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# APPLICATION OF THE INTERFACE CRACK CONCEPT TO PROBLEM OF A CRACK BETWEEN A THIN LAYER AND A SUBSTRATE

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**Abstract**. *In this paper the problem of a crack lying along the interface of a thin film and a substrate is considered. For thin layers, the residual tensile stresses that develop in the layer are of major importance. In the present paper an interpretation of this problem by the concept of linear elastic fracture mechanics is presented. The energy release rate and the stress intensity factor are determined in terms of a dimensionless factor,* $\omega$*, that is function of specimen geometry and elastic material properties. The problem of thin layers under conditions of residual tensile stresses is of utmost importance in applications related to composite materials manufacturing, electronic devices design, protective coating, among others.*

**Key words**: *Interface crack consept, thin film, substrate, residual tensile stresses, stress intensity factors*

## 1. INTRODUCTION

The most interesting problem of thin films is that of films subjected to residual tensile stresses. Films in tension can decohere from the substrate by relaxing the residual stress in the film above the interfacial crack. Decohesion takes place when energy release rate of interface $G_{int.}$, exceeds the interface debonding energy $G_{cleav.}$, i.e.,

$$G_{\text{int.}} \geq G_{cleav.} \tag{1}$$

However, interface-debonding energy may be a strong function of the mode mixity $\psi$. It is not sufficient to know only the energy release rate of the interface but the mode mixity $\psi$ must also be calculated.

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## 2. INTERFACE CRACK

The specific problem of crack lying along bimaterial interface of two linearly elastic isotropic materials, is presented in Figure 1.

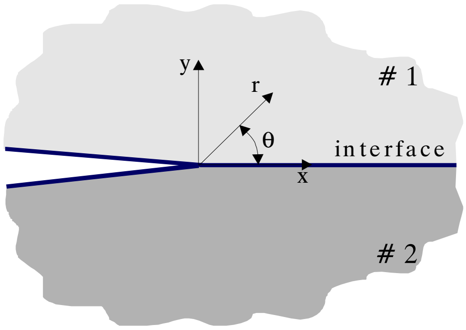


Fig. 1. Interface crack between two dissimilar materials

Let a material with elastic properties $E_1$, $\nu_1$, $\mu_1$ occupy the upper half - plane, $y > 0$, and material with elastic properties $E_2$, $\nu_2$, $\mu_2$ occupy the lower half - plane, $y < 0$. The two materials 1 and 2 are bonded along the positive part of the $x$ - axis.

The near tip stress field for an interfacial crack between dissimilar isotropic bimaterials is a linear combination of two types of fields. The first is coupled oscillatory field defined by a complex stress intensity factor $K$, while the second is non - oscillatory field scaled by a real mode III stress intensity factor $K_{III}$. The near tip stress field for an interface crack has the form:

$$\sigma_{\alpha\beta} = \frac{1}{\sqrt{2\pi r}}[\mathrm{Re}(Kr^{i\varepsilon})\Sigma^{I}_{\alpha\beta}(\theta,\varepsilon) + \mathrm{Im}(Kr^{i\varepsilon})\Sigma^{II}_{\alpha\beta}(\theta,\varepsilon) + K_{III}\Sigma^{III}_{\alpha\beta}(\theta)]\,, \tag{1}$$

here $r$ and $\theta$ are polar coordinates and indices $\alpha$, $\beta$ refer to coordinates $x$, $y$, $z$. $\Sigma^{I,II,III}_{\alpha\beta}(\theta)$ are the angular functions which correspond to tensile tractions, in - plane shear tractions and anti - plane shear tractions across the interface, respectively, so that the tractions, at distance r ahead of the crack tip, take the form:

$$\begin{aligned}(\sigma_{yy} + i\sigma_{xy})_{\theta=0} &= \frac{Kr^{i\varepsilon}}{\sqrt{2\pi r}}\\ (\sigma_{yz})_{\theta=0} &= \frac{K_{III}}{\sqrt{2\pi r}}\end{aligned} \tag{2}$$

and in this sense may be said that $\Sigma^{I,II,III}_{\alpha\beta}(\theta)$ correspond to modes I, II and III of crack growth, [3].

