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THE EFFECT OF STRESS RATIO ON THE BEHAVIOUR OF SHORT FATIGUE CRACKS IN ALUMINIUM ALLOYS

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Abstract. *Based on the principles of the Fatigue Damage Map, the effect of the stress ratio, R, on the boundaries of short crack growth is investigated. The analysis reveals that the tendency of the material to short crack growth is governed by the relation between the fatigue limit,* σ_{FL}*, and the cyclic yield stress,* σ_{cy}*. Materials with low values of* $w=\sigma_{FL}/\sigma_{cy}$ *loaded at high values of* R *exhibit negligible amount of growth irregularities, allowing therefore the accurate representation by a Paris-Erdogan power law.*

INTRODUCTION

For almost thirty years, short crack growth is on the forefront of structural integrity research [1-3]. The growth rate of short fatigue cracks has been shown experimentally to be significant faster than that of a long cracks under the same nominal stress intensity factor range ΔK_I [4-7]. This is because the crack tip plastic zone of a short crack is significantly larger than that predicted by the Linear Elastic Fracture Mechanics (LEFM). The above represents the tendency of short cracks to propagate along persistent slip bands, the size of which is dictated by the stacking fault energy of the material.

Several modifications and adaptations of LEFM have been reported as an attempt to predict the fatigue behaviour of short fatigue cracks. The most known are, the crack closure approach [8], the crack deflection approach [9] and the J-integral [10]. These models have been reasonably successful for short cracks several times longer than the microstructural dimension defined by the length of a grain [11]. Furthermore, most of the above models acknowledge the fact that different materials are likely to show different short crack growth tendencies. In [12] Rodopoulos and de los Rios, using the concept of the Fatigue Damage Map (FDM), proposed that the tendency of the material to exhibit

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short crack propagation depends on the ratio between the dominant cyclic properties of the material, namely, fatigue limit and cyclic yield stress.

In this work, the approach of scrutinising the tendency of the material to short cracking by it's cyclic properties is extended to incorporate the effect of the stress ratio. The analysis is based on several popular aluminium alloys.

DEFINING THE LIMITS OF SHORT CRACK GROWTH

In [13-16] it was proposed that the arrest of a crack takes place when the dislocation pile-up, representing the crack tip plastic zone, is unable to overcome the constraining provided by a dominant microstructural barrier, i.e. grain boundary, twin boundary, pearlite zone, etc. Based on the principles of the Navarro-Rios model [14-16], crack arrest was modelled by,

$$\sigma_{arrest} = \frac{m_i}{m_1}\frac{\sigma_{FL} - \sigma_1}{\sqrt{2a/D}} + \sigma_1 \tag{1}$$

where σ_{arrest} is the crack arrest or threshold stress, a is the crack length, σ_{FL} is the fatigue limit of the material (N>10^7cycles), D is the average transverse grain size, σ_1 is the crack closure stress and m_i is the grain orientation factor. It should be noted that m_i increases monotonically with crack length from a value of 1 until m_i reaches the saturated Taylor value of 3.07 (truly polycrystalline behaviour). The hypothesis m_1=1 is rationalised by the fact that, crack nucleation takes place in grains that are most favourably oriented in relation to the applied stress and consequently the resolved shear stress can easily reach maximum value.

The threshold/arrest stress defined by Eq.(1), identifies two controlling parameters: a) the strength of the grain boundary which is part of σ_{FL} (fatigue limit) and b) the grain orientation, m_i. Both parameters reflect the influence of microstructure on crack arrest. The first, by relating the strength of the boundary to the threshold stress for crack propagation and the latter, by incorporating the effect of the increasing number of grains transversed by the crack front as the crack grows. The latter reflects the increasing probability of a "hard" grain being included in the plastic zone.

In 2002, Kujawski [17] proposed that the effect of the R-ratio on the threshold stress intensity factor range is given by,

$$\frac{\Delta K_{th}}{\Delta K_{th,0}} = (1-R)^{\alpha} \quad \text{for} \quad R \geq 0 \tag{2a}$$

and

$$\frac{\Delta K_{th}}{\Delta K_{th,0}} = (1-R) \quad \text{for} \quad R \leq 0 \tag{2b}$$

where $\Delta K_{th,0}$ is the threshold ΔSIF corresponding to $R = 0$ and α is a fitting parameter ranging between 0 and 1 (a value of $\alpha = 0.5$ was suggested for aluminium alloys and martensitic steels).

