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SIMILARITIES BETWEEN SMALL-SCALE YIELDING INTERFACIAL FREE-EDGE JOINT AND CRACK-TIP FIELDS

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Abstract. *The problem of the small-scale yielding (SSY) plane-strain asymptotic fields for the interfacial free-edge joint singularity is examined in detail and comparisons are made with the interfacial crack tip. The geometries are idealised as isotropic elasto-plastic materials with Ramberg-Osgood power-law hardening properties bonded to a rigid elastic substrate. A fourth-order Runge-Kutta numerical method provides solutions to fundamental equations of equilibrium and compatibility. Asymptotic fields for stress and displacement are developed for a range of hardening parameters and comparisons made between the joint and crack geometries. It is shown that there is some significant similarity between the asymptotic fields particularly for perfectly plastic behaviour.*

INTRODUCTION

Interface-controlled fracture is one of the most important microscopic events leading to ultimate macroscopic rupture in many polycrystalline, composite, and ceramic materials. A fracture mechanics' description of the critical state prior to separation using continuum-based mechanics usually involves the elastic traction-free solution of Williams [1] for the interface crack characterised by the complex stress intensity factor $K = K_1 + iK_2$. This solution models the elastic stress singularity at the interfacial crack tip of the form $Kr^{-0.5+i\varepsilon}$ (where ε is a bimaterial parameter), and thus produces unbounded tractions and crack tip stress and strains. It is of limited use for describing materials that yield and undergo inelastic deformation at high strains. The elasto-plastic interfacial crack problem has received considerable attention in the last decade enabling a thorough understanding to be developed. Numerical solutions involving elasto-plastic behaviour at a traction-free crack tip for a Ramberg-Osgood hardening material have been developed by Shih and Asaro, and Zywicz and Parks [2-4] amongst others.

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Failure of interfacial systems frequently initiates, however, at the free-edge joint between two materials, where a stress singularity also exists, leading to the development and propagation of an interface crack. The analysis of such interfacial free-edge stress fields is just as important, therefore, to our understanding of crack initiation and growth though in comparison to the interface crack it has received far less attention. Further, no direct link has been established between the asymptotic singular fields for the interfacial free-edge joint and the interfacial crack-tip to enable the process of crack initiation at the joint to be completely understood. Part of the reason for this is thought to be that a description of the process leading to crack initiation assuming purely elastic behaviour is complicated by the difference in stress singularity orders and fields. Indeed, it has been shown by Klingbeil and Beuth [5] that conflicting solutions are obtained if designing to prevent debond of the interfacial free-edge joint and/or to prevent propagation of an interfacial crack. Furthermore, the same limitations of the elastic solution apply to the interfacial free-edge joint, i.e. the stress and strains are unbounded. Relatively little effort has been paid to the elasto-plastic behaviour of the free-edge singularity except for the determination of plastic zone size and shape [6,7]. There appears to have been no attempt made to understand the asymptotic elasto-plastic behaviour of the interfacial free-edge joint and then to compare it with the interfacial crack.

In this paper, the asymptotic structure of the elasto-plastic stress field at the interfacial free-edge joint is considered for a quarter of a Ramberg-Osgood hardening material and a rigid elastic material bonded perfectly to form a half plane. The asymptotic structure of the stress and displacement field developed at the joint are obtained using an approach similar to that of Sharma and Aravas [8] and are compared with those for the interface crack. A highly focused finite element (FE) analysis provides some corroborative solutions. It is not the intention of this paper to establish a direct link between the interfacial free-edge joint and crack-tip. However, some remarkable similarities between the structures of the elasto-plastic asymptotic stress fields between the interfacial joint and crack are presented and should enable a description of the process leading to crack initiation and growth to be much more accommodating than with purely elastic behaviour.