There is no unique physical interpretation for bimaterial interfacial crack, such as in homogeneous materials. Namely, symmetry and anti - symmetry modes are entirely separated for homogeneous material. For interface crack, symmetry and anti - symmetry modes are coupled together. However, $\Sigma_{\alpha\beta}^{I,II}(\theta)$ also depend on elastic properties of bimaterial combination through the parameter ε. The parameter ε is called the bielastic constant or the oscillatory index, and is given by:

$$\varepsilon = \frac{1}{2\pi}\ln\left(\frac{1-\beta}{1+\beta}\right). \tag{3}$$

Here β is one of two Dundurs parameters, [4]:

$$\alpha = \frac{\mu_2(\kappa_1+1)-\mu_1(\kappa_2+1)}{\mu_2(\kappa_1+1)+\mu_1(\kappa_2+1)}\ ,\ \ \beta = \frac{\mu_2(\kappa_1-1)-\mu_1(\kappa_2-1)}{\mu_2(\kappa_1+1)+\mu_1(\kappa_2+1)}\ . \tag{4}$$

where: $\mu_i$ is the shear modulus, $\kappa_i = 3 - 4\nu_i$ for plane strain and $\kappa_i = (3 - \nu_i)/(1 + \nu_i)$ for plane stress and $\nu_i$ is the Poisson's ratio and subscripts 1 and 2 refer to materials 1 and 2, respectively.

In equation (1) $K_{III}$ presented the mode III stress intensity factor, which has the same form as for homogeneous solid. As opposite to homogeneous material, where mode I and II factors are separated, $K_I$ and $K_{II}$, for the interface crack there is a single complex stress intensity factor K for in - plane modes. Those two stress intensity factors have different dimensions, $K$ = [stress][length]$^{1/2-i\varepsilon}$ and $K_{IWI}$ = [stress][length]$^{1/2}$.

The complex stress intensity factor is a property of interface crack. That complex stress intensity factor has the generic form:

$$K = YT\sqrt{L}\,L^{-i\varepsilon}e^{-i\psi}\ , \tag{5}$$

where $T$ is a stress magnitude due to load applied to specimen (load), $L$ - a characteristic length (crack length, layer thickness), $Y$ - a dimensionless real positive quantity, and ψ is a phase angle of $Ka^{li\varepsilon}$. It is often called the phase angle of the complex stress intensity factor, or the phase angle of the applied load. Both $Y$ and ψ are dependent on applied load in general, on the ratio of elastic moduli and of characteristic dimensions of the cracked body.

Considering equations (1) and (2), one may conclude that for bimaterial case $K_I$ and $K_{II}$ are not constant. In fact, they are defined as functions of r and denoted as $K_1$ and $K_2$:

$$\begin{aligned} K_1 = K_I(r) \equiv \mathrm{Re}(Kr^{i\varepsilon}) = YT\sqrt{L}\cos[\psi - \varepsilon\ln(L/r)] \\ K_2 = K_{II}(r) \equiv \mathrm{Im}(Kr^{i\varepsilon}) = YT\sqrt{L}\sin[\psi - \varepsilon\ln(L/r)] \end{aligned}\ . \tag{6}$$

For ε = 0 the mode mixity ψ can be defined in the usual way, [5]. When all three modes are present, the mode mixity is fully specified by two solid angles, ψ and ϕ, in the space of the interface traction vector **t** = {$\sigma_{yx}$, $\sigma_{yy}$, $\sigma_{yz}$}, [3]:

$$tg\psi = \left(\frac{\sigma_{yx}}{\sigma_{yy}}\right)_{r\to 0}\ ,\qquad \cos\phi = \left(\frac{\sigma_{yz}}{|\mathbf{t}|}\right)_{r\to 0}\ . \tag{7}$$

An equivalent definition can be given in ($K_I$ , $K_{II}$, $K_{III}$) space:

$$tg\psi = \frac{K_{II}}{K_I}, \qquad \cos\phi = \frac{K_{III}}{\sqrt{K_I^2 + K_{II}^2 + K_{III}^2}} \tag{8}$$

For $\varepsilon \neq 0$ tension and shear effects, near the interface crack tips, are inseparable. A measure of the relative proportion of shear to normal tractions (or mode II to mode I) requires the specification of a characteristic length quantity $\hat{L}$. For oscillatory fields the mode mixity is uniquely specified by:

$$tg\hat{\psi} = \left(\frac{\sigma_{yx}}{\sigma_{yy}}\right)_{r \to \hat{L}}, \qquad \cos\phi = \left(\frac{\sigma_{yz}}{|\mathbf{t}|}\right)_{r \to 0}. \tag{9}$$

The length $\hat{L}$ is arbitrary but must it be constant for a material pair, i.e., $\hat{L}$ must be independent of the overall specimen size and type. A length, which is between the inelastic zone size and the specimen size, depends on selection of $\hat{L}$. For example, $\hat{L} = 100\ \mu m$ is suitable for many brittle bimaterial specimens in laboratory research.

