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Invited Paper

ON WEAKLY SYMMETRIC STRUCTURES ON A RIEMANNIAN MANIFOLD

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Abstract. *This is a survey article on weakly symmetric structures on a Riemannian manifold.The study of weakly symmetric and weakly projective symmetric Riemannian manifold were initiated byTamassy and Binh. Later on several authors studied weakly symmetric Riemannian manifold and analogous structures, viz. weakly Ricci symmetric, weakly projective symmetric and weakly conformally symmetric Riemannian manifolds. Here we present a brief survey of results on weakly symmetric structures on a Riemannian manifold and some applications in the theory of Relativity.*

Key words: *Pseudo symmetric manifold, weakly symmetric manifold, weakly Ricci symmetric manifold, weakly conformally symmetric manifold, weakly projective symmetric manifold.*

1. INTRODUCTION

The notions of pseudo symmetric and pseudo Ricci symmetric Riemannian manifolds have been introduced by M. C. Chaki [1,2]. Later on several authors studied such manifolds and also analogous structures on a Riemannian manifold [3,4,5,6,7,8,...,23].

We recall the definition of these manifolds. Let (M,g) be an n-dimensional Riemannian manifold and U a tensor field of type [1,3] on it. Let X,Y,Z V$\in\chi$(M) be tangent vector fields and A a 1-form on M. Let us consider the relation

$$(\nabla_X U)(Y,Z)V = 2A(X)U(Y,Z)V + A(Y)U(X,Z)V + A(Z)U(Y,X)V + A(V)U(Y,Z)X + g(U(Y,Z)V,X)\rho, \quad (1)$$

where $\rho\in\chi(M)$ is a vector field defined by $g(X,\rho) = A(X)\ \forall X$, ∇ denotes the Levi-Civita connection of (M,g). If [1] holds for $U \equiv R$ (the Riemannian curvature tensor of (M,g)),

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then the manifold is called pseudo symmetric [1]; (1) holds for $U \equiv C$ (the conformal curvature tensor of the manifold (M,g)) is called pseudo comformally symmetric [10]; and (1) holds for $U \equiv W$(the projective curvature tensor of (M,g)), the manifold is called pseudo projective symmetric [7]; A is the associated 1-form. Hence a pseudo symmetric manifold is defined by

$$(\nabla_X R)(Y,Z)V = 2A(X)R(Y,Z)V + A(Y)R(X,Z)V + A(Z)R(Y,X)V + A(V)R(Y,Z)X + g(R(Y,Z)V,X)\rho. \quad (2)$$

If the Ricci tensor S of type (0,2) of (M,g) is non-zero and satisfies the relation

$$(\nabla_X S)(Y,Z) = 2A(X)S(Y,Z) + A(Y)S(X,Z) + A(Z)S(Y,X), \quad (3)$$

then the manifold is called pseudo Ricci symmetric manifold [2]. Pseudo symmetric manifold is denoted by $(PS)_n$ and in analogous way pseudo conformally symmetric, pseudo Ricci symmetric and pseudo projective symmetric manifolds are denoted by $(PCS)_n$, $(PRS)_n$ and $(PWS)_n$ respectively. In case of the vanishing of A the pseudo symmetric Riemannian manifold is symmetric, for in this case $\nabla R = 0$. The class of pseudo symmetric manifolds arose during the study of conformally flat space of class one [24].

Also Prvanović [15] proved that every recurrent manifold satisfies (2). Conversly, if the manifold (M, g) besides (2) satisfies also

$$A(X)R(Y,Z) + A(Y)R(Z,X) + A(Z)R(X,Y) = 0,$$

then (2) reduces to

$$(\nabla_X R)(Y,Z)W = 4A(X)R(Y,Z)W$$

i.e., (2) reduces to a recurrent manifold.

In [16] Ewert-Krzemieniewski proved the existence of the pseudo symmetric manifold satisfying (2).

It may be mentioned that the pseudo symmetry in the sense of Chaki is different from that of R. Deszcz [25].

In 1989 Tamassy and Binh [26] introduced the notion of weakly symmetric and weakly projective symmetric Riemannian manifolds by weakening the condition of symmetry.

A non-flat Riemannian manifold (M,g) of dimension n (>2) is called weakly symmetric if there exists 1-forms A, B, D, E and a vector field F such that

$$(\nabla_X R)(Y,Z)V = A(X)R(Y,Z)V + B(Y)R(X,Z)V + D(Z)R(Y,X)V + E(V)R(Y,Z)X + g(R(Y,Z)V,X)F, \quad X,Y,Z,V \in \chi(M) \quad (4)$$

where R is the curvature tensor of (M, g). A non-flat Riemannian manifold is called weakly projective symmetric if the projective curvature tensor W given by

$$W(X, Y)Z = R(X, Y)Z-(1/n-1)[S(Y,Z)X-S(X,Z)Y]$$

satisfies the relation (4). Weakly symmetric and weakly projective symmetric Riemannian manifolds are denoted by $(WS)_n$ and $(WWS)_n$ respectively. In a subsequent paper Tamassy and Binh [27] introduced the notion of weakly Ricci symmetric manifold. If $B = C = D = (1/2)A$, then the weakly symmetric manifold reduces to a pseudo symmetric manifold.

