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AN ENGINEERING CONCEPT TO ACCOUNT FOR CRACK-TIP CONSTRAINTS

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Abstract. *It is one of the main advantages of engineering fracture mechanics that the user has not to deal with the local stress- and strain-field in detail. Instead, the local loading state and fracture behaviour is characterized by overall parameters like J or K and their critical values. Unfortunately, the transferability of these parameters is restricted to similar conditions concerning crack-tip constraints. However, to take the effects of non-equal constraints into account in a fracture assessment procedure, an elastic-plastic analysis of the crack-tip region has to be performed. This requires a computational effort which is usually beyond the possibilities within an engineering analysis. Therefore, simplified procedures are needed. A promising possibility is suggested in the present paper, where a new constraint parameter Γ is introduced. It has two main advantages: First, it can be obtained easier than the commonly used Q-parameter, and second, it is related to the fracture toughness parameters by closed-form equations.*

1. INTRODUCTION

A sharp notch or a crack in a structural component cause a stress-concentrations that forms a local plastic zone. In the case of small-scale yielding (SSY) it is embedded in an elastic surrounding, in full scale yielding (FSY) in an inhomogeneous plastic strain field. In both cases, the plastic strains in the vicinity of the crack-tip are restrained, creating a triaxial stress-state to occur. This effect is called crack-tip constraints (CTC). Its magnitude depends on the crack length, the loading type and the component size and geometry. In elastic-plastic materials, the CTC result in local stresses that exceed the uniaxial yield stress considerably (Fig. 1).

Due to CTC the non-dimensional maximum stress in the vicinity of the crack-tip, γ, defined as

$$\gamma = \sigma_{y\,\max} / R_p \tag{1}$$

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can be up to 3 for non-hardening and even higher for hardening materials. Through this stress peak the CTC affect the fracture behaviour of a pre-cracked component significantly. The most important effects are:

1. The higher the CTC, the lower the load (in terms of J-integral or K_I) required for initiation of a cleavage fracture.
2. The higher the CTC, the lower the crack-tip opening displacement (CTOD) the crack opening angle (CTOA), and, thus, the fracture energy.
3. The higher the CTC, the higher the ductile-to-brittle transition temperature (DBTT)

Generally speaking, the higher the CTC, the more "brittle" the failure behaviour of a pre-cracked structural part. This effect should be taken into account in a failure assessment analysis. Neglecting it often results in over-conservative failure predictions, since fracture toughness test specimens are designed such that maximum constraints are present.

Conceptually it is suitable to distinguish between out-of-plane constraints (OPC) and in-plane constraints (IPC). The importance of the former is known since the early days of fracture mechanics in the early sixties, when it was found that fracture toughness is significantly dependent on the thickness of the test specimen [1, 2]. This finding resulted in the size requirement of fracture mechanics test specimens. Disregarding a few special cases of "super-constraints", the highest OPC are those corresponding to plane strain. It is interesting to note that it took about twenty years longer until the effect of the IPC on fracture toughness was fully recognized [3-8]. To characterize the IPC several parameters are suggested in the literature. The best known are the so-called T-stress, the Q-factor and the m-factor.

The T-stress quantifies the second term of Williams stress field expansion [9, 10], which - as a homogeneous uniaxial stressfield acting parallel to the crack - obviously contributes to the triaxiality of the near-tip region, for small scale yielding (SSY). Moreover - surprising on a first glance - there is experimental and numerical evidence that T correlates with the constraint effects even in the case of large scale yielding (LSY) and full scale yielding (FSY) [7, 8, 9]. However, in LSY or FSY, T can only serve as an "indexing" parameter, not as a "correcting" one [7].

The Q-factor [5] quantifies the difference between the actual local stress at a certain reference location near the crack tip and the theoretical HRR-stressfield under SSY-conditions. Correspondingly, a detailed local stress analysis by the Finite-Element method (FEM) is required for its determination. So - since the main benefit of engineering fracture mechanics just is its possibility of predicting the local fracture behavior at a crack tip without a detailed local stress analysis - the Q-approach is usually not adequate for practical engineering applications.

The factor m, which appears in the well-known general relation between J and CTOD δ as

$$J = m \cdot R_p \cdot \delta \tag{2}$$

is known to be constraint-dependent, at least as OPC are concerned. Thus m can serve as a parameter to characterize constraints, as suggested in [11]. The factor m can be determined either by a finite-element analysis or experimentally. As a displacement-related quantity it is easier to be determined than the Q-factor. However, like T, m seems to be rather an indexing than a correcting parameter.