FORMULATION OF THE PROBLEM

A thorough analysis of the plane-strain interfacial free-edge joint (Fig. 1a) assuming SSY conditions is presented and solutions obtained are subsequently compared with those for the interfacial crack-tip (Fig. 1b). The constitutive behaviour of the homogeneous isotropic elasto-plastic material is characterised by the J_2 deformation theory for a Ramberg-Osgood uniaxial stress-strain behaviour, i.e.:

$$\varepsilon_{ij} = \frac{1+\upsilon}{E} s_{ij} + \frac{1-2\upsilon}{3E}\sigma_{kk}\delta_{ij} + \frac{3}{2}\alpha\varepsilon_o\left(\frac{\sigma_e}{\sigma_o}\right)^{n-1}\frac{s_{ij}}{\sigma_o} \tag{1}$$

where ε_{ij} is the infinitesimal strain tensor, σ_o is the yield stress, $\varepsilon_o = \sigma_o / E$, and the deviatoric stress is given by:

$$s_{ij} = \sigma_{ij} - \tfrac{1}{3}\sigma_{kk}\delta_{ij} \tag{2}$$

and the Mises equivalent stress is defined as:

$$\sigma_e = (\tfrac{3}{2} s_{ij} s_{ij})^{1/2} \tag{3}$$

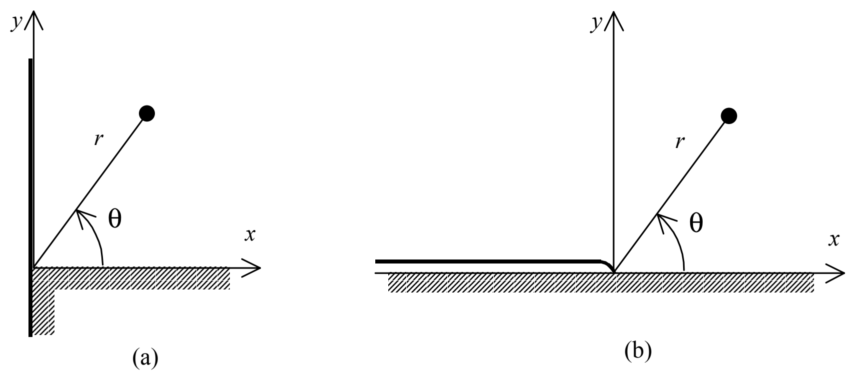

Fig. 1. Schematic (a) interfacial free-edge joint and (b) interfacial crack-tip geometries including polar and Cartesian co-ordinate schemes.

Also, n is the power-law hardening exponent ($1 \le n \le \infty$), E is the Young's moduli, δ_{ij} is the Kronecker delta, and α is a material constant. If $n = 1$ then the behaviour is purely elastic.

A polar (cylindrical) co-ordinate system is adopted and the equilibrium equations for infinitesimal (linear) strain theory can be expressed as:

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{r\theta}}{\partial \theta} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = 0 \tag{4}$$

$$\frac{\partial \sigma_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{\theta\theta}}{\partial \theta} + \frac{2\sigma_{r\theta}}{r} = 0 \tag{5}$$

The strain-displacement equations are written as:

$$\varepsilon_{rr} = \frac{\partial u_r}{\partial r} \tag{6}$$

$$\varepsilon_{\theta\theta} = \frac{u_r}{r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} \tag{7}$$

$$\varepsilon_{r\theta} = \frac{1}{2}\left(\frac{1}{r}\frac{\partial u_r}{\partial \theta} + \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r}\right) \tag{8}$$

where $\boldsymbol{u} = (u_r, u_\theta)$ is the displacement vector. Out-of-plane stresses and strains are assumed to vanish. Finally, the strain-compatibility equation is given by:

$$\left(\frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} - \frac{1}{r}\frac{\partial}{\partial r}\right)\varepsilon_{rr} + \left(\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r}\right)\varepsilon_{\theta\theta} - \left(\frac{2}{r^2}\frac{\partial}{\partial \theta} + \frac{2}{r}\frac{\partial^2}{\partial r \partial \theta}\right)\varepsilon_{r\theta} = 0 \tag{9}$$

For the problem of a homogeneous isotropic elasto-plastic material bonded perfectly to a rigid base, the following boundary conditions apply:

$$\sigma_{\theta\theta}(r,\pi/2)=0\,,\ \sigma_{r\theta}(r,\pi/2)=0 \tag{10}$$

$$u_r(r,0)=0\,,\ u_\theta(r,0)=0 \tag{11}$$

for the interfacial free-edge joint, and:

$$\sigma_{\theta\theta}(r,\pi)=0\,,\ \sigma_{r\theta}(r,\pi)=0 \tag{12}$$

$$u_r(r,0)=0\,,\ u_\theta(r,0)=0 \tag{13}$$

for the interfacial crack tip, where σ and u is the stress tensor and displacement vector respectively, and (r,θ) is the polar co-ordinate system used (Fig. 1).

