CLOSED FORM EXPRESSION FOR FATIGUE LIFE PREDICTION AT COMBINED HCF/LCF LOADING

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Abstract. For the combined HCF/LCF loading, which stress history is simplified in the way that it consists of one LCF stress block with number of cycles equal to the number of start-up in-service operations and one HCF stress block with summed-up all HCF cycles, the closed form expressions are derived for estimating both the crack initiation life and the crack propagation life at combined HCF/LCF loading. As an example of the use, Smith and Haigh diagrams are obtained for the components made of titanium alloy Ti-6Al-4V, which enable the fatigue strength estimation for designed fatigue life, known load ratio and certain number of HCF cycles per one combined stress block.

1. Introduction

Any machine part subjected to substantial load due to start-stop operations has basically a similar stress history consisting of N_B stress blocks (one for each operation) with n_{HCF} high cycle fatigue (HCF) cycles and one low cycle (LCF) cycle (Fig. 1). LCF stresses are actually the "steady" stresses, which result in one cycle for every start-up and shutdown operation [6], and HCF stresses are caused by in-service vibrations. Actually, such type of stress history is usual for all machine parts subjected to substantial load due to start-stop operations. The integrity of these parts is particularly critical, because the usually extremely high cyclic frequencies of in-service loading spectra, causes that the fatigue life of e.g. 10^7 cycles can be reached in few hours. It was one of the reasons that a number of fatigue failures has been detected e.g. in US fighter engines [6]. It is important therefore, to keep looking for the simple procedure enabling designer the reliable estimation of both crack initiation and crack propagation life for a given applied load, or to obtain the (boundary) load (or strain), at which the component would not experience the unpermissible damage during the designed life. This procedure is proposed in this paper.

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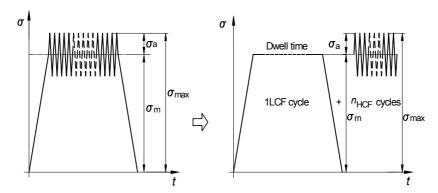


Fig. 1. Common stress history of one combined stress block and its separation in one LCF stress cycle and one HCF stress block.

2. CRACK INITIATION ASSESSMENT AT CA LOADING

When French established his well-known curve [2], and suggested distribution of σ -N diagram field in three regions - the region of damage between Wöhler curve and French curve, the region of failure on the right of the Wöhler curve, and the region of "overload" on the left of the French curve - he didn't know that he actually plotted the CI curve, a years before the Fracture Mechanics was established. The original French procedure of testing consists of cyclic loading that is stopped after a previously determined number of cycles, and after continued at the endurance limit level, or slightly bellow it. If the specimen is fractured after sufficiently long number of cycles, it means that the specimen had been damaged (i.e. cracked) in previous loading. Thus, the unfractured specimens had not been damaged. All the tests resulting in initial crack and all the tests resulting in uncracked specimen, represented by corresponding points, are separated by French curve. So, French curve was nothing but a crack initiation (CI) curve. In strain approach to fatigue design, more suitable to LCF loading, those points are distributed by corresponding CI curve in log N - log ε diagram. Recently, the French procedure is simplified, because the crack initiation is perceived by modern devices, but the name of French is not more in use . In the region of the finite fatigue life, clasping the fatigue lives between the boundary of quasi-static failure N_q and the boundary of the infinite fatigue life region, this curve is well described by the Wöhler type equation [5,6]

$$N_i \sigma^{m_i} = C_i \,, \tag{1}$$

where N_i is the crack initiation life for the certain stress level σ , and m_i and C_i are material constants.

At steady loading (N = 1/4), the CPT equals the ultimate strength σ_U , and for the sufficiently long fatigue life, which can be taken e.g. N_{gr} , it equals the endurance limit σ_0 , meaning the entire fatigue life consists of the crack initiation life.