Navarro et al [18] suggested that the plain fatigue limit is related to the threshold SIF through,

$$K_{th} = Y\sigma_{FL}\sqrt{\pi\frac{D}{2}} \qquad \textbf{(3)}$$

where Y is the crack correction factor. Using a similar argument for the fatigue limit, the effect of R-ratio on the fatigue limit is given by,

$$\frac{\Delta\sigma_{FL}}{\sigma_{FL(R=0)}} = (1-R)^{\alpha} \quad \text{for} \quad R \geq 0 \qquad (4a)$$

and

$$\frac{\Delta\sigma_{FL}}{\sigma_{FL(R=0)}} = (1-R) \quad \text{for} \quad R \leq 0 \qquad (4b)$$

Using Eqs.(1,4), the effect of R-ratio on the crack arrest can be written as,

$$\Delta\sigma_{arrest} = \frac{1}{Y}\frac{m_i}{m_1}\frac{(1-R)^{\alpha}\sigma_{FL(R=0)} - \sigma_1}{\sqrt{2a/D}} + \sigma_1 \quad \text{for} \quad R \geq 0 \qquad (5a)$$

and

$$\Delta\sigma_{arrest} = \frac{1}{Y}\frac{m_i}{m_1}\frac{(1-R)\sigma_{FL(R=0)} - \sigma_1}{\sqrt{2a/D}} + \sigma_1 \quad \text{for} \quad R \leq 0 \qquad (5b)$$

Figure 1 shows a plotting of Eq.(5) for 2024-T351 aluminium alloy. It should be noted that the approach is accurate for opening mode growing cracks.

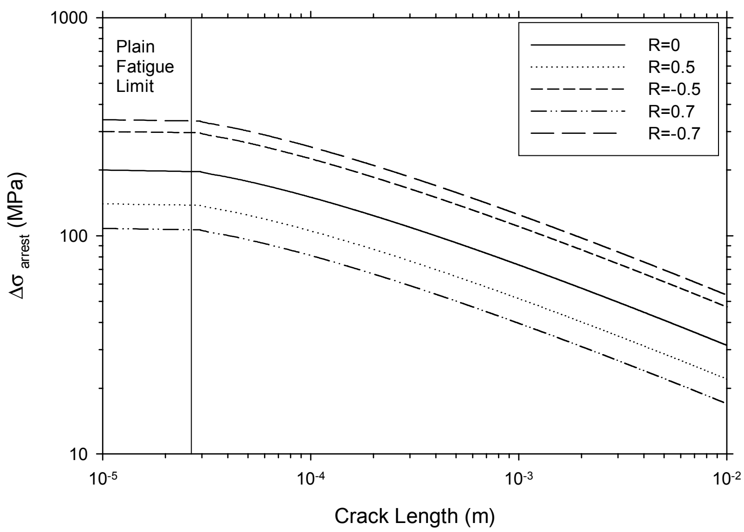

Fig.1. The effect of stress ratio on crack arrest stress range for 2024-T351 according to Eq.(5). The mechanical and physical properties used for the calculations are: $\sigma_{FL(R=0)}$=200MPa [13], m_i=1+0.35ln(2a/D) [19], D=52μm [20], α=0.5 [17], Y=1 and σ_1=0.

In [12,13,21] it was proposed that Stage I crack growth terminates when the crack tip stress field is able to initiate plasticity on two successive grains without further growth of the crack. This assumption is rationalised considering that when the crack is small a single family of slip planes can accommodate the crack tip plasticity. At longer crack lengths, crack tip plasticity is more intense and can only be accommodated by multiple or cross slip. The above was validated by Yoder et al [22], who examined the transition from Stage I to Stage II for a number of materials. According to the above, the transition co-ordinates (applied stress/crack length) from Stage I to Stage II (long) crack growth, is mathematically expressed as,

$$\sigma_{I\rightarrow II} = \frac{1}{Y}\left[\frac{2}{\pi}(\sigma^{c}{}_{y} - \sigma_{1})\sqrt{\frac{4D}{a+2D}} + \sigma_{1}\right] \tag{6}$$

where $\sigma_{I\rightarrow II}$ is the applied stress level at the transition and $\sigma^{c}{}_{y}$ is cyclic yield stress. In the case of a closure free cracks Eq.(6) is written as,

$$\sigma_{I\rightarrow II} = \frac{1}{Y}\left(\frac{2}{\pi}\sigma^{c}{}_{y}\sqrt{\frac{4D}{a+2D}}\right) \tag{7}$$

A typical Stage I to Stage II transition for several aluminium alloys is shown in Figure 2.