Selection of the upper material for this investigation is a comparatively arbitrary choice though that used throughout has elastic properties typical of modern adhesives. That its singularity order under elastic conditions is approximately half that of the interfacial crack means the re is sufficient difference to make for an interesting comparison. The key parameter in the investigation, as with those studies on the interfacial crack tip, is the hardening exponents and values $n = 1.1, 5, 10, 50, \text{and}\,\infty$ (elastic-perfectly-plastic) are considered. The value $n = 1.1$ was chosen as it is just above unity and, therefore, just into the elasto-plastic regime.

BACKGROUND KNOWLEDGE

A singularity at the interfacial free-edge (Figure 1a) is predicted by Bogy [9] to be of order depending on the elastic constants of the materials. In this geometry, the singularity field is of the form:

$$\sigma_{ij}(r,\theta)=Hr^{\lambda-1}f_{ij}\,(\lambda,\theta) \tag{14}$$

where H is its intensity, λ-1 is the singularity order, and f_{ij} are known non-dimensional functions of (λ,θ). The intensity H depends on the far-field geometry and loading. The order of the singularity λ is dependent on two material mismatch parameters first used by Dunders [10] given by:

$$\beta_1=\frac{\mu_1(\kappa_2+1)-\mu_2(\kappa_1+1)}{\mu_1(\kappa_2+1)+\mu_2(\kappa_1+1)}\,,\ \beta_2=\frac{\mu_1(\kappa_2-1)-\mu_2(\kappa_1-1)}{\mu_1(\kappa_2+1)+\mu_2(\kappa_1+1)} \tag{15}$$

where $\kappa_j = 3 - 4\nu_j$ for plane strain, $\kappa_j = (3 - \nu_j)/(1 + \nu_j)$ for plane stress, and μ_j and ν_j, $(j = 1,2)$ are the shear modulus and Poisson's ratios of the constituent materials. Values of λ have been calculated by several investigators for different material combinations and geometries, e.g. Bogy [9], and values of H have been investigated for different geometries by Akinsaya [11]. It has also been shown by Akinsaya and Fleck [12] that an interface crack embedded in a free-edge singularity-dominated zone has its own field amplified by the H-field.

For a rigid elastic material 2, i.e. $\mu_2 = \infty$, equation (15) becomes:

$$\beta_1 = -1\,,\ \beta_2 = -\frac{(\kappa_1 - 1)}{(\kappa_1 + 1)} \tag{16}$$

It is observed that the material mismatch parameters are no longer dependent on the shear moduli; the second depends only on the condition of plane strain (or plane stress). The chosen material for the upper half of the joint in this investigation has $\upsilon_1 = 0.39$. Its singularity order under purely elastic conditions is found to be $\lambda = 0.72$ using the analysis of Bogy [9], i.e. the singular stresses vary according to $\sigma \to \sigma\,(r^{-0.28})$. Put another way, the singularity order used for the joint is approximately half the magnitude expected for the crack.

In general, the interface crack between two isotropic materials suffers a singularity stress field characterised by the complex stress intensity factor, $K = K_1 + \mathrm{i}K_2$, and is of the form:

$$\sigma_{ij}(r,\theta) = Kr^{-0.5+\mathrm{i}\varepsilon} g_{ij}(\varepsilon,\theta) \tag{17}$$

where ε is the oscillatory index given by:

$$\varepsilon = \frac{1}{2\pi}\ln\left(\frac{1-\beta_2}{1+\beta_2}\right) \tag{18}$$

and g_{ij} are known non-dimensional functions of (ε,θ).
The singularity order of the interfacial crack tip under elastic conditions is always λ=0.5 in comparison to the free-edge joint that varies according to the elastic properties of the two materials. However, the mode mix of loading is complicated by the oscillation of the stress components due to the $\mathrm{i}\varepsilon$ term in (17).