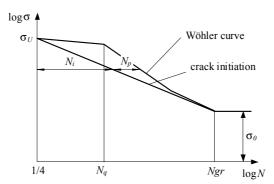


Fig. 2. Approximate design of the CI curve

It is reasonable to assume that there is a unique CPT curve between these two points, because the crack initiation mechanism is similar for both HCF and LCF loading. Therefore, the curve of the CPT could be represented in the log σ - log N diagram (Fig. 2.), with the straight line passing the points (1/4; σ_U) and (N_{gr} ; σ_0). It is easy now to calculate the slope of this curve:

$$m_i = \frac{\log(4N_{gr})}{\log(\sigma_U/\sigma_0)} \tag{2}$$

This expression is found to be in good correlation with experimentally obtained values. For example, the fatigue strength exponent b of steel 42 Cr Mo 4V (after DIN) for initiation life at r = -1 loading, was found to equal 0,0692 [3], meaning $m_i = 1/b = 14,5$. Exactly the same value was obtained after Eq. (2) for $N_{gr} = 3 \cdot 10^7$. It is also in line with novel investigations of Singh (2002).

In the same way as the Wöhler curves for various stress ratios are used for designing the fatigue strength plots in Smith diagram, the S-N curve for crack initiation is used in order to obtain the CI curve for any stress ratio r, in the same diagram. So, the Goodman plot of the fatigue strength is N_f = const plot in the same time, and Goodman CI curve plot is in the same time N_i = const plot, and exhibits also the boundary of crack initiation for any r. Just like the most frequently used fatigue strength plot is Goodman straight line, the best approximation of the CI curve is the Goodman plot again, which is also the straight line (Fig.3), passing the points $(\sigma_{0N,i}/2; \sigma_{0N,i})$ and $(\sigma_{U}; \sigma_{U})$. Indeed, Nicholas and Zuiker [1] declare that this plot is the straight line by definition (?). Thus, any straight line of the certain slope in Smith diagram passing the point $(\sigma_{U}; \sigma_{U})$ is the constant initiation life plot and in the same time the constant fatigue life plot. Of course, these lives are different.

Therefore, for the same lives $N_i = N_f$, these plots are different (Fig. 3.). Whereas the N_i = const curve is the straight line passing the point (σ_U ; σ_U), it is enough to know only the one point more to determine it. For any $N_q < N_i \le N_{gr}$, this point is obtained from the (only one) CI curve (1), usually for the stress ratio r = -1 or r = 0. For the purpose of this paper, the French curve at r = 0 is used, which enables determining the level of the pulsating stress at the CPT for certain N_i , by knowing the crack initiation life N_{gr} at the endurance limit level:

$$\sigma_{0N,i} = \sigma_0 (N_{gr} / N_i)^{1/m_i}$$
(3)

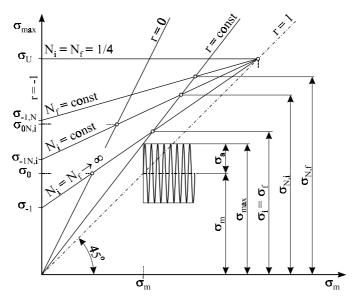


Fig. 3. Crack initiation curves in Smith diagram

Whereas the maximum stresses in Smith diagram change along the load line r = const, the fatigue limit for a certain stress ratio is determined as the ordinate of the intersection point between the fatigue limit plot $N_f = \text{const}$ and load line [6, 7, 8], the maximum value of the stress at the CPT, for the same stress ratio, is analogously obtained as the intersection point between the crack initiation plot $N_i = \text{const}$ and the load line. At common circumstances, when no pre-load stress is applied, the equation of the load line is

$$\sigma_{\max} = k\sigma_m, \tag{4}$$

where k = 2/(1+r) is the slope of the load line. The equation of the Goodman plot for the crack initiation life N_i = const is

$$\sigma_{N,i} = \sigma_{0N,i} + k_{\sigma}(\sigma_m - \sigma_{0N,i}/2),$$
 (5)

where $k_{\sigma} = (\sigma_U - \sigma_{0N,i})/(\sigma_U - \sigma_{0N,i}/2)$ is its slope. The maximum stress at the crack initiation boundary for the arbitrary stress ratio r is now obtained

$$\sigma_{N,i} = \frac{2 - k_{\sigma}}{2 - k_{\sigma}(1 + r)} \sigma_{0N,i}, \qquad (6)$$

which represents a maximum stress level at stress vratio r required for crack initiation after N_i cycles. By means of Eq. (1), it is easy now to obtain the crack initiation life for the maximum stress that reaches the $\sigma_{N_i,i}$ stress boundary:

$$N_i = N_{gr} \left(\frac{2 - k_{\sigma}}{2 - k_{\sigma}(1 + r)} \frac{\sigma_0}{\sigma_{N,i}} \right)^{m_i}. \tag{7}$$