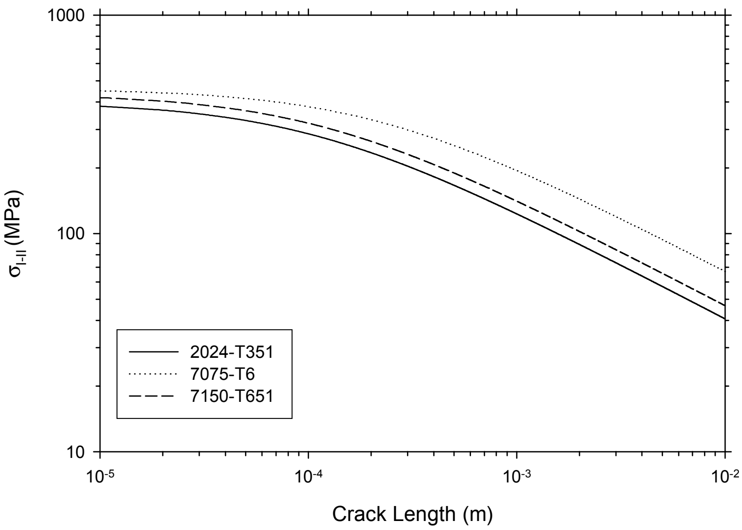

Fig. 3. Stage I to Stage II transition curves for three aluminium alloys. In all three cases a closure free crack and Y=1 was considered. The mechanical properties are: a) 2024-T351- $\sigma^{c}{}_{y}$=450MPa, D=52μm; b) 7150-T651 – $\sigma^{c}{}_{y}$=490MPa [19], D=58μm and c) 7075-T6 – $\sigma^{c}{}_{y}$=517 MPa, D=108μm [12].

In several works [11,14,15,23], it was suggested that the propagation rate of a fatigue crack can be was modelled through the elasto-plastic crack-tip opening displacement, δ_{tip},

$$\frac{da}{dN} = A\delta_{tip}{}^{b} \tag{8}$$

where the fitting parameters A and b depend on the material, stress-state, R-ratio, etc. The parameter δ_{tip} is given by Dugdale [47],

$$\delta_{tip} = \frac{8a}{\pi W}\sigma^{c}{}_{y} \ln\left[\sec\left(\frac{\pi\sigma}{2\sigma^{c}{}_{y}}\right)\right] \tag{9}$$

where W is either the shear modulus in the case of a short, mode II, crack or the elastic modulus in the case of a long, mode I, crack. By substituting Eq.(7) into Eq.(9), the δ_{tip} for the transition from Stage I to Stage II is given by,

$$\delta_{tip} = \frac{8a}{\pi G}\sigma^{c}{}_{y} \ln\left[\sec\left(\frac{\pi\sigma_{I\to II}}{2\sigma^{c}{}_{y}}\right)\right] \tag{10}$$

Mathematical analysis indicates that for $\sigma_{I\to II} << \sigma_{c}{}^{y}$, δ_{tip} tends to a constant $\delta^{c}{}_{tip}$ value [24]. The above comes as a verification to the fact that a constant size crack tip plastic zone corresponds to a constant SIF and consequently to a constant δ_{tip}.

In [25], Allen suggested that the effect of the ratio-R on the crack growth rate is given by,

$$\frac{da}{dN} = A\left(\frac{1+R}{1-R}\right)\delta_{tip}{}^{b} \tag{11}$$

where A, b are fitting parameters with A being a constant and b changing with R. In Eq. (11) the parameter b is for R = 0 ($\delta_{tip} = \Delta\delta_{tip}$). The expression 1 + R/(1−R) represents a mathematical regulator of δ_{tip} that allows the use of the same set of fitting parameters autonomously to R.

Considering that $\delta^{c}{}_{tip}$ depends implicitly on the material and the geometry of the triangle stress/crack/component, it is rational to assume that the crack growth rate at the transition from Stage I to Stage II is also a constant given that the parameter b is a function of R. Modelling of the above rationale in terms of the R adjustment provided by Eq. (11) gives,

$$\Delta\delta^{R}{}_{tip} = \left(\frac{1-R}{1+R}\right)\delta^{c}{}_{tip} \tag{12}$$

where $\Delta\delta^{R}{}_{tip}$ is the adaptation of $\delta^{c}{}_{tip}$ in terms of R. Equation 12, using Eqs.(8,10) can be also expressed in terms of the transition stress $\Delta\sigma_{I\to II}$,

$$\Delta\sigma^{R}{}_{I\to II} = \frac{1}{Y}\left(\frac{1-R}{1+R}\right)\left(\frac{2}{\pi}\sigma^{c}{}_{y}\sqrt{\frac{4D}{a+2D}}\right) \tag{13}$$

In Figure 4 the effect of R on the transition from Stage I to Stage II is illustrated.