In [11] and [12], attempts are made to include constraint effects in engineering failure assessment procedures. However, relying basically on T and Q, the same drawbacks as discussed above apply. In the present paper, a different approach for engineering purposes is suggested. As a constraint parameter an estimation of γ, denoted by Γ, is suggested by the author in [13]. The main ideas behind it are outlined in the present paper. As shown below, Γ is easier to be obtained than Q, and there are analytical relations to fracture toughness.

2. ESTIMATION OF LOCAL STRESS PEAK

To obtain an approximation of γ defined in (1), the magnitude of the local stress σ_{ymax} (see Fig. 1) has to be estimated. For SSY and elastic-perfectly plastic material under plane strain conditions, it was determined in [15] by slip line theory to be

$$\sigma_{y\,\max} = 0.5 \cdot (2+\pi) \cdot R_p \tag{3}$$

where R_p denotes the yield stress. Eq. (3) holds for Tresca's yield criterion, whereas for the von-Mises yield criterion the factor 0.5 has to be replaced by 0.577. By the same model, the relation between J-integral and CTOD δ was obtained in [15] to be

$$\delta = \frac{4}{(2+\pi)} \cdot \frac{J}{R_p} \tag{4}$$

From eq. (2), (3) and (4) it follows that

$$\sigma_{y\,\max} \cong 2 \cdot m \cdot R_p \tag{5}$$

The adjacent elastic surrounding requires a three-axial stress-state to be capable to match with these stress. Assuming a plane strain situation, i.e.

$$\varepsilon_z = \frac{1}{E} \cdot (\sigma_z - \nu \cdot (\sigma_x + \sigma_y) = 0 \tag{6}$$

and a hydrostatic stress state in the x-y-plane, i.e.

$$\sigma_x(x) \cong \sigma_y(x) \tag{7}$$

the application of Tresca's yield criterion

$$\sigma_y - \sigma_z < R_p \tag{8}$$

in a formal, over-all sense, leads to

$$\sigma_{y\,\max} = \frac{R_p}{1-2\nu} \cong 2 \cdot m \cdot R_p \quad (\text{for } T = 0) \tag{9}$$

Eq. (9) represents the maximum stress that can act on the boundary of the plastic zone. For $\nu = 0.3$ its value is quite close to the one given in (3), so with (5) the second equation in (9) results. It is well known that the assumption (7) is true for the elastic stressfield in the vicinity of the crack-tip near the x-axis in case of a pure Mode I-loading and $T = 0$. If a negative T-stress is present, then (7) has to be replaced by

$$\sigma_x = \sigma_y + T \tag{10}$$

Repeating the estimation procedure (6) – (9) with (10) instead of (7), one finds instead of (9)

$$\sigma_{y\max} = \frac{R_p + \nu \cdot T}{1 - 2\nu} \quad (\text{for } T < 0) \tag{11}$$

From comparison with (9) one obtains

$$\sigma_{y\max} \cong 2 \cdot m \cdot R_p + \frac{\nu}{1 - 2\nu} \cdot T \quad \text{for plane strain, } T < 0 \tag{12a}$$

If $T > 0$, then $\sigma_{y\max}$ given by (11) exceeds the one acting in the slip-line zone as given by (3), so the latter is decisive. This means that

$$\sigma_{y\max} \cong 2 \cdot m \cdot R_p \quad \text{for plane strain, } T > 0 \tag{12b}$$

For plane stress, T has no influence on $\sigma_{y\max}$ since lateral necking in a strip-yield-zone occurs, which means that

$$\sigma_{y\max} \cong R_m \quad \text{for plane stress} \tag{12c}$$

where R_m denotes the tensile strength.

The above considerations hold for SSY only. Nevertheless, there are reasons to assume that (12a) - (12c) can be extended to estimate the local stress well beyond SSY. One them is that the dominating term in (12a) and (12b), m, is defined in LSY and FSY in the same way as in SSY, so consistency between SSY and FSY is expected. Furthermore, the second parameter in (12a), the T-stress, is known to be relevant not only in SSY, but also in large- and even in full-scale yielding. This is explicable since the same features of the geometry and the stress-field that cause a high T-stress also tend to cause high constraints in full plasticity.