A non-flat Riemannian manifold is called weakly Ricci symmetric if the Ricci tensor S is non-zero and satisfies the condition

$$(\nabla_X S)(Y, Z) = A(X)S(Y,Z) + B(Y)S(X,Z) + D(Z)S(Y,X) \quad (5)$$

where A, B, D are 1-forms. Such a manifold is denoted by $(WRS)_n$.

Let $g(X,\alpha) = A(X)$, $g(X, \beta) = B(X)$, $g(X,\gamma) = D(X)$ and $g(X,\delta) = E(X)$, $\forall X \in \chi(M)$.

Then $\alpha, \beta, \gamma, \delta$ and $F \in \chi(M)$ will be called the basic vector fields of weakly symmetric structures on a Riemannian manifold. If (4) holds for $R \equiv C$ where C is the conformal curvature tensor defined by

$$C(X,Y)Z = R(X,Y)Z-1/(n-2)[S(Y,Z)X-S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY] \\ + r/((n-1)(n-2))[g(Y,Z)X-g(X,Z)Y],$$

r denotes the scalar curvature of the manifold and Q is the symmetric endomorphism corresponding to the Ricci tensor S defined by $S(X,Y) = g(QX,Y)$, then the manifold is called weakly conformally symmetric manifold [28] and such a manifold is denoted by $(WCS)_n$. Thus a $(WCS)_n$ is defined by

$$(\nabla_X C)(Y,Z)V = A(X)C(Y,Z)V + B(Y)C(X,Z)V + D(Z)C(Y,X)V \\ + E(V)C(Y,Z)X + g(C(Y,Z)V,X)F. \quad (6)$$

Later in 1994 M. C. Chaki [29] generalized the notion of pseudo symmetric manifold and called it generalized pseudo symmetric manifold.

A non-flat Riemannian manifold is called generalized pseudo symmetric if the curvature tensor R satisfies

$$(\nabla_X R)(Y,Z)V = 2A(X) R(Y,Z)V + B(Y) R(X,Z)V + D(Z) R(Y,X)V \\ + E(V) R(Y,Z)X + g(R(Y,Z)V,X)\rho \quad (7)$$

where ρ is a vector field given by $g(X,\rho) = A(X)$ $\forall X$. Such a manifold is denoted by $(GPS)_n$. (4) gives (7) if α and F are related by

$$g(X,F) = \alpha(X) \ \ \forall X.$$

So (7) is a litter stronger assumption than (4). Thus weakly symmetric structures are little weaker than generalized pseudo symmetric structures on a Riemannian manifold.

Subsequently Chaki and Koley [30] introduced the notion of generalized pseudo Ricci symmetric manifld which is defined as follows:

An n-dimensional Riemannian manifold is said to be a generalized pseudo Ricci symmetric manifld if the Ricci tensor S is non-zero and satisfies the condition

$$(\nabla_X S)(Y,Z) = 2A(X) S(Y,Z) + B(Y) S(X,Z) + D(Z) S(X,Y). \quad (8)$$

Such a manifold is denoted by $(GPRS)_n$.

2. Associated 1-forms of a $(WS)_N$ and $(WCS)_N$

The equation (4) can be written as

$$(\nabla_X 'R)(Y,Z,U,V) = A(X)'R(Y,Z,U,V) + B(Y)'R(X,Z,U,V) + C(Z)'R(Y,X,U,V) \\ + D(U)'R(Y,Z,X,V) + E(V)'R(Y,Z,U,X) \quad (2.1)$$

where $'R(Y,Z,U,V) = g(R(Y,Z)U,V)$.

Interchanging Y and Z in (2.1) and then adding with (2.1) we obtain

$$[B(Y)-C(Y)]'R(X,Z,U,V) + [B(Z)-C(Z)]'R(Y,X,U,V) = 0.$$

From this relation M. Prvanović [31] proved that the 1-form B and C are equal. In a similar manner, interchanging U and V in (2.1) we get D = E. Thus the defining relation of a $(WS)_n$ reduces to

$$(\nabla_X{}'R)(Y,Z,U,V) = A(X)'R(Y,Z,U,V) + B(Y)'R(X,Z,U,V) + B(Z)'R(Y,X,U,V) + D(U)'R(Y,Z,X,V) + D(V)'R(Y,Z,U,X). \tag{2.2}$$

Since the conformal curvature tensor satisfies the same skew-symmetric property as the Riemannian curvature tensor, the defining relation of $(WCS)_n$ can be expressed in the following form:

$$(\nabla_X{}'C)(Y,Z,U,V) = A(X)'C(Y,Z,U,V) + B(Y)'C(X,Z,U,V) + B(Z)'C(Y,X,U,V) + D(U)'C(Y,Z,X,V) + D(V)'C(Y,Z,U,X) \tag{2.3}$$

where $'C(X,Y,U,V) = g(C(X,Y)U,V)$.

It may be mentioned that the defining relation of a weakly projective symmetric manifold $(WWS)_n$ can not be expressed in the above reduced form.

3. EXAMPLES OF $(WS)_N$, $(WRS)_N$ AND $(WCS)_N$

In this section we give the examples of a $(WS)_n$, $(WRS)_n$ and $(WCS)_n$.