3. CRACK INITIATION LIFE AT COMBINED HCF/LCF LOADING

For the stress history described in Fig. 1., the crack initiation life expressed in number of stress blocks $N_{B,i}$, is derived on the basis of Palmgren - Miner hypothesis of linear damage accumulation, where the level of damage is defined as

$$D_{i} = \sum_{j=1}^{n_{B}} \frac{n_{j}}{N_{j}} = \sum_{j=1}^{n_{B}} \left(\frac{n_{HCF,j}}{N_{HCF,j}} + \frac{1}{N_{LCF,j}} \right).$$
 (8)

The crack initiation is reached for $D_i = 1$, when number of blocks n_B becomes $N_{B,i}$:

$$\sum_{j=1}^{N_B,i} \left(\frac{n_{HCF,j}}{N_{HCF,j}} + \frac{1}{N_{LCF,j}} \right) = N_{B,i} \left(\frac{n_{HCF}}{N_{HCF,i}} + \frac{1}{N_{LCF,i}} \right) = 1, \tag{9}$$

where from follows the crack initiation life expressed in stress blocks [6]:

$$N_{B,i} = \frac{1}{\frac{n_{HCF}}{N_{HCF,i}} + \frac{1}{N_{LCF,i}}}.$$
 (10)

It is easy now to obtain the total initiation life:

$$N_{i} = N_{B,i}(1 + n_{HCF}) \cong N_{B,i} \cdot n_{HCF} = \frac{1}{\frac{1}{N_{HCF,i}} + \frac{1}{N_{LCF,i}} \cdot n_{HCF}}$$
(11)

The initiation life $N_{LCF,i}$ is obtained after the French curve (3) at r = 0:

$$N_{LCF} = N_{gr} (\sigma_0 / \sigma_m)^{m_i}$$
 (12)

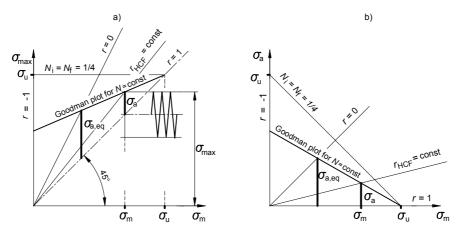


Fig. 4. Reducing the HCF stress amplitude σ_a to an equivalent stress amplitude $\sigma_{a,eq}$ at r = 0 in a) Smith diagram, and b) Haigh diagram.

Since the Palmgren-Miner hypothesis is valid for various stress blocks at the same stress ratio, this equation is also used for the calculation of the HCF initiation life, but by substituting in it an equivalent stress range obtained by reducing a HCF stress range (with stress ratio $r_{HCF} > 0$) to an equivalent stress range at r = 0. It can be done after Fig. 4, or after Eq.(7) by substituting in it the values for the slope $k_{\sigma} = (\sigma_U - \sigma_{max})/(\sigma_U - \sigma_m)$ and for the stress ratio r = 0. It is obtained:

$$\Delta \sigma_{eq} = 2\sigma_{a,eq} = \frac{2\sigma_a \sigma_U}{\sigma_U + \sigma_a - \sigma_m}$$
 (13)

Thus, by substituting Eq. 12 and Eq. 13 in Eq. 12, the explicit formula is obtained for determining the crack initiation life at combined HCF/LCF loading:

$$N_{i} = \frac{N_{gr}\sigma_{0}^{m_{i}}}{\left(\frac{2\sigma_{U}\sigma_{a}}{\sigma_{U} + \sigma_{a} - \sigma_{m}}\right)^{m_{i}} + \frac{\sigma_{m}^{m_{i}}}{n_{HCF}}}$$
(14)

3. CRACK PROPAGATION ASSESSMENT FOR COMBINED HCF/LCF LOADING

3.1 Reshaping the crack growth rate formulae

The fatigue crack growth rate formulae valid in regions II and III of crack growth rate [1, 9], and therefore acceptable for the estimation of the crack propagation life at constant amplitude loading, can be generally noted down as

$$\frac{da}{dN} = f(\Delta K^m, K_{\text{max}}^n) \tag{15}$$

where

$$\Delta K = \Delta \sigma Y \sqrt{\pi a} \tag{16}$$

is the stress intensity range,

$$K_{\text{max}} = \sigma_{\text{max}} Y \sqrt{\pi a} \tag{17}$$

is the upper value of the stress intensity factor, m and n are material constants, $\Delta \sigma = 2\sigma_a$ is a stress range, σ_{max} is a maximum stress, Y is a crack form factor, and a is a crack size.