3. Definition of the Constraint Parameter Γ

We introduce a constraint parameter Γ such that

$$\Gamma \cong \gamma = \frac{\sigma_{y\max}}{R_p} \tag{13}$$

Inspired by (12a) – (12c) and the above discussed extension to LSY, we assume Γ to be a linear combination of m and T in the following form:

$$\Gamma = c_m \cdot m + c_\beta \cdot \beta \tag{14}$$

where

$$\beta = T_{\max} / R_p \tag{15}$$

with $T_{\max}$ being the maximum T-stress at the considered crack, i.e. the T-stress at $K_I = K_{Ic}$ in the case of SSY, or at the plastic limit load in the cases of LSY and FSY, respectively. Based on (12a – 12c), the factors in (14) are expected to be approximately as follows:

for plane strain, $T \leq 0$: $c_m = 2 \quad c_\beta = 0.75$ (16a) for plane stress: $c_m = \sigma_f / R_p \quad c_\beta = 0$ (16b)

As mentioned above, Γ is considerably simpler to be determined than Q, since both its ingredients, T and m, are well known and defined parameters, which can be found in the literature for several systems, at least as approximations. Actually, determination of m according to (2) requires a FEM-calculation, too, but being a displacement-related quantity it is easier to be obtained than Q. Unlike Q, it also can be determined experimentally or - as in the example in section 5 - even analytically as an approximation. Furthermore, as shown in the next section, it can be related to fracture toughness by simple models.

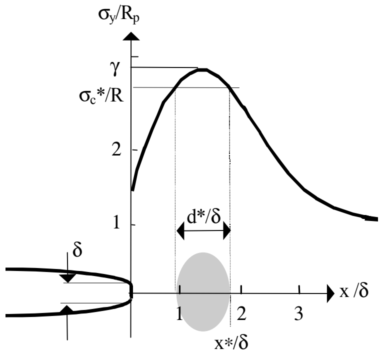

Fig. 1. Non-dimensional representation of the stress distribution in the vicinity of a crack-tip

ESTIMATION OF CONSTRAINT EFFECTS ON FRACTURE TOUGHNESS

4.1. Cleavage Fracture

The critical J- or CTOD-values at onset of cleavage, denoted as J_c and δ_c, respectively, are known to be constraint-dependent. In the following, this dependence is estimated analytically.

Taking pattern from [16], the following two criteria are assumed to govern initiation of unstable cleavage:

i) The maximum stress in the vicinity of the crack tip must exceed the cleavage stress σ_c*, i.e.

$$\sigma_{y\,\max} = \gamma \cdot R_p > \sigma_c^* \tag{17}$$

ii) The elastic energy $W_{el} = \int U_{el} \cdot dV$ stored in a critical Volume V^* of the width d^* (Fig. 1) must be sufficient to produce a cleavage fracture in the area $0 < x < x^*$.
Using the proportionalities

$$U_{el} \propto (\gamma \cdot R_p)^2 \,;\; V^* \propto \delta^2,\, d^* \propto \delta \,;\, x^* \propto \delta \tag{18}$$

one finds with the general relation (2) in criterion ii) and (13),

$$J_c \cdot \frac{\Gamma^2}{m} \cong const \quad \text{for} \quad \Gamma > \sigma_c^*/R_p \tag{19}$$

By using (2), the analogous proportionality can be written in terms of CTOD, i.e.

$$\delta_c \cdot \Gamma^2 = const \quad \text{for} \quad \Gamma > \sigma_c^*/R_p \tag{20}$$

For $\Gamma < \sigma_c^*/R_p$ no cleavage occurs.

4.2. Ductile Tearing

The J or CTOD- values at or near initiation of ductile tearing are denoted in the following by J_{it} and δ_{it}, respectively. They represent near initiation parameters such as $J_{0.2}$, $J_{0.2/Bl}$, J_{Ic} , etc., or the corresponding CTOD-values, respectively. J_{it} and δ_{it} are size independent only if the size requirements corresponding to [17] are met, and if the crack-tip constraint of the test specimen is as high as in the standard test specimens, i.e. deeply cracked bend or CT specimens. Otherwise they are constraint-dependent. This effect is estimated in the following.

The CTOD δ at crack initiation can be assumed to be proportional to the plastic failure strain ε_{pf} in the fracture process zone,

$$\delta_{it} \propto \varepsilon_{pf} \tag{21}$$

ε_{pf} is known to be constraint-dependent [18]. Based on the failure hypothesis of Gillemot [19], which states that ductile failure occurs when the plastic energy density U_p reaches a certain critical value U_{pf}, we simply assume that the product of the true (logarithmic) failure strain and $\sigma_{y\,max} = \gamma \cdot R_p$ is constant at crack initiation, i.e.