We define the metric g in the coordinate space R^n ($n \geq 4$) by the formula

$$ds^2 = \phi(dx^1)^2 + k_{\alpha\beta}\, dx^\alpha dx^\beta + 2dx^1 dx^n, \tag{3.1}$$

where $[k_{\alpha\beta}]$ is a symmetric and non-singular matrix consisting of constants and ϕ is a function of $x^1, x^2, \ldots, x^{n-1}$ and independent of x^n. Here each Latin index run over 1,2,...,n and each Greek index over 2,3,..., n-1.

In the metric considered, the only non-vanishing components of the Christoffel symbols and the curvature tensor R_{hijk} are [32]

$$\begin{Bmatrix} \beta \\ 1 \quad 1 \end{Bmatrix} = -\frac{1}{2}k^{\beta\alpha}\phi.\alpha, \qquad \begin{Bmatrix} n \\ 1 \quad 1 \end{Bmatrix} = -\frac{1}{2}\phi.1, \qquad \begin{Bmatrix} \beta \\ 1 \quad \alpha \end{Bmatrix} = -\frac{1}{2}\phi.\alpha,$$

$$\text{and} \quad R_{1\alpha\alpha\beta} = \frac{1}{2}\phi.\alpha\beta$$

where (.) denotes the partial differentiation and $[k^{\alpha\beta}]$ is the inverse matrix.

An example of a $(WS)_n$ is given by the following:

Theorem 3.1. (De and Bandyopadhyay [33]) Let V^n ($n \geq 4$) be a Riemannian space with a metric of the form

$$ds^2 = \phi(dx^1)^2 + \delta_{\alpha\beta} dx^\alpha dx^\beta + 2dx^1 dx^n,$$

$$\phi = \delta_{\alpha\beta} x^\alpha x^\beta e^{x^1}.$$

Then V^n is a weakly symmetric space which is not symmetric .

Considering the same metric as in Theorem 3.1 we (De and Ghosh [34]) prove the existence of a $(WRS)_n$.

For an example of a $(WCS)_n$ the following theorem is obtained.

Theorem 3.2. (De and Bondyopadhyay [28]) Let V^n ($n\geq 4$) be a Riemannian space with a metric of the form

$$ds^2 = \phi(dx^1)^2 + \delta_{\alpha\beta}dx^\alpha dx^\beta + 2dx^1dx^n,$$
$$\phi = (M_{\alpha\beta} + \delta_{\alpha\beta})x^\alpha x^\beta e^{x^1}$$

where $M_{\alpha\beta}$ are constants and satisfy the relations

$$M_{\alpha\beta} = 0 \text{ for } \alpha\neq\beta;\ M_{\alpha\beta}\neq 0 \text{ for } \alpha = \beta \text{ and } \sum_{\alpha=2}^{n-1} M_{\alpha\beta} = 0.$$

Then V^n is a weakly conformally symmetric space with zero scalar curvature which is neither conformally flat nor conformally symmetric.

4. Decomposable and semi-decomposable $(WS)_N$

A Riemannian space is said to be decomposable if it can be expressed as a product $V_r\times V_{n-r}$ for some r i.e., if coordinates can be found so that its metric takes the form

$$ds^2 = \sum_{a,b=1}^{r} g_{ab}\, dx^a dx^b + \sum_{\alpha,\beta=r+1}^{n} g_{\alpha\beta}dx^\alpha dx^\beta, \qquad (4.1)$$

where g_{ab} are functions of $x^1, x^2, ..., x^r$ and $g_{\alpha\beta}$ are functions of $x^{r+1}, x^{r+2}, ..., x^n$ only; a,b,c, ... are taken to have range 1 to r and α,β,γ, ... to have the range r + 1 to n. The two parts of (3.1) are the metrices of a V_r and V_{n-r} which are called the decomposition spaces of V_n [35].

Considering decomposable $(WS)_n$ with $A\neq 0$, T. Q. Binh [36] proved the following

Theorem 4.1. If a $(WS)_n$ with $A\neq 0$ is a decomposable space $V_r\times V_{n-r}$ (r, n-r$\geq$2), then one of the decomposition spaces is flat and the other is weakly symmetric; and conversly, if in a decomposable $V_n = V_r\times V_{n-r}$ one of the decomposition spaces is flat and the other is weakly symmetric with $A\neq 0$, then V_r is a $(WS)_n$ with $A\neq 0$.

In the same paper [36] T. Q. Binh proved also the following

Theorem 4.2. If a $(WS)_n$ has cyclic Ricci tensor, moreover

$$\Omega = B + C + D + E$$

is not orthogonal to

$$\theta = A + C + D$$

and the cyclic sum $\Sigma A(X)\theta(Y)\theta(Z)$ is not zero for all X, Y, Z, then the space is an (X,Y,Z) Einstein space of zero scalar curvature.

Remark. Theorem 4.1 and Theorem 4.2 generalizes the results of a $(PS)_n$ studied by Chaki and De [4]. Moreover Binh [36] proved the converse part in theorem 4.1.

An n-dimensional (n>2) Riemannian space V_n is said to be semi-decomposable [37] if in some coordinates its metric is given by

$$ds^2 = g_{ij}dx^i dx^j = g_{ab}dx^a dx^b + \sigma \overset{*}{g}_{\alpha\beta}dx^\alpha dx^\beta \qquad (4.2)$$

where i,j,k,... = 1, 2, ..., n ; a,b,c,... =1,2,..., q(q<n), α,β,γ, ... =1,2,...,n; $\overset{*}{g}_{ab}$ and σ are function of $x^1, ..., x^q$ only and $\overset{*}{g}_{\alpha\beta}$ are functions of $x^{q+1}, ..., x^n$ only.