Using the stress field (1), or the tractions (2), the mode mixity $\hat{\psi}$ and $\phi$, can also be defined in the $K$ space as:

$$tg\hat{\psi} = \frac{\mathrm{Im}(K\hat{L}^{i\varepsilon})}{\mathrm{Re}(K\hat{L}^{i\varepsilon})}, \qquad \cos\phi = \frac{K_{III}}{\sqrt{|K|^2 + K_{III}^2}}. \tag{10}$$

The energy release rate is related to K and $K_{III}$ by:

$$G = \frac{1}{ch^2(\pi\varepsilon)} \cdot \frac{|K|^2}{E^*} + \frac{K_{III}^2}{2\mu^*}. \tag{11}$$

## 3. INTERFACE CRACKS IN BILAYERS

The above results are used to analyze a semi-infinite interface crack between two isotropic elastic layers under generalized edge loading conditions, Figure 2, [2]. The problem shown in Figure 2(c) is obtained as superposition of problems shown in Figures 2 (a) and (b).

Force and moment equilibrium dictate that, [6]:

$$P_1 - P_2 - P_3 = 0 \tag{12}$$

$$M_1 - M_2 + P_1\left(\frac{h}{2} + H - \delta\right) + P_2\left(\delta - \frac{H}{2}\right) - M_3 = 0 \tag{13}$$

Only four among these six loading parameters are actually independent. These are $P_1$, $P_3$, $M_1$ and $M_3$. The number of independent load parameters can be further reduced to only two, through superposition (Figure 2). These parameters are force and moment, given by:

$$P = P_1 - C_1 P_3 - C_2 \frac{M_3}{h}$$
$$M = M_1 - C_3 M_3 \tag{14}$$

where the $C$'s are dimensionless numbers. Following calculations in [2], section 2.3.1, one obtains the necessary variables for calculating the complex stress intensity factor. Force and moment parameters are then:

$$P = P_1 - \int_{H-\delta}^{H-\delta+h} \sigma_{xx}(y)dy$$
$$M = M_1 - \int_{H-\delta}^{H-\delta+h} \sigma_{xx}(y) \cdot \left[ y - \left( H - \delta + \frac{h}{2} \right) \right] \cdot dy , \tag{15}$$

and the energy release rate can be computed from the difference between energy stored in the structure per unit length far ahead and far behind the crack tip.

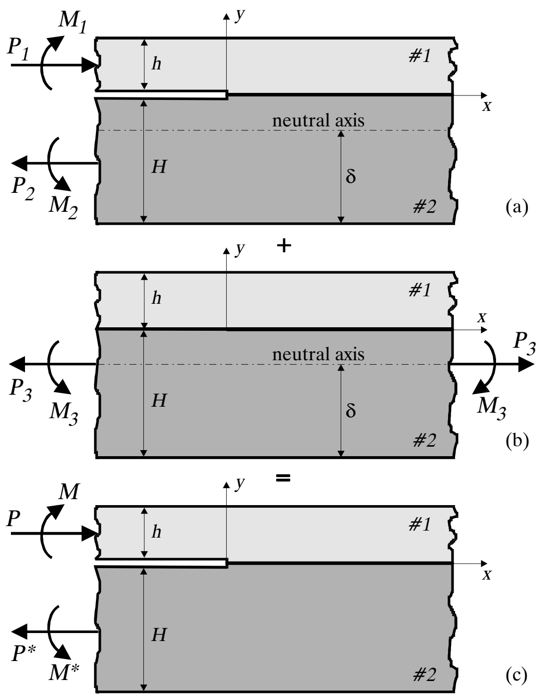


Fig. 2. Superposition scheme for the bimaterial structure with generalized edge loading

$$G = \frac{1}{2E_1} \cdot \left[ \frac{P^2}{Ah} + \frac{M^2}{Ih^3} + 2\frac{PM}{h^2\sqrt{AI}} \cdot \sin\gamma \right], \tag{16}$$

with:

$$A = \frac{1}{1 + \Sigma \cdot (4\eta + 6h^2 + 3\eta^3)}, \; I = \frac{1}{12(1 + \Sigma\eta^3)}, \qquad (17)$$
$$\sin\gamma = 6\Sigma\eta^2 (1+\eta)\sqrt{AI}$$

