot,\varepsilon) : Y \to Y$ in seminorm $\|\cdot\|_Y^\delta$.

The results obtained demonstrate when $\varepsilon \in [-\Lambda,\Lambda]$ the equation (3.1) solution exists and is a single-valued function of parameter ε. Check up continuity of an implicit function $\mathbf{y}(\varepsilon)$ on seminorm $\|\cdot\|_Y^\delta$. One has

$$\left\|\mathbf{y}(\varepsilon) - \mathbf{y}^0\right\|_Y^\delta \leq \left\|\mathbf{Q}(\mathbf{y}(\varepsilon),\varepsilon) - \mathbf{y}^0\right\|_Y^\delta \leq \left\|\mathbf{Q}(\mathbf{y}(\varepsilon),\varepsilon) - \mathbf{Q}(\mathbf{y}^0,\varepsilon)\right\|_Y^\delta + \left\|\mathbf{Q}(\mathbf{y}^0,\varepsilon) - \mathbf{y}^0\right\|_Y^\delta \leq$$
$$\leq q\left\|\mathbf{y}(\varepsilon) - \mathbf{y}^0\right\|_Y^\delta + \left\|A^{-1}\right\|_Y \left\|\mathbf{H}(\mathbf{y}^0,\varepsilon)\right\|_Y,$$

And immediately the estimation is derived

$$\left\|\mathbf{y}(\varepsilon) - \mathbf{y}^0\right\|_Y^\delta \leq \frac{\left\|A^{-1}\right\|_Y \left\|\mathbf{H}(\mathbf{y}^0,\varepsilon)\right\|_Y}{1-q} \to 0$$

when $\varepsilon \to 0$. As a result the following implicit function theorem is obtained.

Theorem 3.1.

For any seminorm $\|\cdot\|_Y^\delta$ there exist such Λ, $\Delta > 0$, that for any $\varepsilon \in [-\Lambda,\Lambda]$ equation (3.1) has in a ball $U_\Delta^\delta(\mathbf{y}^0)$ the unique solution $\mathbf{y}(\varepsilon)$, continuous in a point $\varepsilon = 0$ in seminorm $\|\cdot\|_Y^\delta$.

From the theorem proof follows, that the convergence in any seminorm $\|\cdot\|_Y^\delta$ is provided within the frame of the same iterative process (3.2), defined in space Y. It is known [5], that when $t \to 0,2\pi$ the function $\mathbf{y}^0(t)$ performs infinite number of oscillations, decreasing on amplitude. Therefore it is clear, that when computational resources are limited it is possible to speak on uniform approximation of the precise solution only on subsegment $[\delta, 2\pi - \delta]$ for $\delta > 0$ small enough.

In such circumstances it is necessary to use seminorm $\|\cdot\|_Y^\delta$. If one has in regular area (when $\varepsilon \neq 0$) approximation of the solution $\mathbf{y}(\varepsilon)$ on all segment $[0, 2\pi]$ precise enough, then "moving" on parameter ε to zero point, using, as usual, for this purpose Newton's method, one can derive the limit problem solution approximation on subsegment $[\delta, 2\pi - \delta]$. Concurrently more accurate approximating in a regular case implies more wide subsegment $[\delta, 2\pi - \delta]$ as a result of prolongation.

The convergence in the seminorm $\|\cdot\|_Y^\delta$ automatically results the convergence in the norm of space $L_2[\delta, 2\pi - \delta]$ for phase variables derivatives, that in turn provides a uniform convergence for functions of phase variables. In actual calculations instead of a precise equation (2.3) it is necessary to use its finite-dimensional Galerkin reduction.

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REFERENCES

1. Filippov, A. F., (1988) *Differential equations with discontinuous right-hand sides.* Mathematics and Its Applications: Soviet Series, 18. Dordrecht etc.: Kluwer Academic Publishers. x, 304 p.
2. Beletsky, V. V., (1965) *Motion of an artificial satellite about its center of mass.* Moscow: Science. 416 p. (in Russian).
3. Beletsky, V. V., (1975) *Motion of a satellite about its center of mass in field of gravity.* Moscow: Moscow Univ. Press. 308 p. (in Russian).
4. Maz'ya, V. G., (1985) *Sobolev spaces.* Berlin etc.: Springer-Verlag. xix, 486 p.
5. Bruno, A. D., Varin, V. P., (1997) *The Limit Problems for the Equation of Oscillations of a Satellite*. Celestial Mechanics and Dynamical Astronomy, **67**, 1–40.

METRIKA INTEGRALA SA FUNKCIJAMA TEŽINE DA BI SE REGULISALI SINGULARITETI NUMERIČKI

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Jednačina satelitskih oscilacija oko svog centra masa, koje se istovremeno kreću po zadatim eliptičnim orbitama se razmatra u ovom radu. Rešenje se izračunava za interval vremena koji je potreban da se ostvari jedan obrtaj po orbiti. Evolucija rešenja Košijevog problema sa nepromenljivim početnim podacima je proučena u zavisnosti od parametra e koji predstavlja ekscentričnost orbite. Početni uslovi su smešteni u tački aro-fokusa. Tačka pericentra odgovara levom i desnom kraju opsega gde je rešenje definisano. Anomalija ekscentričnosti se koristi kao nezavisno promenljiva. Parametar ekscentričnosti se menja u opsegu [0,1]. Uslovi neprekidnosti prelaska na ograničenje u prostoru rešenja kada je e=1 se proučavaju u ovom radu. Stoga orbita u graničnom uslovu postaje segment jedne linije i satelit ima sudar sa centrom gravitacije u pericentru. Jednačina satelitskih oscilacija u tački pericentra ima singularnost. Kada je e=1 desna strana diferencijalne jednačine ima neuniformno ograničenje. Uslovi prelaska na ograničenje koje je neprekidno po parametru ekscentričnosti u prostoru rešenja se proučava u ovom radu. Primena proračuna stoga ne zahteva povećanje vremena simulacije kada se približava graničnom slučaju e=1. Da bi se postigla namera navedena na početku, vrši se redukcija sistema sledećih diferencijalnih jednačina. Ovde je e-parametar jednog takvog problema tako da je $e=1-e_2$. Gornji

sistem običnih diferencijalnih jednačina se redukuje na jednu integralnu predstavljenu u prostoru izvoda faznih promenljivih. Ovde xO predstavlja vektor početnih podataka. Rešenje je produženo po parametru postavljenog problema e od neke regularne vrednosti e_o*=O koja se smatra kao neporemećena vrednost. Redukcija opisana putem ove formule zadržava uniformnu topologiju u faznom prostoru. Upotreba integralnih mera za izvode istovremeno omogućava konstrukciju regularnog rešenja algoritama produžavanja po parametru čak i kada regularnost na desnoj strani jednačina kretanja je narušena. S ovom namerom umesto L2([0,2p],R2) neophodno je koristiti prostore težine sa odgovarajućim funkcijama težine koje isčezavaju u singularnim tačkama. Regularizacija algoritama se postiže usled iscrpljivanja topologije prostora rešenja u ovim tačkama. Stoga slično onome kako je to u jednoj teoremi implicitnih funkcija, moguće je konstruisati iterativni proces aproksimacije preciznog rešenja istovremeno.*