The two parts of (1.3) are the matrices of V_q and V_{n-q} which are called the decomposition spaces of V_n. Each object denoted by a bar is assumed to be formed from $\bar{g}_{ab}$ and each object denoted by a star form $\overset{*}{g}_{\alpha\beta}$. If, in particular, $\sigma = 1$, then V_n reduces to a decompoable space. The decomposibility of $(WS)_n$ has been studied by Binh [36].

Concerning the semi-decomposibily of $(WS)_n$ we prove the following:

Theorem 4.3. [38] For a semi-decomposable $(WS)_n$ with non-constant funstion σ, the part V_q (q>2) is a $(WS)_n$ and if A_e is not a gradient vector, then the part V_{n-q} is a space of constant curvature.

From (2.2) we obtain

$$dr(X) = A(X)r + 2S(X,\tilde{P})$$

where $\tilde{P}$, is a vector field defined by $g(X,\tilde{P}) = T(X)$
where $T(X) = A(X) + B(X)$.

Let $V(X) = S(X, \tilde{P})$.

Then the vector field corresponding to the 1-form V shall be called the Ricci-associate of the vector field $\tilde{P}$. Considering the Ricci-associate of $\tilde{P}$ we prove the following

Theorem 4.4. [38] In a $(WS)_n$ (n>2) of constant scalar curvature, the Ricci-associate of P is collinear with the vector field α defined by $g(X,\alpha) = A(X)$ and in a $(WS)_n$ (n>2) of non-constant scalar curvature, the Ricci-associate of $\tilde{P}$ and α can not be both gradient unless the vector dr(X) is collinear with both α and the Ricci-associate of P.

5. Some results of (WS)n

Now we recall the definition of a B-space given by P. Venzi [39]. Let $L(\theta)$ be a vector space formed by all vector θ satisfying

$$\theta_l R_{nijk} + \theta_j R_{nikl} + \theta_k R_{nilj} = 0. \tag{5.1}$$

A Riemannian space is said to be a B-space if dim $L(\theta) \geq 1$.

Concerning B-space M. Prvanović [31] proved the following Theorems.

Theorem 5.1. If a weakly symmetric Riemannian manifold is not pseudo symmetric (in the sence of Chaki), then it is a B-space.

Theorem 5.2. In a B-space there exists a symmetric tensor field T_{ij} such that the curvature tensor has the form

$$R_{hijk} = T_{hk}\theta_i\theta_j + T_{ij}\theta_h\theta_k - T_{hj}\theta_i\theta_k - T_{ik}\theta_h\theta_j,$$

where θ is the basis vector of the space $L(\theta)$. In order that such a space with dim $L(\theta) = 1$ be weakly symmetry, it is necessary and sufficient that T_{ij} and θ_j satisfy some conditions. This weak symmetry is of the form

$$\nabla_r R_{hijk} = F_r R_{hijk} + D_h R_{rijk} + D_I R_{hrjk} + D_j R_{hirk} + D_k R_{hijr}. \tag{5.3}$$

Theorem 5.3. Let us consider a B-space such that dim $L(\theta) = 1$ and the basis for $L(\theta)$ is a unit vector field. In order that such a space be weakly symmetric, it is necessary and sufficient that the Ricci tensor and the basis vector θ satisfy certain conditions. This weak symmetry is of the form (5.3).

Theorem 5.4. Let us consider a B-space characterized by dim $L(\theta) = 2$. In order that this B-space be weakly symmetric, it is necessary and sufficient that the conditions

$$\nabla_r \bar{\theta}_h = \bar{a}_r \bar{\theta}_h + \bar{b}_r \bar{\theta}_h + C_h \bar{\theta}_r$$

and

$$\nabla_r \theta_h = \bar{e}_r \theta_h + \bar{f}_r \theta_h + C_h \theta_r \quad \text{hold}$$

where $\bar{\theta}_i$ is a basis vector and a_r, b_r, c_r, e_r, f_r are vectors.

In 1994 Chaki [29] studied generalized pseudo symmetric manifolds defined by (6) which is similar to the weakly symmetric manifold defined by Tamassy and Binh.

In his paper [29] Chaki obtained the following results which can be stated as follows :

Theorem 5.5. An Einstein $G(PS)_n$ satisfying the condition

$$(2n + 1)A(X) + B(X) + C(X) + D(X) \neq 0 \tag{i}$$

is of zero scalar curvature. Also a $G(PS)_n$ satisfying the condition (i) can not be of constant curvature.

Since a 3-dimensional Einstein manifol is of constant curvature [40], we can state the following corollary of Theorem 5.5.

Corollary. An Einstein $G(PS)_n$ satisfying the condition $7A(X) + B(X) + C(X) + D(X) \neq 0$ does not exist.

In the same paper Chaki studied conformally flat $G(PS)_n$ and obtained a theorem.

6. $(WS)_N$ ADMITTING CERTAIN VECTOR FIELDS

A vector field V is said to be concurrent [45] if

$$\nabla_X V = pX \tag{6.1}$$

where p is a non-zero constant.

If p = 0, the vector field reduces to a parallel vector field.

A vector field V is said to be recurrent [45] if

$$\nabla_X V = \lambda(X)V \tag{6.2}$$

where λ is a non-zero 1-form.