Thus, the corresponding stress intensity factor is:

$$|K|^2 = \left[\frac{P^2}{Ah} + \frac{M^2}{Ih^3} + 2\frac{PM}{h^2\sqrt{AI}} \cdot \sin\gamma\right] \cdot \frac{p^2}{2} \qquad (18)$$

where:

$$p = \sqrt{\frac{1-\alpha}{1-\beta^2}}\,.$$

The linearity and dimensional considerations lead to the following general expression:

$$K = \left[a\frac{P}{\sqrt{Ah}} + b\frac{M}{\sqrt{Ih^3}}\right] \cdot \frac{p}{\sqrt{2}} \cdot h^{-i\epsilon}, \qquad (19)$$

where a and b are dimensionless complex numbers, which can be found by substitution of equation (18) into equation (19), yielding:

$$2\sin\gamma = \bar{a}b + a\bar{b}\,, \qquad (20)$$

such that:

$$a = e^{i\omega} \;\; and \;\; b = -ie^{i(\omega+\gamma)}, \qquad (21)$$

where $\omega$ is a real angular function of $\alpha$, $\beta$ and $\eta$, tabulated by Suo and Hutchinson, [1]. In this paper their results are substituted by Mathematica program simulation of $\omega$ and it is presented by the following expression:

$$\omega = \frac{1-\eta}{1+\eta} \cdot \sqrt{\frac{\beta(1-\alpha)}{\alpha-\beta^2}}\,.$$

Equation (19) can be written as:

$$K = K_1 + iK_2 = \frac{1}{\sqrt{2}} \cdot \left(\frac{P}{\sqrt{Ah}} - ie^{i\gamma}\frac{M}{\sqrt{Ih^3}}\right) \cdot \frac{p}{\sqrt{2}} h^{-i\epsilon} e^{i\omega}\,. \qquad (22)$$

Taking as the reference length the film thickness $h$, one obtains:

$$K_1 = \mathrm{Re}(Kh^{i\epsilon}) = \frac{p}{\sqrt{2}} \cdot \left[\frac{P}{\sqrt{Ah}}\cos\omega + \frac{M}{\sqrt{Ih^3}}\sin(\omega+\gamma)\right]$$
$$K_2 = \mathrm{Im}(Kh^{i\epsilon}) = \frac{p}{\sqrt{2}} \cdot \left[\frac{P}{\sqrt{Ah}}\sin\omega - \frac{M}{\sqrt{Ih^3}}\cos(\omega+\gamma)\right]. \qquad (23)$$

In accordance with equation (10), the mode mixity, at the prescribed length $r = h$ ahead of the crack tip for the planar conditions, is given by:

$$\psi = \text{arctg}\left[\frac{\xi \sin\omega - \cos(\omega+\gamma)}{\cos\omega + \sin(\omega+\gamma)}\right], \tag{24}$$

where:

$$\xi = \frac{Ph}{M}\sqrt{\frac{I}{A}}. \tag{25}$$

The mode mixity $\psi$ is plotted in Figure 3 as function of $\alpha$ for various film/substrate thickness ratios $\eta$. This diagram is obtained with variable $\omega$ calculated by Mathematica program package.

The mode mixity obtained from (24), for various bimaterial systems, is shown in Figure 4. This phase angle is small for bimaterial system, while it takes value $\psi = 50°$ for homogeneous case and thin film/substrate systems.