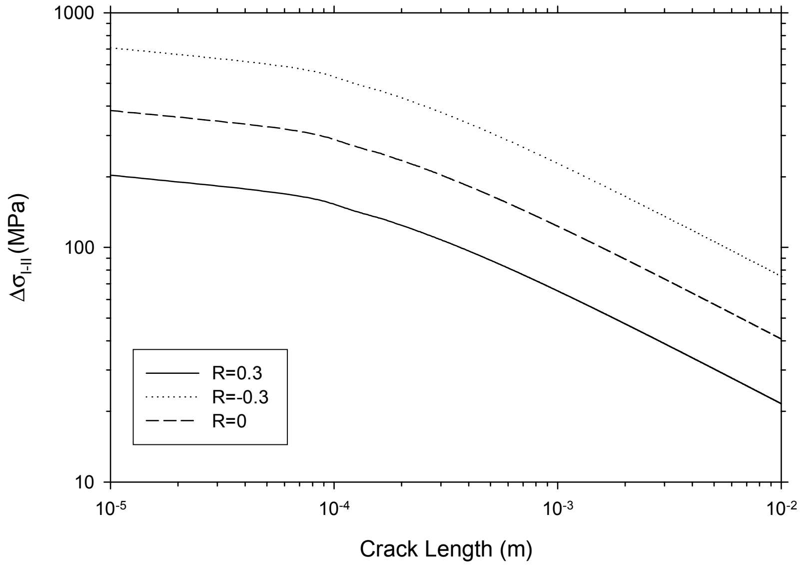

Fig. 4. The effect of R on the transition from Stage I to Stage II for 2024-T351.
The mechanical properties used for the calculations are as before.
The crack correction factor was unity.

Simultaneous plotting of Eqs.(5,13) provides the limits of short crack growth. A typical example in given in Figure 5.

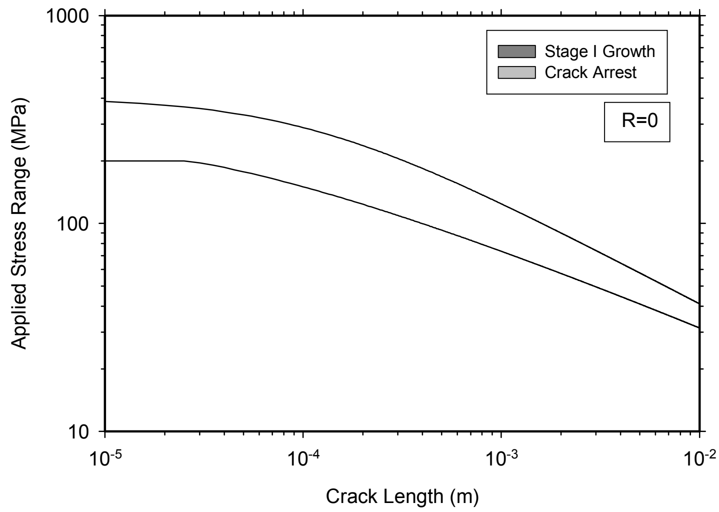

Fig. 5. Typical Stage I crack growth area for 20247-T351 at R=0 and Y=1.
Stage I growth is area between crack arrest and the transition from Stage I to Stage II crack growth.

ANALYSING THE EFFECT OF R ON STAGE I GROWTH

Analysis of the tendency of the material to Stage I cracking can be evaluated by examining the conditions that would allow the interception between crack arrest and Stage I growth. This can be achieved by equating Eq.(5) to Eq.(13),

$$\frac{1}{Y}\frac{m_i}{m_1}\frac{(1-R)^{0.5}\sigma_{FL(R=0)}}{\sqrt{2a/D}}=\frac{1}{Y}\left(\frac{1-R}{1+R}\right)\frac{2}{\pi}\sigma^c{}_y\sqrt{\frac{4D}{a+2D}} \tag{14}$$

Solution of Eq.(14) in terms of a can provide a clear understanding over the effect of R on the tendency of different aluminium alloys to Stage I cracking. In Figure 6 the outcome of such rationale is depicted.