Considering parallel vector field in a $G(PS)_n$ Chaki [29] obtained the following:

Theorem 6.1. If a $G(PS)_n$ admits a parallel vector field which is not orthogonal to the vector field ρ defined by $g(X, \rho) = A(X)$, then the manifold can not be conformally flat.

We have considered concurrent and recurrent vector fields in a $(WS)_n$ and obtained the following:

Theorem 6.2. [38] If a $(WS)_n$ admits a concurrent vector field V given by (6.1), then V is not orthogonal to each of the vector fields corresponding to the associated 1-forms of a $(WS)_n$.

If p is zero in (6.1), then the vector field reduces to a parallel vector field. Hence from Theorem 6.2 we can state the following:

Corollary. If a $(WS)_n$ (n>2) admits a parallel vector field V, then V is orthogonal to each of the associated vector fields corresponding to the associated 1-forms of a $(WS)_n$.

Theorem 6.3. If a $(WS)_n$ admits a non-null recurrent vector field defined by (6.2), then the 1-form λ is closed and V is orthogonal to each of the associated vectors fields corresponding to the associated 1-form.

In [41] De and sengupta studied weakly symmetric manifold admitting a type of semi-symmetric metric connection $\tilde{\nabla}$ whose torsion tensor is given by

$$T(X,Y) = D(Y)X-D(X)Y$$

and whose curvature tensor R and torsion tensor T satisfies the conditions

$$\tilde{R}(X,Y)Z = 0$$

and

$$(\tilde{\nabla}_X T)(Y, Z) = D(X)T(Y, Z)$$

respectively. It is proved that such a $(WS)_n$ reduces to a particular kind of a $(WRS)_n$ with non-zero and non-constant scalar curvature. It is also shown that if a $(WS)_n$ admits a type of semi-symmetric metric connection mentioned above, then the manifold is a subprojective manifold in the sense of Kagan.

7. SUBMANIFOLDS OF A (WS)N

Let $(\tilde{M},\tilde{g})$ be an n-dimensional Riemannian manifold covered by a system of coordinate neighbourhoods (U,y^α). Let (M,g) be an m-dimensional (n<m) submanifold of $(\tilde{M},\tilde{g})$, defined in a local coordinate system by parametric equations $y^\alpha = y^\alpha (x^I)$, where g is the induced metric. Here and in the sequel, Greek indices take the values 1, 2, ..., m and the Latin indices the values 1, 2, ..., n.

Let N_P (P,Q = n + 1, ..., m) be unit mutually orthogonal normals to (M,g). The second fundamental tensor H_{jip} for N_p^α is given by

$$H_{jip} = H_{ji}^{\alpha} N_{p\alpha}, \qquad H_{ji}^{\alpha} = \nabla_j B_i^{\alpha},$$

where $B_i^\alpha = \partial y^\alpha/\partial x^I$ and ∇_j denotes covariant differential with respect to the metric g of (M,g). Let (M,g) be a totally umbilical submanifold. Then $H_{ij}^{\alpha} = g_{ij}H^\alpha$ holds where H^α is the mean curvature vector. The mean curvature H of (M,g) is defined by

$$H^2 = |H_\alpha H^\alpha|.$$

In [42] M. Prvanovic studied totally umbilical submanifold of a $(WS)_n$ and also she obtained some Theorems as a particular case of a $(WS)_n$. As a particular case of a weakly symmetric manifold we obtain a pseudo symmetric manifold in the sense of Chaki or a generalized recurrent manifold named by Prvanovic [15]. In [17] Ewert-Krzemieniewski studied totally umbilical submanifold of a pseudo symmetric manifold.

In 2002 [43] F. Ozen and S. Altay studied totally umbilical hypersurfaces of a weakly symmetric Riemannian manifold and obtained the following

Theorem 7.1. Let (M^n,g) be a totally umbilical hypersurface of a weakly symmetric space. If (M^n,g) is weakly symmetric, then it is also pseudo symmetric or totally geodesic hypersurface.

Theorem 7.2. Let (M^n,g) be a totally umbilical hypersurface of a weakly Ricci symmetric space. (M^n,g) is weakly symmetric if and only if the mean curvature vanishes.

Theorem 7.3. Let (M^n,g) be a totally umbilical hypersurface of a pseudo symmetric space. If (M^n,g) is a pseudo symmetric space, then (M^n,g) is totally geodesic or the associated vector satisfies the condition

$$H_{,k} - A_k H = o \quad \text{for all k.}$$

8. SOME RESULTS ON WEAKLY RICCI SYMMETRIC MANIFOLD

Weakly Ricci symmetric manifold is defined by (5). A weakly Ricci symmetric manifold is denoted by $(WRS)_n$. Generalized pseudo Ricci symmetric manifold is same as $(WRS)_n$ which is denoted by $G(WRS)_n$ and studied by Chaki and Koley [30] . De and De [44] studied conformally flat $G(PRS)_n$ and De and Ghosh [34] studied conformally flat $(WRS)_n$. Considering conformally flat $(WRS)_n$ we obtain the following results.

Theorem 8.1. [34] In a $(WRS)_n$ with $\delta(X) = B(X)-D(X) \neq 0$, the scalar curvature can not be zero and Ricci tensor will be of the form $S(X,Y) = rT(X)T(Y)$ where $T(X) = \delta(X)/\sqrt{\{\delta(\nu)\}}$ and $g(X,\rho) = T(X)$, ρ is a unit vector, $g(X, \nu) = \delta(X)$.

