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LETTERS TO EDITOR

ASYMPTOTIC CORRESPONDENCE OF PHYSICAL THEORIES

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INTRODUCTION

Almost any physical theories, formulated in mathematical terms in a general way, are extremely complicate. Thus both in creating a theory and in its further development, of paramount importance are the simplest limiting cases that admit an analytical solution. Commonly, for them that the number of equations is decreased, the equations' order becomes smaller (is reduced), the non-linear equations are replaced by the linear ones, the original system is subjected to a kind of averaging, etc. Behind the above idealizations, however diverse they may seem, lies a high degree of symmetry inherent in a mathematical model of a phenomenon at issue in its limiting situation. An asymptotic approach to a complex, "insoluble" problem consists basically of treating an original insufficiently symmetric - system as approximating to a given symmetric one. It is basically important that determining corrections allow for deviations from a limiting cases is much simpler than a direct study of the original system. At first sight, the potentialities of such an approach are limited by a narrow range of system parameter variations. Experience gained in the study of various physical problems has shown, however, that in the case of system parameters varying considerably and the system itself departing from one limiting symmetric pattern, another limiting system, often with a less pronounced symmetry generally exists and a perturbed solution can now be formed for the latter one. This enables the system's behavior defined over the entire range of parameter variations using a finite number of limiting cases. Such an approach uses the most of the physical intuition and contributes to its further development, while leading to the forming of new physical concepts. Thus the boundary layer, an important concept in the fluid mechanics, is of pronounced asymptotic nature and is related to the localization at the boundaries of a streamlined body in the zone where fluid viscosity cannot be neglected. In the mechanics of deformable rigid body and in the electricity theory, similar phenomena are termed the "edge effect" and "skin effect" respectively. These questions were clarified in [1-3].

ASYMPTOTIC APPROACHES AND PROGRESS IN MODERN PHYSICS

Frequently the progress in modern physics appears to be closely connected with typical (small or large) asymptotic parameters. In particular, smallness of famous "thin structure constant" $a = e^2/h$ (e is electron charge, h is Plank constant, c is velocity of light) gives us possibility to investigate the photons' and electrons' interaction in the quantum electrodynamics with high precision. This non-dimensional parameter defines the intensity of elec-

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tromagnetic interactions. All fundamental results of quantum electrodynamics, describing experimental data with surprising accuracy, have been obtained due to the use of perturbation theory [4]. In this theory the solutions are represented by expansions in powers of a. Similar parameters for the case of strong interaction particles - hadrons (for example, neutrons and protons) exceed an in value multifold. This is the main cause of essential difficulties on the way of development of strong interaction theory. Only the discovery of quantum quark structure of hadrons and of the phenomenon of "asymptotic freedom" [5] (the essence of this phenomena is in relaxation of interaction between quarks and interconnective gluons at the short distances) immediately changed situation and gave birth of new theory of strong interaction - quantum chromodynamics. Often the possibilities and ways of using small or large parameters, natural for some physical theory, may be completely realized only in a long run. For example, dependence of rigid characteristics in some point of elastic body upon its position (nonhomogeneity) or chosen direction (anisotropy) arises serious difficulties for investigators. Many methods of solution of isotropic media problems, based on specific symmetries of such media, can't be used for anisotropic or nonhomogeneous case. But in the case of strong anisotropy (or nonhomogeneity) one can invent the set of small or large parameters (as a rule, ratio of rigidity in the various directions)and, using suitable asymptotic methods, achieve the fast development of such media theory. Sometimes the simplified equations of anisotropic media appear to be less - complicated than ones of isotropic and homogeneous model [6]. Let us consider another example. It is known that the difficulties of analysis of molecular systems grow fast with increase of size and mass of molecules. Meantime, in the case of macromolecules of polymers, there are natural small and large parameters. The first evident large parameter for such systems is the number of atoms in the chain, N. This parameter allows to investigate even the single polymer molecule as macroscopic system, to apply effective averaging procedure commonly used in statistical physics. One of the main problems of modern physics of polymers - study of asymptotic behavior of polymer systems for $N \to \infty$. In particular, such fundamental characteristic as average size of polymer ball during dissolving or melting of polymer, b is defined by relations $r \sim N^{\beta}$, where exponent β depends upon physical conditions of polymer system [7]. Furthemore, polymer systems possess a set of small parameters, subjected to the interaction hierarchy. Covalent interaction (canonical bond) of atoms along the chain is stronger than any other ("physical") kind of interaction. It gives us possibility to assume atom sequence along the chain to be fixed. Intensity of physical interactions of other types also differs significantly. The simplest of asymptotics corresponds to neglecting of all physical interactions (assuming the lengths of bonds fixed). Next, anyone can take into account the physical interaction between links of the polymer chain, depicting chain resistance to bending and torsion (as earlier, lengths of bonds are supposed to be fixed). At last, interaction between spatially adjacent (but not neighboring along the chain!) link of twisted polymer chain may be taken into account.