Using a J-integral argument Rice and Rosengren [13], and Hutchinson [14] have shown that crack problems under SSY conditions result in a $1/r$ singularity in the strain-energy density. That is, the product of stress and strain is given by:

$$\sigma_{i,j}\varepsilon_{i,j} \to \frac{\text{some function of }\theta}{r} \quad \text{as } r \to 0 \tag{19}$$

Then the strain singularity for a power law hardening material of index n is given by:

$$\varepsilon_{i,j} \to r^{-n/s}\mathrm{E}_{i,j}(\theta) \tag{20}$$

and the stresses have a singularity order given by:

$$\sigma_{i,j} \to r^{-s}\Sigma_{i,j}(\theta) \tag{21}$$

noting that the singularity order s is given by $-1/(1+n)$ [13,14]. In equations (20) and (21) $\mathrm{E}(\theta)$ and $\Sigma(\theta)$ are functions of θ. The same states of stress and strain for interfacial cracks under SSY conditions have been confirmed [8]. The same investigators have shown that asymptotic solutions for $\mathrm{E}(\theta)$ and $\Sigma(\theta)$ correspond to a dominating tensile mode scaled by the multiplicative constant, J, even for hardening exponents n slightly greater than unity. It does not appear that asymptotic elasto-plastic solutions exist for an arbitrary mode mix ratio, $\sigma^{(0)}_{r\theta}/\sigma^{(0)}_{\theta\theta}$, at the interface [8,15]. Consequently, if the leading-order mode-mix parameter $M^{(0)}_p$ is defined as [16]:

$$M_p^{(0)} = \frac{2}{\pi}\tan^{-1}\left(\lim_{r\to 0}\frac{\tilde{\sigma}_{\theta\theta}^{(0)}}{\tilde{\sigma}_{r\theta}^{(0)}}\right) \tag{22}$$

then typically $M_p^{(0)}$ has values of unity. The authors [17] have obtained whole-field experimental data using digital photoelasticity to verify the existence of the dominating tensile mode at interfacial crack tips.

In comparison to the interface crack, the behaviour of the interfacial free-edge joint under elasto-plastic conditions has not been considered in full. The only known attempt to determine singularity orders for materials with hardening is due to Rudge and Tiernan [18]. Several aspects of the elasto-plastic behaviour of the interfacial free-edge joint are unclear and the authors are not aware of any published solutions for the resulting asymptotic distributions of stress and displacement. Consequently, this study aims to redress the balance with a view to considering in the future the mechanisms involved in the development of a crack from the interface joint.

ASYMPTOTIC SOLUTIONS FOR THE INTERFACIAL FREE-EDGE JOINT

The fundamental questions to be posed are: "what is the asymptotic structure of the interfacial free-edge joint under elasto-plastic conditions?" and "how does it compare with the interfacial crack-tip?". To arrive at some answers the numerical results of an asymptotic analysis for the interfacial free-edge joint are presented and compared with those of the interfacial crack tip in the next section. Numerical calculations assume the upper domain is a Ramberg-Osgood hardening material that is perfectly bonded to a rigid elastic substrate. Field solutions to the plane-strain interfacial free-edge joint problem were obtained using the asymptotic approach of Sharma and Aravas [8] and by FE analysis for hardening cases n = 1.1,5,10,50, and ∞.

According to Sharma and Aravas [19] the leading order problem that defines the asymptotic behaviour in terms of stress and displacement fields satisfying (4)-(9) consists of five non-linear ordinary differential equations:

$$\begin{aligned}
(s+1)\tilde{\sigma}_{rr}^{(0)} - \tilde{\sigma}_{\theta\theta}^{(0)} + \frac{\mathrm{d}\tilde{\sigma}_{r\theta}^{(0)}}{\mathrm{d}\theta} &= 0 \\
\frac{\mathrm{d}\tilde{\sigma}_{\theta\theta}^{(0)}}{\mathrm{d}\theta} + (s+2)\tilde{\sigma}_{r\theta}^{(0)} &= 0 \\
(sn+1)\tilde{u}_r^{(0)} - \frac{3}{2}\tilde{\sigma}_e^{(0)n-1}\tilde{s}_{rr}^{(0)} &= 0 \\
\tilde{u}_r^{(0)} + \frac{\mathrm{d}\tilde{u}_\theta^{(0)}}{\mathrm{d}\theta} - \frac{3}{2}\tilde{\sigma}_e^{n-1}\tilde{s}_{\theta\theta}^{(0)} &= 0 \\
\frac{1}{2}\left(\frac{\mathrm{d}\tilde{u}_r^{(0)}}{\mathrm{d}\theta} + sn\tilde{u}_\theta^{(0)}\right) - \frac{3}{2}\tilde{\sigma}_e^{(0)n-1}\tilde{\sigma}_{r\theta}^{(0)} &= 0
\end{aligned} \tag{23}$$

A fourth-order Runge-Kutta solution to the equations in (23) was obtained for different values of the hardening exponent n using the proprietary software *Mathcad* (v.2000), distributed by Adept Scientific Ltd.). An iteration scheme was used to determine the solution s to the non-linear eigenvalue problem and the subsequent distributions for the stresses and displacements that satisfy (23) above, and the conditions (10) and (11).