Theorem 8.2. [44] In a conformally flat $(WRS)_n$ $(n>3)$ with $\delta\neq 0$, the vector field ρ defined by $g(X, \rho) = T(X)$ is a proper concircular vector field [45].

It is known [46] that if a conformally flat (M^n, g) $(n>3)$ admits a proper concircular vector field, then the manifold is a subprojective manifold in the sense of Kagan. Since a conformally flat $(WRS)_n$ with $\delta\neq 0$ admits a proper concircular vector field, namely the vector field ρ, we obtain the following :

Theorem 8.3. [44] A conformally flat $(WRS)_n$ $(n>3)$ with $\delta\neq 0$ is a subprojective manifold in the sense of Kagan.

K. Yano [47] proved that in order that a Riemannian space admits a concircular vector field, it is necessary and sufficient that there exists a coordinate system with respect to which the fundamental quadratic differential form may be written in the form

$$ds^2 = (dx^1)^2 + e^q \overset{*}{g}_{\alpha\beta} dx^\alpha dx^\beta$$

where $\overset{*}{g}_{\alpha\beta} = \overset{*}{g}_{\alpha\beta}(x^\gamma)$ are the function of x^γ only ($\alpha, \beta, \gamma, \delta = 2, 3, ..., n$) and $q = q(x^1) \neq$ constant is a function of x^1 only. Thus if a $(WRS)_n$ is conformally flat i.e.,

$$(\nabla_X S)(Y, Z)-(\nabla_Y S)(X, Z) = 1/\{2(n-1)\}[dr\,(X)g(Y, Z)-dr\,(Y)g(X, Z)],$$

it is a warped product $IX_{e^\wedge q}\overset{*}{M}$, where $(\overset{*}{M},\overset{*}{g})$ is an (n-1)-dimensional Riemannian manifold. Using Gebarawski's result [48] we obtain the following :

Theorem 8.4. [34] A conformally flat $(WRS)_n$ $(n>3)$ with $\delta\neq 0$ can be expressed as a product $IX_{e^\wedge q}\overset{*}{M}$ where $\overset{*}{M}$ is an Einstein manifold.

Theorem 8.5. [34] A conformally flat $(WRS)_n$ $(n>3)$ with $\delta\neq 0$ is a manifold of quasi-constant curvature [49].

If we consider a $(WRS)_n$ space-time manifold i.e., a 4-dimensional Lorentzian manifold, then we prove the following:

Theorem 8.6. [34] A conformally flat $(WRS)_n$ space-time is the Robertson-Walker space-time [50].

Next we mention a theorem concerning special conformally flat $(WRS)_n$ $(n>3)$. The notion of a special conformally flat manifold which generalizes the notion of subprojective manifold was introduced by Chen and Yano [51]. According to them a conformally flat manifold is said to be a special conformally flat manifold if the tensor H of type (0,2) defined by

$$H(X,Y) = -1/(n-2)S(X,Y) + r/\{2(n-1)(n-2)\}g(X,Y),$$

is expressible in form

$$H(X,Y) = -(\alpha^2/2)g(X,Y) + \beta(X\alpha)(Y\alpha),$$

where α and β are two scalars such that α is positive.

Theorem 8.7. [44] A conformally flat $(WRS)_n$ is a special conformally flat manifold.

It is known from a theorem of Chen's and Yano's paper that every simply connected special corformally flat manifold can be isometrically immersed in a Euclidean space E^{n+1} as a hypersurface.

Hence from the above theorem we obtain

Theorem 8.8. [44] Every simply connected conformally flat $(WRS)_n$ (n>3) can be isometrically immersed in an Euclidean space E^{n+1} as a hypersuface.

9. WEAKLY CONFORMALLY SYMMETRIC MANIFOLD

In 1988 [13] Prvanović introduce the notion of conformally quasi-recurrent manifold.

A non-flat Riemannian manifold is called conformally quasi-recurrent if the conformal curvature tensor C of type (0,4) satisfies the condition

$$(\nabla_U C)(X,Y,Z,V) = 2A(U)C(X,Y,Z,V) + A(X)C(U,Y,Z,V) + A(Y)C(X,U,Z,V) \\ + A(Z)C(X,Y,U,V) + A(V)C(X,Y,Z,U) \quad (9.1)$$

where A is a non-zero 1-form. If A = 0, the manifold reduces to a conformally symmetric manifold. De and Biswas [10] called such a manifold a pseudo conformally symmetric manifold.

She obtained many interesting results on conformally quasi-recurrent manifold which are stated below.

Theorem 9.1. Any conformally quasi-recurrent manifold satisfies

$$(\nabla_Z \pi)(X,Y)-(\nabla_Y \pi)(X,Z) = 0$$

where $\pi(X,Y) = S(X,Y)-r\cdot g(X,Y)/\{2(n-1)\}$.

Theorem 9.2. A conformally quasi-recurrent manifold in which the 1-form A is closed, can always be locally conformally related to a conformally symmetric manifold and converse is also true.

Some other results have also been obtained in the same paper [13].

K. Buchner and W. Roter [52] also studied conformally quasi-recurrent manifold.

In [53] De and Mazumder studied proper conformal motion in a pseudo conformally symmetric manifold.