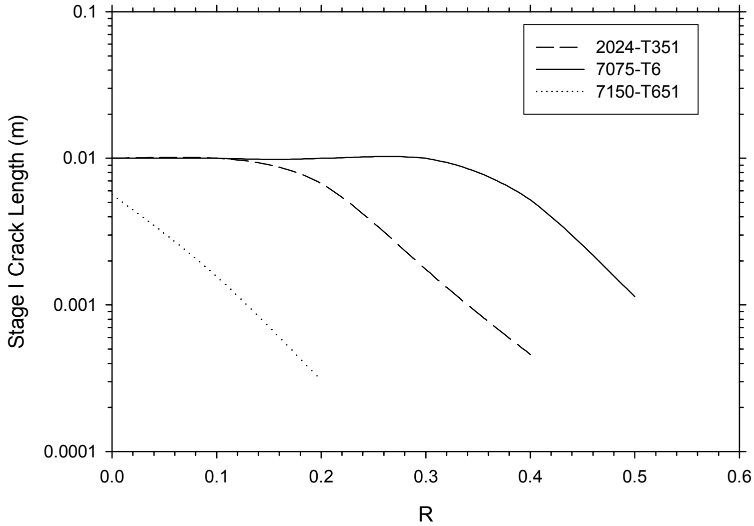

Fig. 6. The extend to Stage I cracking for three common aerospace aluminium alloys for positive stress ratios. The bound at 10mm represents run-outs. For the 7150-T651, an average grain size of D=58μm, $\sigma^c{}_y$=490 MPa and $\sigma_{FL(R=0)}$=300 MPa were used. The ratio of $\sigma_{FL(R=0)}/\sigma^c{}_y$ was 0.61 for 7150-T651, 0.44 for 2024-T351 and 0.36 for 7075-T6.

DISCUSSION AND CONCLUSIONS

This work represents an attempt to incorporate the effect of the stress ratio R into the FDM. The effect of R on crack arrest was modelled through the Kujawski's parameter α. The approach provides the necessary flexibility to encounter for changes caused by different materials, etc. Stress ratio effects on the transition from Stage I to Stage II growth was modelled by making use of the Allen's model. The analysis is possible due to the fact that transition conditions are characterised by a constant crack tip plastic zone and consequently by a constant value of δ_{tip}. The above allows the use of the matching crack propagation concept for several values of R.

Knowledge of the effect of R on crack arrest and on the transition from Stage I to Stage II growth allows the characterisation of the tendency of the material to exhibit Stage I cracking. Such information is vital, in cases where the accuracy of the Paris-Erdogan power law to predict fatigue life is in question. In general, the fatigue life of materials with limited tendency to Stage I crack growth is expected to be better determined by such approach. In contrast, the prediction error increases for cases where the size of Stage I crack growth is significant.

The current work reveals that life prediction error is expected to become smaller with stress ratio. This is because the tendency of the material to Stage I growth diminishes with R. Numerical predictions based on three popular aerospace aluminium alloys showed that the effect of R on Stage I crack growth increases with the ratio between the fatigue limit and the cyclic yield stress.