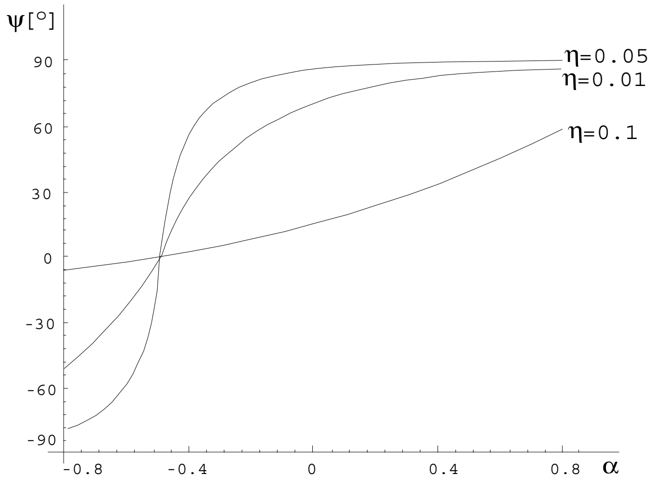


Fig. 3. Mode mixity versus parameter $\alpha$

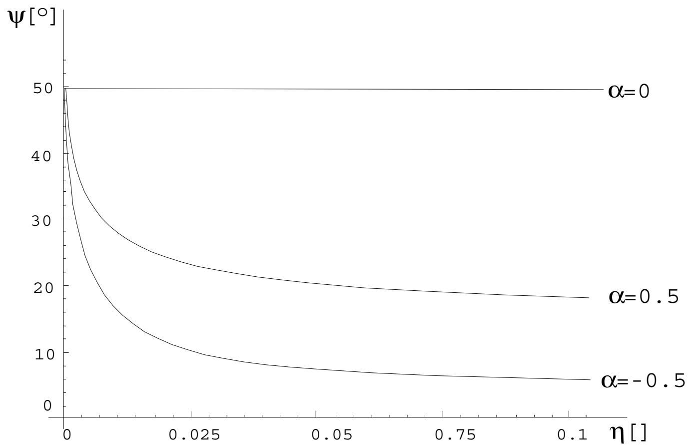


Fig. 4. Variations of the mode mixity with thickness ratio $\eta$

Let us denote the energy release rate for semi - infinite crack lying along the interface as $G_i$, (given by equation (11) for planar problem), and energy release rate for steady - state substrate as $G$, (given by equation (16)). The ratio $G/G_i$ is shown in Figure 5 as a function of $\alpha$ for various values of the ratio $\eta$ of the film to substrate thickness. This ratio is relatively independent of the bimaterial system properties and it varies between 0.55 and 0.83.

Let $G_C$ be the substrate toughness and let $G_{iC}$ be the interface toughness. If

$$\frac{G_C}{G_{iC}} > \frac{G}{G_i}, \tag{26}$$

the system is more likely to fracture by interface then substrate failure and visa-versa.

## 4. Conclusion

In this paper the problem of the crack lying along the interface between two layers and the problem of the crack on joining of thin film and the substrate are considered, using the linear elastic fracture mechanics concept of a crack along an interface. The energy release rate can be calculated from equation (16). Comparing those values with the values of the energy release rate for the substrate or thin film, we can define where the crack is going to propagate: into the substrate, into the thin film or along the interface. Values of the energy release rate, the stress intensity factor and mode mixity parameter are determined in terms of only one dimensionless factors $\omega$, which is a function of sample geometry and materials elastic properties. The thin layers problem, under conditions of residual tensile stresses, gives the appropriate model for solving problems in the area of composite materials manufacturing, electronic devices design, protective coatings problems, as well as for other applications.

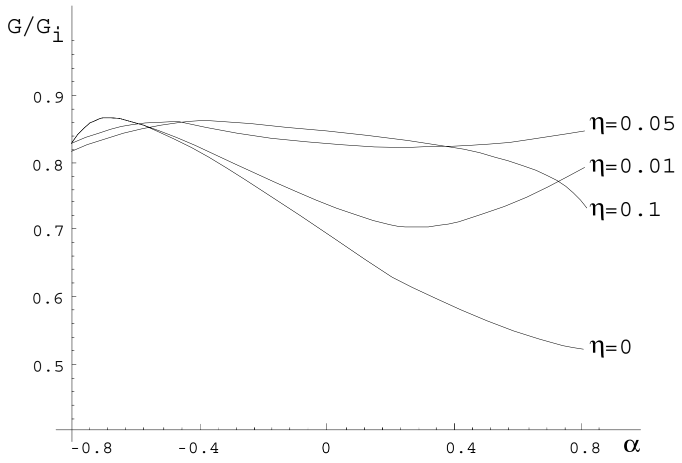


Fig. 5. Energy release rate ratio as a function of parameter $\alpha$

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# PRIMENA KONCEPTA INTERFEJSNE PRSLINE NA PROBLEM PRSLINE IZMEDJU TANKOG SLOJA I OSNOVE

**Jelena M. Veljković, Ružica R. Nikolić**

*U radu se razmatra problem prsline na spoju tankog filma i osnove. Za tanke slojeve najvažniji je problem slojeva izloženih zaostalim zateznim naponima. U radu je prikazana interpretacija ovog problema primenom koncepta linearne elastične mehanike loma za prslinu na intefejsu. Odredjene su veličine brzine oslobadjanja energije i faktora intenziteta napona u zavisnosti od samo jednog bezdimenzionog faktora* $\omega$*, koji je funkcija geometrije uzorka i elastičnih karakteristika materijala. Problem tankog sloja, u uslovima zaostalih zateznih napona, predstavlja adekvatan model za rešavanje problema u oblasti proizvodnje kompozitnih materijala, projektovanja elektronskih uredjaja, problema zaštitnih prevlaka i drugih.*

Ključne reči: *koncept interfejsne prsline, tanak film, zaostali naponi, faktori intenziteta napona.*