$$\gamma \cdot R_p \cdot \ln(1+\varepsilon_{pf}) = U_{pf} \tag{22}$$

The logarithmic strain is used because ε_{pf} is in general not small enough to be linearized. According to [20] U_{pf} can be roughly obtained from a uniaxial tensile test as the area under the true stress-true strain diagram in the necking area, which is approximately

$$U_{pf} \cong \frac{\sigma_f \cdot Z}{1-Z} \tag{23}$$

where $\sigma_f = (R_p + R_m) / 2$ denotes the flow stress and Z the standard reduction of area of a uniaxial tensile test. With (22), (23), and $\Gamma \cong \gamma$, equation (21) leads to

$$\frac{\delta_{it}}{\exp\left[\dfrac{\sigma_f \cdot Z}{R_p \cdot \Gamma \cdot (1-Z)}\right] - 1} = const \tag{24}$$

Using (2) in (24) delivers the analogous proportionality in terms of J:

$$\frac{J_{it}}{m \cdot \left\{\exp\left[\dfrac{\sigma_f \cdot Z}{R_p \cdot \Gamma \cdot (1-Z)}\right] - 1\right\}} = const \tag{25}$$

5. EXAMPLE

For edge-cracked beams under bending (Fig. 2), the constraints are known to depend on the crack-length [7, 9, 20, 21]. For this reason this system is used to check the validity of Γ as a constraint parameter. Under FSY the following relation was analytically obtained in [22]:

$$m = \frac{c_p \cdot \eta}{4 \cdot c_{CR}} \tag{26}$$

Herein, $c_{CR} = r_{CR} / b$ is the non-dimensional distance between the crack-tip and the center of rotation, and η the well known eta-factor that relates J to plastic energy. c_p denotes the plastic constraint factor that appears in the plastic limit load as

$$M_p = \frac{c_p \cdot R_p \cdot b^2}{4} \tag{27}$$

The values of these parameters are taken from the literature (Fig. 3). For the considered system the parameters m according to (26) and β as defined in (15), calculated from the T-stress given in [8] at the plastic limit load given in (27), are shown in Fig. 4, as well as the Γ resulting therefrom by means of (14) – (16).

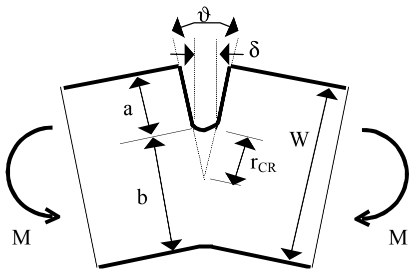

Fig. 2. Edge-cracked beam under a bending moment M

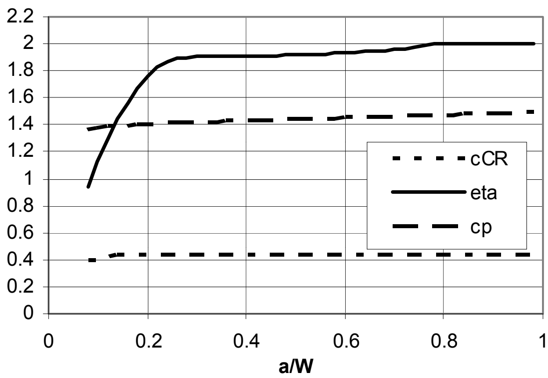

Fig. 3. Factors appearing in eq. (26), as functions of a/W (c_p from [20], η from [21]; c_{CR} represents the trend extracted from [20] and [23].)

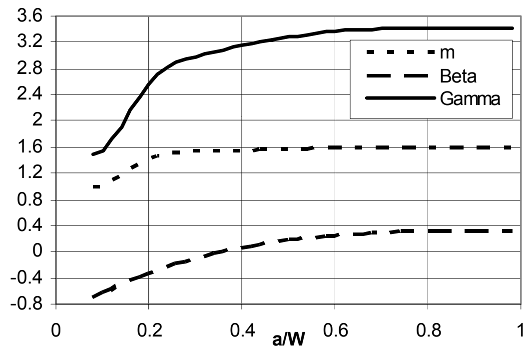

Fig. 4. m, β (Beta) and Γ (Gamma) for an edge cracked bend specimen as a function of crack length

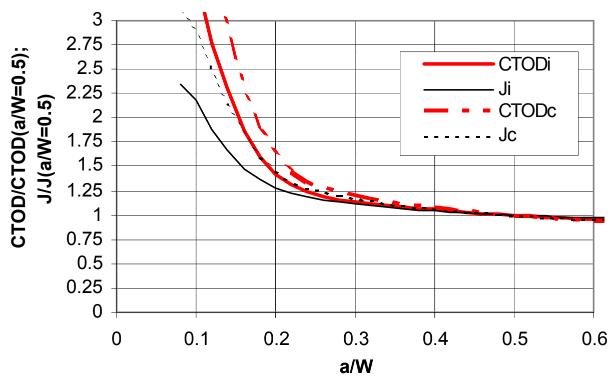

Fig. 5. Ratios of apparent fracture toughness to the corresponding values at $a/W = 0.5$ as a functions of non-dimensional crack length.