A Riemannian manifold (whose metric need not be positive definite) is said to admit an infiniesimal conformal motion if there exists a vector field ξ such that

$$(\underset{\xi}{L}\, g)(X, Y) = 2\sigma g(X, Y),$$

where L denotes the Lie derivation with respect to ξ and σ is a scalar function. For a proper conformal motion σ is non-constant. If σ is constant, the motion is called homothetic.

De and Mazumder [53] obtained the following:

Theorem 9.3. If a pseudo conformally symmetric manifold admits a proper conformal motion with respect to a scalar field σ, then the manifold is either conformlly flat or the vector field corresponding to $\nabla_X \sigma$ is null.

Theorem 9.4. If a pseudo conformally symmetric space-time $(PRS)_4$ admit a proper conformal motion, then $(PRS)_4$ is either of type 0 or N. In case the manifold is of type N and the Einstein tensor is invarient under the conformal vector field, $(PCS)_4$ represents plane-fronted gravitational waves with parallel rays.

In 1997 De and De [54] introduced the notion of generalized pseudo conformally symmetric manifold which generalizes the notion of pseudo conformally symmetric manifold or conformally quasi-recurrent manifold. On the otherhand we have mentioned earlier that generalized pseudo conformally symmetric manifold and weakly conformally symmetric manifold are the same notion. In [54] De and De obtained some results which generalized the results of Prvanović [14].

A Riemannian or pseudo-Riemannian manifold is said to be of harmonic conformal curvature [55] if $n \geq 4$ and the condition (div C)(X,Y)Z = 0 holds. We obtained the following: [54].

Theorem 9.5. Every generalized pseudo conformally symmetric manifold $G(PCS)_n$ ($n>3$) is of harmonic conformal curvature if and only if A(C(X,Y)Z) = 0 holds.

Theorem 9.6. In a $G(PCS)_n$ with A(C(X,Y)Z) = 0, the scalar curvature is constant if and only if the Ricci tensor is a Codazzi tensor.

Theorem 9.7. A $G(PCS)_n$ ($n>3$) satisfies

$$(\nabla_X T)(Y,Z) = (\nabla_Y T)(X,Z) \qquad \text{if} \quad A(C(X,Y)Z) = 0$$

where $T(X,Y) = S(X,Y) - r \cdot g(X,Y) / \{2(n-1)\}$.

Also in the same paper we have studied conformal tranformations of $G(PCS)_n$ and we proved the following:

Theorem 9.8. If a $G(PCS)_n$ is tranformed into a $G(PCS)_n$ with the same associated 1-form by a conformal tranformation; $\overset{*}{g} = \bar{\rho}^2 g$ then either the manifold is conformally flat or the tranformation is homothetic (ρ = constant) .

Theorem 9.9. In order that a $G(PCS)_n$ which is not conformally flat is tranformed into another $G(PCS)_n$ with the same associated 1-form by a conformal transformation $\overset{*}{g} = \bar{\rho}^2 g$, it is necessary and sufficient that $\bar{\rho}$ is constant.

Recently in a paper [56] we have studied Ricci-recurrent weakly conformally symmetric manifold $(WCS)_n$. An n-dimensional Riemannian manifold is said to be Ricci-recurrent if the Ricci tensor is non-zero and satisfies the condition

$$(\nabla_Z S)(X,Y) = \alpha(Z)S(X,Y)$$

where α is a non-zero 1-form.

We prove that

Theorem 9.10. In a weakly conformally symmetric Ricci-recurrent manifold with non-zero scalar curvature, the 1-form α is equal to A + B where A and B are associated 1-forms of $(WCS)_n$.

Finally we prove that a $(WCS)_n$ satisfying second Bianchi indentity can be endowed with a uniquely determined semi-symmetric metric connection with respect to which the conformal curvature tensor is a $(WCS)_n$.

10. WEAKLY PROJECTIVE SYMMETRIC MANIFOLD

A Riemannian manifold (M, g) is called weakly projective symmetric [26] if there exist 1-form A, B, C, D and a vector field F such that

$$(\nabla_X W)(Y,Z)V = A(X)W(Y,Z)V + B(Y)W(X,Z)V + C(Z)W(Y,X)V + D(V)W(Y,Z)X + g(W(Y,Z)V,X)F,$$

where W is the projective curvature tensor of (M, g). Tamassy and Binh [26] obtained necessary and sufficient conditions for a weakly symmetric manifold to be a weakly projective symmetric or conversly, with the same associated 1-form.

They obtained the following theorems:

Theorem 10.1. If an (M, g) ($n\neq4$) is weakly symmetric and also weakly projective symmetric with the same associated 1-forms A, B, C, D and associated vector field F (where the vanishing of F is also allowed), then

a) BS = CS, b) S, c) CS, d) DS are totally symmetric and

$[A(X) + C(X) + D(X)]S(Z,V) = (\nabla_X S)(Z,V)$ holds where S is the Ricci tensor.

Theorem 10.2. A Riemannian manifold (M,g) ($n\geq4$) is weakly symmetric and also weakly projective symmetric with the same associated 1-forms A, B, C, D and associated vector field $F\neq0$ iff the Ricci tensor S vanishes.

From theorem 10.2 the theorem of Chaki and Saha [7] follows.

