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NONLINEAR PHENOMENA IN DYNAMICS OF CAR MODEL

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Abstract. *In this paper research results of influences, of masses debalances of car system and of rough spot (prominence) in the way on which the car is in move, at nonlinear dynamics properties of car are presented. Also, the properties of nonlinear dynamics of car model are investigated by using the corresponding equations of phase trajectories of corresponding basic scleronomic nonlinear model to the rheonomic car dynamics model. Particularly we were analyzed homoclinic orbits and their transformation shaped by number eight, there appear and disappear are caused by changing some parameters of system. By using Math-Cad program for drawing families of phase portraits visualization of nonlinear phenomena in dynamic of car model are presented, also on that graphics it is barely noticeable a influence of masses debalances parameters like as of rough spot (prominence) in the way at nonlinear dynamics features of car.*
It is observe one system of tree degree of mobilities and with one degree of freedom and we narrow our problem on research of following nonlinear differential equation:

$$\ddot{\varphi}\left(1+k\frac{\lambda_1^2}{\lambda}+\frac{J}{mr^2}+\frac{J_1}{\lambda^2 mr^2}\right)+\frac{g}{r}\left(\sin\varphi+\frac{\lambda_1}{\lambda}k\sin\frac{\varphi}{\lambda}\right)=-\ddot{x}\left(-\cos\varphi+\frac{\lambda_1}{\lambda}k\cos\frac{\varphi}{\lambda}\right)\frac{1}{r}-$$
$$-\ddot{z}\left(\sin\varphi+\frac{\lambda_1}{\lambda}k\sin\frac{\varphi}{\lambda}\right)\frac{1}{r}+\dot{\varphi}\,\dot{x}\left(\sin\varphi+\frac{\lambda_1}{\lambda^2}k\sin\frac{\varphi}{\lambda}\right)\frac{1}{r}-\dot{\varphi}\,\dot{z}\left(\cos\varphi+\frac{\lambda_1}{\lambda^2}k\cos\frac{\varphi}{\lambda}\right)\frac{1}{r}$$

like as homogenous equation appropriate to this equation:

$$\ddot{\varphi}\left(1+k\frac{\lambda_1^2}{\lambda}+\frac{J}{mr^2}+\frac{J_1}{\lambda^2 mr^2}\right)+\frac{g}{r}\left(\sin\varphi+\frac{\lambda_1}{\lambda}k\sin\frac{\varphi}{\lambda}\right)=0$$

From characteristics visualizations we can noticeable the phenomena of trigger of coupled singularities and homoclinic orbits shaped by number eight like as double number eight. Analyzing the properties of basic nonlinear system we comes to conclusion that with modification of parameters of system appears a separation of one homoclinic orbits in more, like as that becomes to bifurcation of relative rest position in rheonomic system, apropos in equivalent scleronomic system which correspond to him.

Key words: *Car model, rough spot (prominence) in the way, nonlinear dynamics, phase portrait, trigger of coupled singularities, homoclinic orbit, layering of homoclinic orbit.*

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INTRODUCTION

Series of articles (see Refs. listed from [5] to [12]), by first author of this article and hers collaborators, presents results of original researches of nonlinear dynamic of mechanical systems with properties of periodic exchanges, which has application in engineering systems. Volume [5] presents reviews of the basics methods of theory of Ljupanov's stability applying them in nonlinear oscillations with special chapters consecrate to the method of phase plane underlining importance of phase trajectories and singularities in researching qualitative properties of nonlinear system dynamics.

Like as basic properties of a linear oscillatory system with one and more degrees of oscillatory freedom are the own frequencies of discreet material particle system who has linear oscillatory motion, also for nonlinear system with one degree of freedom we may considered phase portrait with singularity structure, which gives nonlinear dynamic's properties and phenomena of system in phase plane. Therefore for nonlinear dynamic's systems and their subsystems it's important to study a structure of phase portraits, their stability like as their transformations and transformations of phase trajectories which we obtained exchanging any parameters of systems. Except in very often-mentioned monographs (see Refs. [1], [2], [3], [13] and [14]), contributions to this knowledge's we can find also in articles listed by [5], [6], [7], [8], [9], [10], [11] and [12] with theory and applied results. There was studied a nonlinear dynamic's of rotors, by influence of deviation properties on phase portraits, coupled rotors, like as planetary reductor and gyrorotors, ad also nonlinear dynamics of the systems with coupled rotations.

First author defines a series trigger of coupled singularities theorems and existence of homoclinic orbit and their transformation shaped by number eight, like as theirs application on systems relevant for technical practice in hers articles [6] and [8], also she constructs phase portraits and particularly considers phenomena of homoclinic orbits transformations and their disintegration, appearance and disappearance of these homoclinic orbits shaped by number eight, like as trigger of coupled singularities.