Γ inserted in (19) and (24), or (20) and (25), respectively, gives the prediction of the constraint effect on the apparent fracture resistance in terms of J or δ. Fig. 5 shows the effect of crack length on the apparent fracture toughness. These predictions are compared in Fig. 6 and 7 with experimental fracture toughness data reported in literature. Comparison of cleavage data reported in [7] with the J_c- curve in Fig. 5 exhibits a similar agreement. Regarding the typical scatter in the experimental data, the agreement between predicted and experimental data is satisfactory.

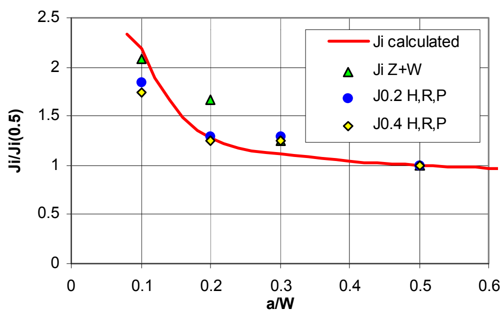

Fig. 6. Comparison of J_{it} predicted by (20) with experimental J_{it} ($Z + W$ from [20], H,R,P from [9]).

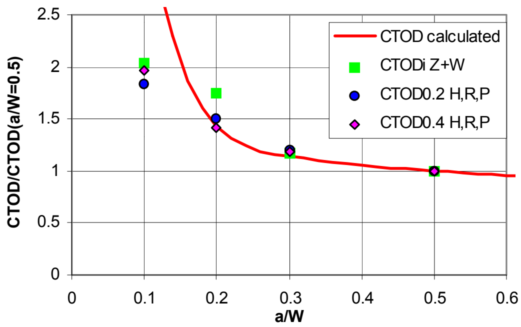

Fig. 7. Comparison of δ_{it} predicted values with experimental δ_{it} ($Z + W$ from [21], H,R,P from [9]).

6. Discussion and Conclusions

Actually, since crack-tip constraints are a feature of the local elastic-plastic stress-field, there is no simple way of quantifying them without a detailed local stress analysis. Simplifications like the one suggested in the present paper are always trade-offs with accuracy. The presented approach is an attempt to reduce the required modeling and computational effort to an engineering level. A key point to make this possible is the assumption that the non-dimensional stress peak γ or its approximation Γ, respectively, is a linear combination of the parameters m and T. Qualitatively this assumption seems to be reasonable. Besides its relative simplicity, the main advantage of the proposed parameter Γ is its ability to describe effects of IPC as well as OPC by closed-form relations that could be derived by simple analytical models.

There are other examples where the effect of constraints can be estimated by similar simple procedures as the one shown above, e.g. the thickness effect on fracture toughness. For general cases, however, Γ needs to be determined numerically based on (14) – (16), which means that T, J and CTOD as well as the plastic limit load has to be calculated. Anyway, determination of these parameters does not require a detailed local model as to evaluate Q, and the actual constitutional law can be replaced by a simple perfectly plastic one.

However, further comparisons with experimental and theoretical data are necessary to assess the accuracy and consistency of the presented formulas. The factors c_m and c_β, which are chosen here just on the basis of some crude theoretical considerations of SSY, might be optimized by fitting experimental results.

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TEHNIČKI KONCEPT OBJAŠNJENJA NAPREZANJA VRHOVA PRSLINA

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Jedna od glavnih prednosti tehničke mehanike loma je to što korisnik ne mora da se detaljno bavi lokalnim oblastima napona i dilatacije. Umesto toga, lokalno stanje opterećenja i ponašanje loma okarakterisani su globalnim parametrima poput J i K i njihovim kritičnim vrednostima. Nažalost, prenosivost ovih parametara ograničena je na slične uslove u vezi sa vezama vrhova prslina. Medjutim, da bi se posledice nejednakih veza uzele u obzir u proceduri procene prsline, mora se izvršiti elastično-plastična analiza oblasti vrha prsline. To zahteva računski napor koji obično prevazilazi mogućnosti tehničke analize. Stoga su potrebne pojednostavljeni postupci. Obećavajuca mogućnost se predlaže u ovom radu, gde se uvodi novi parametar veza I. On ima dve glavne prednosti: prvo, do njega se može doći lakše nego do Q-parametra koji se obično koristi, i drugo, on je povezan sa parametrima žilavosti loma putem jednačina sa zatvorenom formom.