De and De [57] studied generalized pseudo projective symmetric manifold $G(PWS)_n$. A weakly projective symmetric manifold is little weaker than a $G(PWS)_n$. De and De [57] obtained the following theorems.

Theorem 10.3. A $G(PWS)_n$ ($n>2$) is of zero scalar curvature if and only if $W(X,Y)Q = R(X,Y)Q$ holds provided $T\neq0$ where $T(X) = A(X) + B(X)$ and $g(Q,X) = T(X)$.

Theorem 10.4. In a $G(PWS)_n$ satisfying $A(W(X,Y)Z) = 0$, the Ricci tensor S is of Codazzi type i.e., the Ricci tensor satisfies $(\nabla_X S)(Y,Z) = (\nabla_Y S)(X,Z)$.

Theorem 10.5. If a $G(PWS)_n$ satisfies $A(W(X,Y)Z) = 0$, then either the manifold is of constant curvature or the associated vector ρ defined by $g(X, \rho) = A(X)$ is orthogonal to the vector $\tilde{\rho}$ defined by $g(X,\tilde{\rho}) = G(X)$ where $G(X) = 2A(X)-B(X)-C(X)$.

Theorem 10.6. If the vector field ρ defined by $g(X,\rho) = A(X)$ is a parallel vector field in an Einstein $G(PWS)_n$, then $G(PWS)_n$ reduces to a $G(PS)_n$ provided the vector fields corresponding to the 1-form A and B are not codirectional.

11. CONTACT STRUCTURE OF $(WS)_N$ AND $(WRS)_N$

Tamassy and Binh [27] in their paper studied weakly symmetric and weakly Ricci symmetric Sasakian manifold.They proved the following

Theorem 11.1. There exist no weakly symmetric Sasakian manifold M^n (ϕ,η,ξ,g) ($n>2$) if A + B + D is not everywhere zero.

Theorem 11.2. There exists no weakly Ricci symmetric manifold M^n (ϕ,η,ξ,g) ($n>2$) if A + B + C is not everywhere zero.

It is known that every Sasakian manifold is K-contact, but the converse, is not true in general. However a 3-dimensional K-contact manifold is Sasakian. De, Binh and Shaikh [58] studied weakly symmetric and weakly Ricci symmetric K-contact manifold to generalize the result of Tamassy and Binh [27]. Also De and Ghosh [59] studied generalized pseudo Ricci symmetric Sasakian manifold.

12. KÄHLER STRUCTURE OF $(WS)_N$ AND $(WRS)_N$

A Kähler manifold is an even-dimensional manifold M^{2n} with a complex structure J and a positive definite metric g satisfies the following condition

$$J^2 = -I \quad g(\bar{X},\bar{Y}) = g(X,Y), \ \bar{X} = JX \quad \text{and} \quad \nabla J = 0,$$

where ∇ means the covariant derivative according to the Levi-Civita connection.

Tamassy, De and Binh [60] studied weakly symmetric and weakly Ricci symmetric Kähler manifolds.

The following theorems are obtained.

Theorem 12.1. In a weakly symmetric Kähler manifold,

(a) if the scalar curvature is a non-zero constant, then the sum of the associated 1-forms is zero.

(b) α, $J\alpha$, β, $J\beta$, γ, $J\gamma$ are the eigen vector of the Ricci tensor S with the same eigen value (r/2) where α, β, γ are the vector fields corresponding to the associated 1-form A, B and D respectively.

Theorem 12.2. Let M^{2n} be a weakly symmetric Kähler manifold of dimension 6 and let α, $J\alpha$, β, $J\beta$, γ, $J\gamma$ be linearly independent. Then the manifold is Ricci flat.

Concerning the weakly Ricci symmetric Kähler manifold we obtained the following :

Theorem 12.3. In a weakly Ricci symmetric Kähler manifold with non-zero constant scalar curvature, the 1-forms of $(WRS)_n$ are all equal.

13. APPLICATIONS

In Theorem 8.6. we mention that a conformally flat weakly Ricci symmetric space-time manifold is the Robertson-Walker space-time.

Also in Theorem 9.4. we see that if a 4-dimensional pseudo conformally Symmetric space $(PRS)_n$ admit a proper conformal motion then $(PRS)_n$ is either of type 0 or N. In case the manifold is of type N and the Einstein tensor is invariant under the conformal vector field, $(PCS)_n$ represents plane fronted gravitational waves.

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O SLABOSIMETRIČNIM STRUKTURAMA RIEMANIAN MNOGOSTRUKOSTI

U.C. De

U ovom radu je dat pregled o slabosimetričnim strukturama Riemanian mnogostrukosti. Studirane su slabo simetrične i slabo projektivne simetrične Riemannian mnogostrukosti koje su započei Tamassy i Binh. Kasnije jedan broj autora je studirao slabosimetričnu Riemanian mnogostrukost i nalogne strukture, kao i slabe Riccijeve simetrčne, slabo projektivne simetrične i slabo konformno simetrične Riemannian mnogostrukosti. Ovde, u vom raduje prikazan pregled rezultata o slabosimetričnim strukturama na Riemannian mnogostrukosti i neke primene u teoriji relativnosti.

Ključne reči: *Psoudo simetrična mnogostrukost, slabosimetrična mnogostrukost, slabo Ricci-jeva simetrična mnogostrukost, slabo konformna simetrična mnogostrukost, slabo projektivna simetrična mnogostrukost.*