In this paper research results of influences, of masses debalances of car and of rough spot (prominence) in the way on which the car is in move, at nonlinear dynamics qualitative properties of car are presented. Also, the properties of nonlinear dynamics of car model are investigated by using the corresponding equations of phase trajectories of corresponding basic scleronomic nonlinear model to the rheonomic car dynamics model. Particularly we were analyzed homoclinic orbits and their transformation shaped by number eight, there appear and disappear are caused by changing some parameters of system. By using Math-Cad program for drawing families of phase portraits visualization of nonlinear phenomena in dynamic of car model are presented, also on that graphics it is barely noticeable a influence of masses debalances parameters like as of rough spot in the way at nonlinear dynamics phenomena of car.

As an example of practical application theorem of trigger of coupled singularities and homoclinic orbits shaped by number eight we used researched results of nonlinear dynamic of car model. This theorem is published in article [8].

THE BASIC EQUATIONS OF CAR MODEL DYNAMICS

Figure 1. shows one simple car model, which has following geometric and kinematics parameters: J_1 and J - mass axial moments of inertia of geared; r and λr - radius of cog-wheels. Assumption that discs and shafts are nonhomogeneous rigid discs with nonhomogeneous "points", which can be present like material mass particles with masses m and km being on distances of r and $\lambda_1 r$ from their centers. Mentioned model is in the field of gravity and persistent forces are unattended, and is in move on bumpy with the low of rough spot (prominence) in the way given on following equation:

$$z(t) = \frac{h}{2}(1 - \cos 2\Omega t) \qquad \text{where is} \quad \Omega = \frac{\dot{x}\pi}{l}. \tag{1}$$

It is observe one system of tree degree of mobilities and with one degree of freedom and for generalized coordinate we chose the angle of relative rotation φ. With $z(t)$ and $x(t)$ we denote rheonomic coordinate, as a known kinematical, depending of time, perturbation to the system.

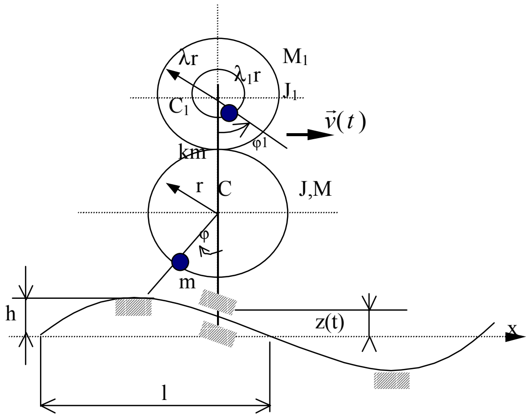

Fig. 1. One simple model of nonlinear dynamics of car: The influence of coupled rotation of debalances masses and their deviations properties.

Assumption that cog-wheel band is coulisse which slide on bumpy with neglect of friction force we obtained expression of the kinetic energy of mentioned system in following form: $2\mathbf{E_k} = mv_m^2 + kmv_{km}^2 + \mathbf{J}\dot{\varphi}^2 + \mathbf{J}_1 \frac{\dot{\varphi}^2}{\lambda^2} + Mv_C^2 + M_1 v_{C1}^2$, $\tilde{k} = k + \frac{M + M_1}{m}$ or

$$\begin{aligned} 2\mathbf{E_k} &= mr^2\dot{\varphi}^2\left(1 + k\frac{\lambda_1^2}{\lambda^2} + \frac{J}{mr^2} + \frac{J_1}{\lambda^2 mr^2}\right) + (1+\tilde{k})m\dot{x}^2 + (1+\tilde{k})m\dot{z}^2 + \\ &+ 2mr\dot{x}\dot{\varphi}\left(-\cos\varphi + \frac{\lambda_1}{\lambda}k\cos\frac{\varphi}{\lambda}\right) + 2mr\dot{z}\dot{\varphi}\left(\sin\varphi + \frac{\lambda_1}{\lambda}k\sin\frac{\varphi}{\lambda}\right) \end{aligned} \tag{2}$$

because $\varphi_1 = \frac{\varphi}{\lambda}$. We can see that system is rheonomic, kinematicaly perturbed.