By introducing into the formula (11) the damage ratio $D = a/a_c$, where a_c is a critical crack size and fracture toughness

$$K_c = \sigma_{\text{max}} Y \sqrt{\pi a_c} , \qquad (18)$$

it can be reshaped in the form

$$\frac{dD}{dN} = \frac{1}{a_c} f_1 \left[\left(\Delta K / K_c \right)^m, \left(K_{\text{max}} / K_c \right)^n \right] = \frac{D_0}{a_0} f_1 \left[\left(2(1-r)D^{\frac{1}{2}} \right)^m, D^{\frac{n}{2}} \right] = f_2(D)$$
 (19)

where a_0 is initial crack size, $D_0 = a_0/a_c$ is an initial damage ratio and $r = \sigma_{\min}/\sigma_{\max} = K_{\min}/K_{\max}$ is a load (stress intensity) ratio. By integrating this formula, the damage ratio after N propagating cycles is obtained:

$$D = f_3(N) \tag{20}$$

where from it is easy to determine the crack propagation life at constant amplitude loading - by substituting in it D=1.

The equation (19) can be used also in fatigue assessment at variable amplitude loading [7], but in such a case a_c changes, if σ_{max} changes. Thus, Eq. (19) must be reshaped:

$$\frac{dD}{dN} = \frac{1}{a_c} \frac{da}{dN} - \frac{a}{a_c^2} \frac{da_c}{dN} = f_2(D) + \frac{D}{D_0} \frac{dD_0}{dN} \,. \tag{21}$$

This expression is appropriate for the crack propagation assessment at any loading conditions, including non-regular ones, where maximum stress, crack form factor and load ratio change.

3.2. Explicit expression for approximate estimation of the crack propagation life at combined HCF/LCF loading

Herein, the formula (19) is applied for the crack propagation life estimation in the gas turbine and compressor discs and blades made of the titanium alloy Ti-6Al-4V, at combined HCF/LCF loading. If the stress history is simplified in the way that it consists of one LCF stress block with $N_{\rm LCF} = N_{\rm B}$ cycles at maximum stress σ_m and load ratio r = 0, followed by one HCF stress block with $n_{\rm HCF} \cdot N_{\rm B}$ cycles at maximum stress $\sigma_{\rm max}$ and load ratio $r = (\sigma_{\rm max} - 2\sigma_a)/\sigma_{\rm max}$, then the initial damage ratio is

$$D_0 = \pi a_0 \left(\frac{Y_{Lc} \sigma_m}{K_c} \right)^2 \tag{22}$$

where Y_{Lc} is a crack form factor at K_c stress intensity of the LCF loading. After Raju and Newman [8], the form factor is approximated by

$$Y = 0.78 \left(1 + \frac{a}{d} \right) \tag{23}$$

where d is a bar diameter.

As most appropriate for the purpose of this paper, the Ritchie formula [9] for the crack growth rate

$$\frac{da}{dN} = C\Delta K^m K_{\text{max}}^n \tag{24}$$

is applied for determining the damage ratio. For titanium alloy Ti-6Al-4V, the following values of material constants were obtained: $C = 5.2 \cdot 10^{-12}$, m = 2.5 and n = 0.67. The damage ratio growth rate is obtained now

$$\frac{dD}{dN} = \frac{B}{a_c} (1 - r)^m D^{\frac{m+n}{2}}$$
 (25)

where $B = 2^m C K_c^{m+n}$ is a material constant. By integrating this equation, it is easy now to determine the damage ratio at the end of LCF stress block:

$$D_{LCF} = \frac{D_0}{\left[1 - D_0^{\frac{m+n}{2}} - 1 \frac{B}{2a_{cL}} (m+n-2)N_B\right]^{\frac{2}{m+n-2}}}.$$
 (26)

where a_{cL} is the critical value of the srack size at LCF loading. This value can be determined by solving its equation:

$$a_{cL} = \frac{1}{\pi} \left[\frac{K_c}{Y(a_{cL})\sigma_m} \right]^2 \tag{27}$$

where $K_c = 50$ MPa m^{1/2} for Ti-6Al-4V alloy, [9].