ASYMPTOTIC CORRESPONDENCE

Of no less importance is the fact the asymptotic method assists in relating different physical theories with one another. Albert Einstein would point out that "the happiest lot of a physical theory is to serve as a basis for a more general theory while remaining a limiting case thereof'. Naturally, most impressive and rich in content examples of the asymptotic correspondence appear in revealing the relations among the fundamental physical theories. Each new theory, brought in by advancement of science, used to be considered as the negation of the preceding one, i.e. the incompatibility of the old and the new ideas and concepts, that had come to replace them, was pushed into the foreground. Only the formulation of Bohr's correspondence principle and creation of quantum mechanics brought the successiveness of physical theories under thorough investigation of physicists and philosophers. Despite the variety of viewpoints, even of alternative ones, upon the correlation of successive physical theories, the mathematical link between these theories may be easily revealed. Such link may be expressed in the terms of asymptotic correspondence, which becomes apparent in various, very often in implicit forms. There are, so to speak, various types of 'limit transitions' from a new theory to the old one, as a rule, for zero or infinite value of some parameters or variables. New theory may be considered as generalization of the previous one (one can recall Einstein words cited in), but this generalization is not only qualitative, but also a quantitative one. That's why new theory includes possibilities totally unpredictable before. Often such possibilities become explicitly apparent in the contrarily limiting cases, when parameters, presumed small, become large and vice versa. The effects, formally principal, become negligible, and new contents of physical theory shows out in explicit form. We'll try to retrace this correspondence for various physical theories.

FROM ARISTOTLE TO NEWTON

At first, in this context let's try to discuss the transition from Aristotelian forced motion theory to Newton's mechanics, which is a good example of radical change of scientific conceptions, views and methods predominating during long time. Meantime even in such a case the asymptotic relation comes to light showing the applicability of Aristotle's model for high friction resistant motion. It seems not so surprising because Aristotles' reasoning had relied on the intuitive representations following from everyday observation of moving objects in the restricted range of external conditions and certainly containing a seed of truth [8]. The investigations of psychologists confirm that even some our contemporaries, being not acquinted enough with modern theory, easily come to Aristotle's kind of conclusions and explanations [9,10]. There are conceptions of force as a cause of motion, of arrest of movement due to running out of driving force - "impetus", of vertical downfall of body thrown from horizontally moving object, at last - of different downfall time of the bodies with different weight. In this investigations the surprising similarity between views of antique or medieval philosophers and our contemporaries' ones was found. As a rule, in this investigations the special emphasis is laid on the unconsistency of Aristotelian views with Newton mechanics. Meantime in the field of usual human experience, namely under the earth conditions, the endurance of these representations may be explained from Newton's mechanics point of view, namely, by asymptotic relation mentioned above. This relation may be put up despite the deepest ideological distinctions between old and new theories and essential contradictions in philosophical concepts which it originate from. To confirm above mentioned let us consider the motion of body subjected to the constant force F through the media with friction coefficient h. Aristotle had not considered the friction force itself; he considered it as natural and unremovable attribute of the motion. He had not also formulated the motion law in the mathematical language also. Aristotle's "motion law" (in the case of linear dependence of resistance force upon the velocity) may be written as

$$F = h\, v$$

If the force is constant the velocity will be constant too. The increase of the force gives rise to the increase of the velocity. These conclusions in general are in the accordance with observations of motion under the Earthly conditions if the resistance is large enough. According to Newton the resistance force is the external one and the motion law for a point-line particle under the conditions mentioned above has a form:

$$M\, \mathrm{d}v/\mathrm{d}t = F - hv$$

In the absence of initial velocity one has

$$V = (F/h)(1 - \exp(-ht/m))$$

For the case of high frictional resistance, the second (transitional) term diminishes rapidly, "switching off the force" (presuming long enough observation period).The remaining term corresponds to Aristotle's mechanics. Obviously, observations of motion at small friction levels should show at once the essential variability of velocity and its slow approach to the stabilization value. The everyday experience of ancient Greeks apparently lacked such kind of observations. Only 2000 years later Galilei made the idealized mental experiment and came to understanding of the inertial motion as one of the main initial concepts of the New Times' physics [8]. From the physical point of view the Aristotle's approach preserves its importance as asymptotics of the motion for the long time span, the larger friction the earlier time point at which this approximation will be applicable. From mathematical point of view we meet with singular perturbations here: the velocity v increases not smoothly but infinitely quickly. In such a case the additional asymptotics exists which may be easily found studying if the behavior of exact solution at small values of exponent power:

$$V = Ft/m$$

It describes the nonuniform motion of the body with the constant acceleration induced by the constant force in the absence of resistance. This solution is correct for any friction level, if observation time is small enough. Smaller the friction, wider the solution's applicability (and the later is the time-point of reaching Aristotle's asymptotics). The equation of motion corresponding to the small times

$$M\mathrm{d}v/\mathrm{d}t = F$$

represents the mathematical note of the well-known Newton's law. Here we enter in the field of the conservative, or Hamiltonian systems (the constancy of the mechanical energy is valid), which allows the forms of motion absolutely alien for Aristotle's mechanics: vibrations, periodic rotations. Theory of the conservative systems is the most important part of the Newton's mechanics because the motion modes described by it (in particular, periodic and nearly periodic ones) for many physical systems are the very good approximations to the reality. Nevertheless, Aristotle's approximation has its own field of applicability, when the friction becomes large enough, as, for example, for the motion of the polymeric molecules in the solutions. Such systems are called overdamped and corresponding dynamic processes relaxation, that is transition to equilibrium.