These asymptotic solutions were verified by a FE analysis performed using the software *Lusas* (v13.2, distributed by FEA Ltd., Kingston, UK). Highly-focused, refined meshes for the interfacial free-edge joint were prepared using four-noded linear elements until satisfactory convergent results were obtained. Preliminary trials to perfect the mesh were performed using the interface crack tip geometry that was compared with known solutions. The eventual FE mesh used for the interface joint problem consisted of a quarter-circle domain with boundary displacements applied calculated using the asymptotic elastic solution to the singularity problem. Logarithmic seeding was used in the radial direction consisting of 10 nodes per decibel for $-5 \le \log r/r_p \le 1$. In the circumferential direction, the region was separated into 6 elements of uniform spacing between 0° and 9°, and 28 elements of uniform spacing between 9° and 90°. The displacements on the 0° radial were zeroed to simulate bonding to a rigid substrate and traction-free boundary conditions were assumed on the 90° radial. The authors would have preferred to use nine-noded Lagrangian elements and the B-bar approach to elasto-plastic analysis that seems to have been accepted in the literature as conventional.

Figures 2 and 3 show the angular variation of the polar components of stress and displacement, respectively, for the four cases of hardening n=1.1,5,10, and 50. Nodal values of the polar components of stress and displacement from the FE analysis are plotted as white circles for the radius $\log(r/r_p) = -2$. The results are normalised so that the maximum value of the equivalent stress σ_e in the angular variation is unity, i.e.:

$$[1.5\tilde{s}_{ij}^{(0)}\tilde{s}_{ij}^{(0)}]_{\max}^{1/2} = 1 \tag{24}$$

The fourth-order Runge-Kutta solution to the angular variations from equation (23) has been superimposed onto the FE results to enable them to be validated. In all cases of the polar components of stress and displacement, the agreement is excellent between the asymptotic solution and the FE results.

COMPARISONS OF INTERFACIAL FREE-EDGE JOINT AND CRACK-TIP ASYMPTOTIC FIELDS

Firstly, the singularity orders for the interfacial free-edge joint obtained by the asymptotic analysis are shown plotted against hardening exponent n in the graph of Figure 4.a as white squares. The thick black line represents the solution for the crack given by $s = -1 / (1 + n)$. It is seen that for a given n the singularity order for the joint is of slightly smaller magnitude that for the crack. This confirms that the singularity order for the interface joint itself varies with n as in the case of the crack. Figure 4(b) shows values for the mode mix parameter M_p calculated from solutions from the asymptotic analysis using (22). These are denoted by the white squares in Figure 4.b. The solution for the crack has been superimposed using values from the study by Sharma and Aravas [8]. Attempts were made to obtain solutions for a range of mode mix parameters. As with the case for the crack [8] there only appears to be a narrow band of mode-mix ratio solutions available for the interfacial joint even for n slightly greater than one. However, in general, the mode-mix parameters for the joint involve more "Mode II" than for the crack that is predominantly "Mode I".