The damage ratio $D_{\rm LCF}$ is the initial damage ratio for HCF stress block. Thus, the damage ratio at the end of the HCF stress block, as the final damage ratio, is now

$$D_{HCF} = \frac{D_{LCF}}{\left[1 - D_{LCF}^{\frac{m+n-2}{2}} \frac{B}{2a_{cH}} (1 - r)^m (m + n - 2) n_{HCF} N_B\right]^{\frac{2}{m+n-2}}}.$$
 (28)

The fatigue fracture occurs when this damage ratio reaches the value of one. Then, from the equations (26) and (28), it is not difficult to solve for the N_B and consequently for the entire crack propagation life:

$$N_p = 2 \frac{D_0^{1 - \frac{m+n}{2}} - 1}{B(m+n-2)[(1-r)^m n_{HCF} / a_{cH} + a_{cL}^{-1}]} n_{HCF}.$$
 (29)

Thus, the explicit expression is derived, enabling the estimation of the crack propagation life at combined HCF/LCF loading, for certain values of the stress levels σ_{max} and σ_{m} , which are hidden in a_{cH} and a_{cL} .

3.3. A more precise procedure for the crack propagation assessment at combined HCF/LCF loading

Assumption that stress history consists of one LCF cycle followed by one HCF stress block consisting of $n_{\rm HCF}$ cycles, followed by one LCF cycle etc. (Fig. 1) is much closer to real operational conditions. Thus, after one LCF cycle, the damage ratio, according to (20), is

$$D_{LCF,1} = \frac{D_0}{\left[1 - D_0^{\frac{m+n}{2} - 1} \frac{B}{2a_{cL}} (m+n-2)\right]^{\frac{2}{m+n-2}}}.$$
 (30)

At the end of the HCF stress block, the damage ratio becomes

$$D_{HCF,1} = \frac{D_{LCF,1}}{\left[1 - D_{LCF,1}^{\frac{m+n-2}{2}} \frac{B}{2a_{cH}} (1-r)^m (m+n-2) n_{HCF}\right]^{\frac{2}{m+n-2}}}.$$
 (31)

which is an initial damage ratio for the next combined stress block, etc.

The fatigue fracture occurs at the moment when damage ratio reaches the value of one. Then, the reached life becomes the fatigue life for certain, input values of stress levels σ_{max} and σ_{m} . For the subject material, Ti-6Al-4V, the fatigue lives determined in such a way were not significantly different from fatigue lives obtained by explicit formula (29) confirming known thesis that order of loading doesn't significantly influence the fatigue behaviour.

4. FATIGUE LIMITS FOR COMBINED HCF/LCF LOADING

In fatigue design generally, and especcially in design of components subjected to combined HCF/LCF loading, the Smith (or Haigh) diagram is a very useful tool, presenting the areas, i.e. the stress levels at which the required fatigue life will not be reached. The corresponding curves obtained, enable damage tolerant design, i.e. they divide the diagram area into two zones: the zone of stress states resulting in allowable and unallowable fatigue lives, that is in allowable and unallowable damage level. The procedure is the same as described in previous chapter, but for the fatigue life as input data. Thus, for certain values of fatigue lives, the fatigue strength curves are obtained indicating the stress levels in Smith diagram causing the fatigue failure after $N_f = C_f$ cycles. The calculations are carried out for various values of C_f , and for a number of HCF cycles per one stress block $n_{\rm HCF} = 10^2...10^5$. The fatigue limit curves obtained precisily exibit the reduction of the design area in Smith diagram compared to HCF loading only, the more so as the share of LCF loading is greater.