NEWTON'S MECHANICS AND PARTIAL THEORY OF RELATIVITY

The creation of the theory of relativity broke the notions, considered the only possible ones, deeply implanted and of Newton's mechanics - about, independence of space and time, about absolute time and so on. But Newton's mechanics, as it should be, was not rejected by the partial theory of relativity and had become its asymptotic limit [11]. Asymptotic relation between these two theories may be illustrated by the example of the particle with mass m which 0 undergoes the action of constant force F beginning from the time $t = 0$. It can be easily shown, in the partial theory of relativity the particle's velocity in unmovable coordinate system will be

$$v = V/C$$

where $V = Ft/m$, C is square root from R, R is sum 1 and square of V/c.

The solution in the framework of Newton's mechanics corresponds to asymptotics for the small times or velocities ($V/c << 1$). First correction to this solution is very small:

$$v = V(1 - 0.5R)$$

In the theory of relativity there is an additional asymptotics of "the large times", irrelevant to Newton's mechanics. Indeed, taking into account the first correction, expression for velocity yields:

$$v = c(1 - 0.5/R)$$

It brings us new concepts of simultanuity, absence of absolutely rigid bodies and so on, reflecting the deepness of ideological revolution accomplished by the theory of relativity.

GEOMETRIC AND WAVE OPTICS

The study of relationship between wave and geometric optics is of importance both itself and for understanding of the relationship between classical and quantum mechanics. For a long time it was accepted for good that the elementary geometric constructions form the basis of geometric optics. After discovering of light diffraction the wave theory became generally accepted, while geometrical optics, seemed kind of homespun prescription, which doesn't reflect fundamental laws of the Nature. Only in the twenties of our century it was clearly determined that transition from wave to geometric optics is connected with small wavelength $l \to 0$. Since $l = 10^{-7}$ m for visible light geometric optics is the good approximation in many cases [12-14]. From mathematical viewpoint transition to geometric optics is performed in the framework of so called WKB-method (named after Wentzel, Kramers and Brillouin). In the space point with coordinates (x,y,z) every characteristic of electromagnetic field in the light wave $U(x,y,z)$ is represented in the form:

$$U = A(x,y,z,l)\exp(q(x,y,z)/l),$$

where A is wave amplitude and q is wave phase. Then A and q are represented in the form of power series of $1/l$. After substitution of this expression into wave equation and sorting off the terms with equal powers of *1*, the nonlinear differential equation for phase φ may be obtained (eikonal equation). Namely, this one corresponds to geometrical optics. For determination of the expansion coefficients the recurrent sequence of linear differential equations may be obtained (so called transfer equations). In the geometrical optics it is supposed that light rays propagate along certain curves. The edge of the beam seems very sharp, but in reality the intensity of light boundary changes although quickly but continuously in boundary layer which thickness has the order of wave length *l*. Asymptotics describing purely wave phenomenon of diffraction can be constructed using boundary layer concept.

CLASSICAL AND QUANTUM MECHANICS

The relationship between classical and quantum mechanics in certain sense is similar to that which exist between geometric and wave optics. In quantum mechanics the wave function W of quasiclassical, that is almost classical physical system may be represented in the form $W = A\ \exp(S/h)$, where S is so called action. The small parameter here is the ratio h/S. Transition from quantum to classical mechanics formally is described by the WKB-method at h tends to 0. The essence of such transition is that the center of localized wave packet which is the initial probability distribution of the particle coordinates moves then in accordance with laws of classical mechanics. But the quasiclassical approach loses its sense for very small particle momentums. This occurs, for example, in the vicinity of "turning points", where presuming classical mechanics is valid the particle should stop and reverse its movement. But in the quantum mechanics the principally nonclassical phenomenon becomes possible "tunneling" of the particle over the potential barrier. This phenomenon can be described by the asymptotics using just the smallness of the momentum. In the process of creation of quantum mechanics heuristic role of asymptotic correspondence was manifested outstandingly. This role increases especially nowadays when the attempts of construction of the theory uniting all fundamental interactions are performed. In the framework of the such theory concepts of electromagnetic, weak, strong and gravitational interactions themselves have to be asymptotic, having the sense only for small energy levels. Let's emphasize that constructive role of asymptotic relationship between classical and quantum mechanics reflected in - the famous Bohr's "correspondence principle" in the special physical and philosophical literature. Meantime we include this example in our topic for the sake of completeness of the picture.

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