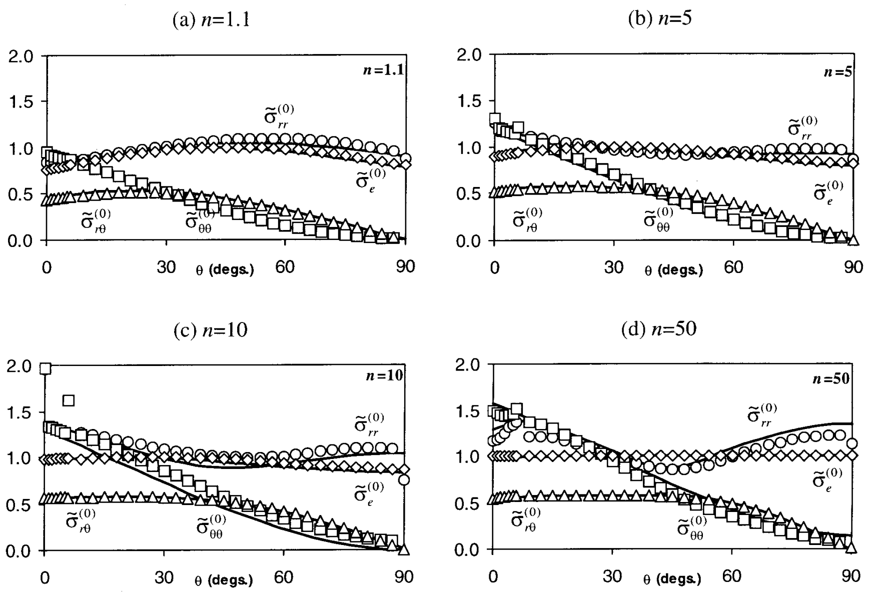

Fig. 2. Angular variation of the asymptotic normalised plane-strain polar stress components for the interfacial free-edge joint (lines=asymptotic, markers=FE).

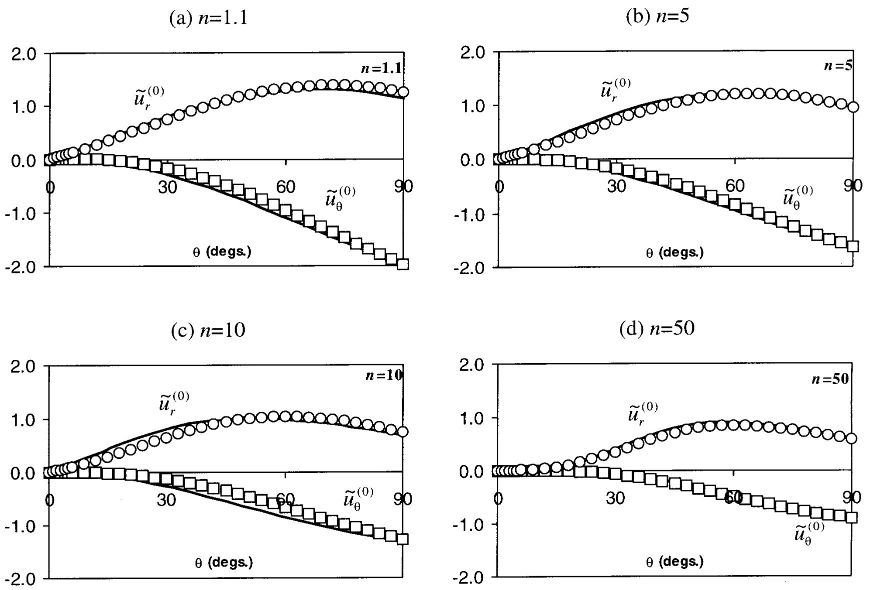

Fig. 3. Angular variation of the asymptotic normalised plane-strain polar displacement components for the interfacial free-edge joint (lines=asymptotic, markers=FE).

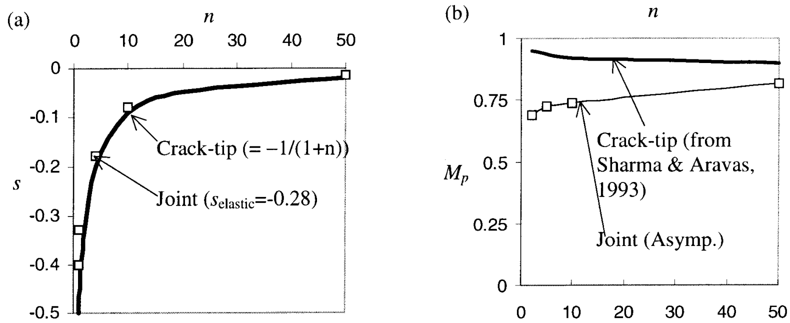

Fig. 4. Comparison of (a) stress exponent s and (b) mode mix M_p with hardening exponent n for the interfacial crack-tip and free-edge joint.