As an example, the resulting $N_f = 10^7$ curves for titanium alloy Ti-6Al-4V, and for $n_{\rm HCF} = 10^2...10^5$, are exhibited in Smith diagram, Fig. 3. In view of these curves, which share the diagram space on the safe and the unsafe one, it is observed:

- These curves are located between Goodman line and σmax = σm straight line, the higher the nHCF the higher curve position. At the region of lower mean stresses, they make one with Goodman line, then separate from it, reach maximum and finaly fall down at constant mean stresses. Thus, presence of the LCF component restricts the safe design space compared to that in case of pure HCF, the more so as the share of the LCF component is greater.
- -Between the curves of constant fatigue life based on the initial crack sizes of 0,1 mm and 0,05 mm was not observed a significant difference.
- The curves of constant fatigue life obtained on the basis of the derived closed form fatigue life formula, and those obtained on the basis of growth increments computed for one LCF cycle, nHCF cycles, next LCF cycle, etc., does not differ significantly.

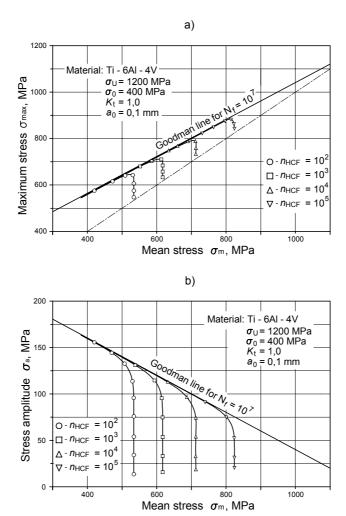


Fig. 5. Fatigue strengths in Smith (a) and Haigh (b) diagram for a combined HCF/LCF loading of a titanium alloy Ti-6Al-4V.

5. SUMMARY AND CONCLUSIONS

It is derived the closed form expression for estimation of the crack initiation life at combined HCF/LCF loading is derived, and the way of reshaping the crack growth rate formulae in the form enabling their use in fatigue design at non-stationary loading is demonstrated. Herein, the reshaped crack growth rate formula is applied for the fatigue design of aircraft components made of titanium alloy Ti-6Al-4V and subjected to combined HCF/LCF loading. For the stress history simplified in the way that it consists of one LCF stress block with $N_{\rm LCF} = N_{\rm B}$ cycles at maximum stress $\sigma_{\rm max}$ and load ratio r = 0, followed by one HCF stress block with $n_{\rm HCF} \cdot N_{\rm B}$ cycles at maximum stress $\sigma_{\rm max}$ and load

ratio $r = (\sigma_{\text{max}} - 2\sigma_a) / \sigma_{\text{max}}$, the closed form expression is derived for estimating the crack propagation life at combined HCF/LCF loading.

Smith and Haigh diagrams as design tools for estimating the fatigue strengths for designed fatigue life, known load ratio and various number $n_{\rm HCF}$ cycles, are obtained for the parts made of titanium alloy Ti-6Al-4V and subjected to combined HCF/LCF loading. However, the results of this research should be taken as a guide because

- The small crack behaviour has not been taken into account,
- The presence of other damage mechanisms like creep fatigue, oxidation and other environmental effects are ignored,
- The residual stresses have not been handled,
- The stress concentration has been ignored,
- Technology faults, material quality and operating conditions (like elevated temperature), have not been taken into account,
- Linear damage summation rule has been applied, although more precise techniques exist,
- The presence of inclusions and the service-induced damages could not be clasped,
- The reliability aspect of the design has been ignored.

At the same time, these imperfections are the sign-posts in the direction of building an expert system for the fatigue design of the aircraft components subjected to combined HCF/LCF loading.

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IZRAZ U ZATVORENOJ FORMI ZA PREDVIDJANJE TRAJANJA PRI ZAMORU KOD KOMBINOVANOG HCF/LCF OPTEREĆENJA

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Za kombinovano HCF/LCF opterećenje, čija je istorija napona pojednostavljena na takav način da se sastoji od jednog LCF naponskog bloka sa brojem ciklusa koji je jednak broju početnih radnji unutrašnjeg servisa, i jednog HCF naponskog bloka u kome su sumirani svi HCF ciklusi, izrazi u zatvorenoj formi izvode se za procenu kako trajanja začetka prsline, tako i trajanja prostiranja prsline kod kombinovanog HCF/LCF opterećenja. Kao primer upotrebe dobijeni su Smit i Hej dijagrami za komponente nacinjene od titanijumske legure Ti-6Al-4V, koje omogućavaju procenu snage pri zamoru za konstruisano trajanje pri zamoru, pozanatom odnosu opterećenja i odredjenom broju HCF ciklusa po jednom kombinovanom naponskom bloku.