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REFERENCES

1. Pearson S (1975) Initiation of fatigue cracks in commercial aluminium alloys and the subsequent propagation of very short cracks, Engng Fract. Mech. 7, pp. 235-247.
2. Morris W. L. (1976) The early stage of fatigue crack propagation in Al 2048, Metall. Trans., A84, pp. 589-596.
3. Miller K. J. (1982) The short fatigue crack problem, Fatigue Fract. Engng. Mater. Struct., 5, pp. 223-232.
4. Cooper T. D and Kelto C. A. (1978) Fatigue in machines and structures-aircraft, In: Fatigue and Microstructure, ASM, Metals Park, Ohio.
5. McCarver J. F and Ritchie R. O. (1982) Fatigue crack propagation thresholds for long and short cracks in Rene' 95 nickel-base superalloy, Mater. Sci. Engng., 55, pp. 63-67.
6. Tanaka K and Nakai Y. (1983) Propagation and non-propagation of short fatigue cracks at a sharp notch, Fatigue Fract. Engng. Mater. Struct., 6, pp. 315-327.
7. de los Rios E. R, Tang Z and Miller K. J. (1984) Short crack fatigue behaviour in a medium carbon steel, Fatigue Fract. Engng. Mater. Struct., 7(2), pp. 97-108.
8. Morris W. L and Buck O. (1977) Crack closure load measurements for microcracks developed during the fatigue of Al 2219-T851, Metall. Trans., A8, pp. 597-601.
9. Suresh S. (1983) Crack deflection: implications for growth of long and short fatigue cracks, Metall. Trans., A14, pp. 2375-2385.
10. El Haddad M. H, Dowling M. E., Topper T. H and Smith Jr. K. N. (1980) J-integral applications for short-fatigue-crack modelling, Int. J. Fracture, 16, pp. 15-30.
11. de los Rios E. R. and Navarro A. (1990) Considerations of grain orientation and work hardening on short-fatigue-crack modelling, Phil. Mag., 61, pp. 435-449.
12. Rodopoulos C. A. and de los Rios E. R. (2002) Theoretical Analysis on short fatigue cracks, Inter. J. of Fatigue, 24, pp. 719-724.
13. Rodopoulos C. A. and Yates J. R. (2002) Fatigue damage tolerant analysis using the fatigue damage map, In the 14th US National Congress of Theoretical and Applied Mechanics, eds. R. C. Batra and E. G. Henneke, Virginia Tech, 197-208.
14. Navarro A and de los Rios E. R. (1992) Fatigue crack growth modelling by successive blocking of dislocations. Proc. R. Soc., A437, pp. 375-390.
15. de los Rios E. R. (1998) Dislocation modelling of fatigue crack growth in polycrystals. Engng. Mech., 5, pp. 363-368.
16. de los Rios E. R. and Navarro A. (1990) Considerations of grain orientation and work hardening on short-fatigue-crack modelling, Phil. Mag., 61, pp. 435-449.
17. Kujawski D. (2001) A fatigue crack driving force parameter with load ratio effects, Inter. J. of Fatigue, 23, pp. S236-S246.

18. Navarro A, Vallellano C, de los Rios E. R. and Xin X. J. (1999) Notch sensitivity and size effects described by a short crack propagation model, In: Engineering Against Fatigue, J. H. Beynon, M. W. Brown, T. C. Lindley, R. A. Smith and B. Tomkins, Balkema, Netherlands, pp. 63-72.
19. Curtis S. A., Solis Romero J, de los Rios E. R, Rodopoulos C. A and Levers A (2002) Predicting the Interfaces Between Fatigue Crack Growth Regimes in 7150-T651 Aluminium Alloy Using the Fatigue Damage Map, Mater. Sc. Engng., in press.
20. de los Rios E. R, Trooll M and Levers A (1999) Improving the fatigue crack resistance of 2024-T351 aluminium alloy by shot peening, In: Proc. Ceas Forum, Churchill College Cambridge, pp. 261-268.
21. Rodopoulos C. A, de los Rios E. R, Yates J. R and Levers A (2002) A fatigue damage map for the 2024-T3 aluminium alloy, Fatigue Fract. Engng. Mater. Struct., in press.
22. Yoder G. R, Cooley L. A and Crooker T. W (1982) On microstructural control of near-threshold fatigue crack growth in 7000-series aluminium alloys, Scr. Metall., 16, pp. 1021-1025.
23. de los Rios E. R (1998) Dislocation modelling of fatigue crack growth in polycrystals. Engng. Mech., 5, pp. 363-368.
24. Choi J-H (2002) The effect of stress ratio on the fatigue behaviour of 2024-T351 aluminium alloy, MPhil Thesis, University of Sheffield.
25. Allen R. J (1980) Fatigue crack growth characterisation by LEFM-Part II, Fatig. Fract. Engng. Mat. Struct., 11(2), 71-108.

UTICAJ ODNOSA NAPONA NA PONAŠANJE PRSLINA SA KRATKIM ZAMOROM KOD ALUMINIJUMSKIH LEGURA

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Na osnovu principa Mape oštećenja zamorom, istražen je uticaj odnosa napona, R, na granice napredovanja kratke prsline. Analizom je otkriveno da se tendencija materijala ka napredovanju kratke prsline rukovodi odnosom izmedju granice zamora, σ_{FL}*, i cikličnog napona popustljivosti,* σ_{cy}*. Materijali sa niskim vrednostima* $w = \sigma_{FL}/\sigma_{cy}$ *koji se opterećuju pri visokim vrednostima R pokazuju zanemarljivu količinu nepravilnosti u napredovanju, čime omogućavaju tačno predstavljanje putem Paris-Erdoganovog zakona snage.*