Further comparisons between solutions for the interfacial free-edge joint and the crack-tip are made by plotting results of the asymptotic analysis obtained for both geometries for $n = 1.1, 5, 10$, and 50. The solutions obtained for the crack corroborate those existing in the literature. Asymptotic behaviour for both geometries is compared in Figures 5 and 6 through the angular distribution for the polar stress components and displacements respectively. The results are again normalised so that the maximum value of the equivalent stress in the angular variation is unity. It is interesting to observe that the fields are in closer agreement than one might imagine. They appear to belong to the same 'family' of curves if one ignores for the moment that the crack involves some higher values for tangential components of stress and displacement (Figures 5.b and 6.b). The only exception appears to be in the case of the radial stress components in Figure 5.a.

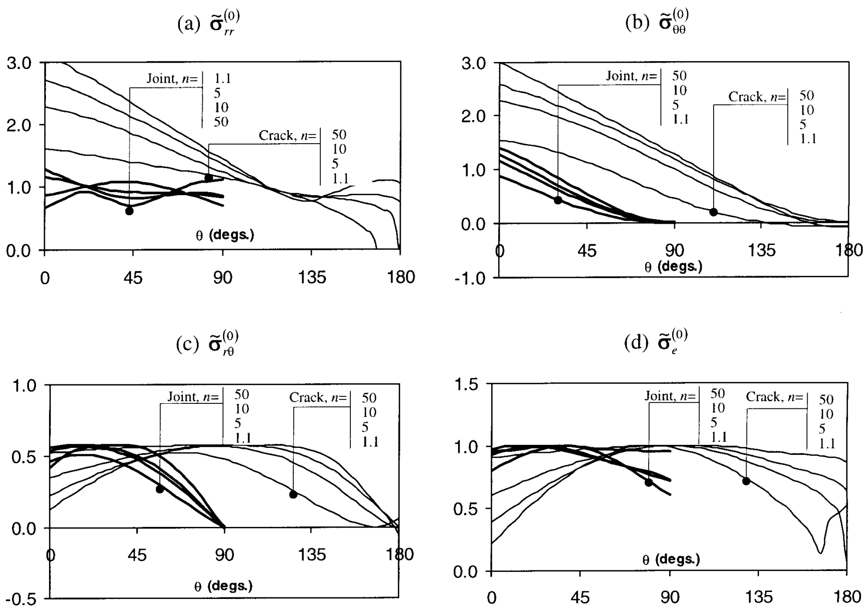

Fig. 5. Comparison of the asymptotic normalised plane-strain polar stress components and equivalent stress for the interfacial free-edge and crack-tip for hardening n=1.1,5,10, and 50.

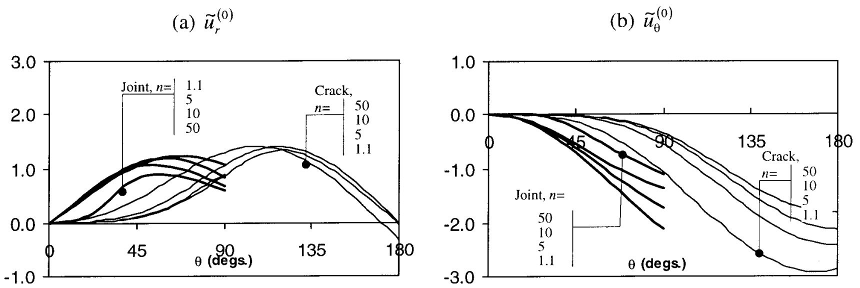

Fig. 6. Comparison of the asymptotic normalised plane-strain polar displacement components for the interfacial free-edge and crack-tip for hardening n=1.1,5,10, and 50.

A more important comparison arises when plotting the angular distribution of stresses for the limiting case $n = \infty$, i.e. an elastic-perfectly-plastic material. It was considered here that there may be some similarity between the inelastic behaviour at the interfacial free-edge joint and the interfacial crack if the latter case includes the elastic sector predicted by Zywicz and Parks [4] between $90^0 \leq \theta \leq 180^0$, i.e. both are inelastic between $0^0 \leq \theta \leq 90^0$. The reason for this is that the inelastic behaviour is confined to the same quarter in both cases. Solutions for both geometries were obtained by performing the asymptotic analysis as used elsewhere in the paper. Strictly speaking, the solutions are approximate as a large but finite value for n has to be input to get a solution and so $n \cong \infty$. The results for the polar components of stress are shown normalised in Figure 7(a). There are already some close similarities between the two sets of curves, however, a more obvious similarity is then obtained when the deviatoric stresses are calculated using (2) and plotted as in Figure 7(b). Here the two sets of curves cannot be distinguished for the region $0^0 \leq \theta \leq 45^0$. One may surmise that upon development of an interfacial crack from an interfacial free-edge joint the asymptotic deviatoric stress field does *not* have to change its distribution in the region $0^0 \leq \theta \leq 45^0$ for an elastic-perfectly-plastic material. In other words, upon attaining yielding stress levels a 'pseudo' crack-tip already exist at the interfacial free-edge joint upon yielding in terms of the deviatoric stresses field. The authors hope that this similarity opens avenues to formulating a direct link between the two geometries to enable crack initiation at a free surface in interfacial material systems to be better understood.

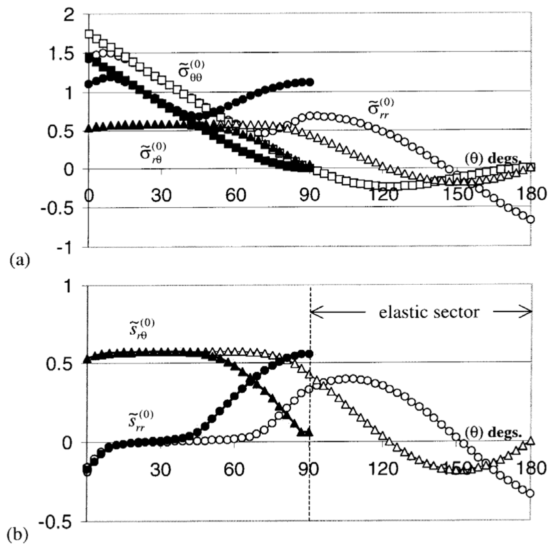

Fig. 7. Comparison of the asymptotic normalised plane-strain (a) polar and (b) deviatoric stress components for the interfacial free-edge and crack-tip under elastic-perfectly-plastic slip conditions.

CONCLUSIONS

For an isotropic elasto-plastic material bonded to a rigid substrate the SSY asymptotic plane-strain behaviour at the interfacial free-edge joint has been identified for several values of the hardening exponent *n*. Using an asymptotic analysis the polar components of stress and displacement have been determined and confirmed by a highly-focused FE analysis. Similarly, the singularity orders under elasto-plastic behaviour were identified and shown to be dependent on the hardening exponent *n* of the material. As in the case for crack, there only exist asymptotic solutions for a narrow range of mode mixes at the interfacial free-edge joint under elasto-plastic behaviour.

Comparisons of the asymptotic structure of the stress and displacement fields between the joint and crack show that they appear to belong to the same family of curves. In comparing elastic-perfectly–plastic behaviour between the interfacial free-edge joint and the crack tip, involving an elastic sector between $90^0 \le \theta \le 180^0$, it is seen that the normalised deviatoric stress field is indistinguishable between the two geometries for the region $0^0 \le \theta \le 45^0$. Consequently, it would appear that the asymptotic deviatoric stress field that drives many forms of crack propagation does not need to change structure upon initiation of an interfacial crack at the free-edge joint.

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SLIČNOSTI IZMEDJU INTERFACIJALNIH POLJA NISKOG NIVOA POPUSTLJIVOSTI SPOJENIH PREKO SLOBODNIH IVICA I POLJA SA VRHOM PRSLINE

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Problem dilatacije asimptotskih polja niskog nivoa popustljivosti za interfacijalnu osobenost spojenosti preko slobodnih ivica proučen je do detalja, i izvršena su poredjenja sa interfacijalnim vrhom prsline. Geometrije su idealizovane kao izotropski elasto-plastični materijali sa svojstvima otvrdnjavanja po Ramberg-Ozgudovom zakonu snage koji su vezani za rigidni elastični substrat. Runge-Kuta numerički metod četvrtog reda daje rešenja za osnovne jednačine ravnoteže i kompatibilnosti. Asimptotska polja za napon i pomeranje razvijena su za niz parametara otvrdnjavanja i izvršena poredjenja izmedju geometrija spoja i prslina. Pokazano je da postoji značajna sličnost izmedju asimptotskih polja, naročito za savršeno plastično ponašanje.