Exchange of potential energy of system compose of exchange of potential energy of heavy particles in the field of gravity, due to car motion by bumpy with rough spot (prominence) in the way on which the car is in move, define with rheonomic coordinate $z(t)$, as a kinematical perturbation on the car system, which we measure from x axis. Expression of the exchange of potential energy of system we obtain in the following form:

$$\mathbf{E_p} = mg(1+\widetilde{k})z(t) + mgr(1+\lambda_1 k) - mgr(cos\,\varphi + \lambda_1 k\, cos\frac{\varphi}{\lambda}) . \tag{3}$$

We observe one system of tree degree of the mobilities and with one degree of freedom and by using extended system of Lagrange second kind equations, and for generalized coordinate φ we obtain the nonlinear differential equation in following form:

$$\begin{gathered}\ddot{\varphi}\left(1+k\frac{\lambda_1^2}{\lambda}+\frac{J}{mr^2}+\frac{J_1}{\lambda^2 mr^2}\right)+\frac{g}{r}\left(\sin\varphi+\frac{\lambda_1}{\lambda}k\sin\frac{\varphi}{\lambda}\right)=-\ddot{x}\left(-\cos\varphi+\frac{\lambda_1}{\lambda}k\cos\frac{\varphi}{\lambda}\right)\frac{1}{r}-\\ -\ddot{z}\left(\sin\varphi+\frac{\lambda_1}{\lambda}k\sin\frac{\varphi}{\lambda}\right)\frac{1}{r}+\dot{\varphi}\dot{x}\left(\sin\varphi+\frac{\lambda_1}{\lambda^2}k\sin\frac{\varphi}{\lambda}\right)\frac{1}{r}-\dot{\varphi}\dot{z}\left(\cos\varphi+\frac{\lambda_1}{\lambda^2}k\cos\frac{\varphi}{\lambda}\right)\frac{1}{r}\end{gathered} \tag{4}$$

Homogenous nonlinear differential equation correspond to the previous equation (4) is in following form:

$$\ddot{\varphi}\left(1+k\frac{\lambda_1^2}{\lambda}+\frac{J}{mr^2}+\frac{J_1}{\lambda^2 mr^2}\right)+\frac{g}{r}\left(\sin\varphi+\frac{\lambda_1}{\lambda}k\sin\frac{\varphi}{\lambda}\right)=0 \tag{5}$$

Original model of the car system dynamics is rheonomic system, and last differential equation corresponds to the basic scleronomic system, which correspond to the original one.

EQUATION OF PHASE TRAJECTORIES AND INTEGRAL OF ENERGY

If we denote $A=\left(1+k\frac{\lambda_1^2}{\lambda}+\frac{J}{mr^2}+\frac{J_1}{\lambda^2 mr^2}\right)$ the last differential equation becames:

$$\ddot{\varphi}+\frac{g}{rA}\left(\sin\varphi+\frac{\lambda_1}{\lambda}k\sin\frac{\varphi}{\lambda}\right)=0 \tag{6}$$

For assignment a phase trajectory equation in phase plane $(\dot{\varphi},\varphi)$ let's multiple bought of two sides of last equations with $2\dot{\varphi}dt$ and after integration in interval from φ_0 till φ we obtained:

$$\dot{\varphi}=\pm\sqrt{\dot{\varphi}_0^2-\frac{2g}{rA}\left[-\cos\varphi+\cos\varphi_0+\lambda_1 k(-\cos\frac{\varphi}{\lambda}+\cos\frac{\varphi_0}{\lambda})\right]} . \tag{7}$$

The integral of system's energy is: $E_k + E_p = const$, apropos

$$\dot{\varphi}^2+\frac{2g}{rA}\left\{-\cos\varphi-\lambda_1 k\cos\frac{\varphi}{\lambda}\right\}=h , \tag{8}$$

and presents equation of curves of constant energy in phase plane.

CONFIGURATIONS OF SYSTEM'S RELATIVE EQUILIBRIUM POSITIONS

Researching conditions of definite for form of expression of the potential energy exchange of the corresponding basic scleronomic conservative system and defining their extreme and stationary values, we appropriate a positions of system's equilibrium and relative rest, their stability or instability, and with that the structure of phase portrait, too. Because of that let's appoint first and second derivatives, with respect to generalized coordinate φ, of the expression of potential energy exchange of the basic system:

$$\frac{d\tilde{E}_p}{d\varphi} = mgr\left[\sin\varphi + \frac{\lambda_1 k}{\lambda}\sin\frac{\varphi}{\lambda}\right] = 0 \tag{10}$$

$$\frac{d^2\tilde{E}_p}{d\varphi^2} = mgr\left[cos\,\varphi + \frac{\lambda_1 k}{\lambda^2}cos\,\frac{\varphi}{\lambda}\right] \begin{matrix}\le\\ \ge\end{matrix} 0 \tag{11}$$

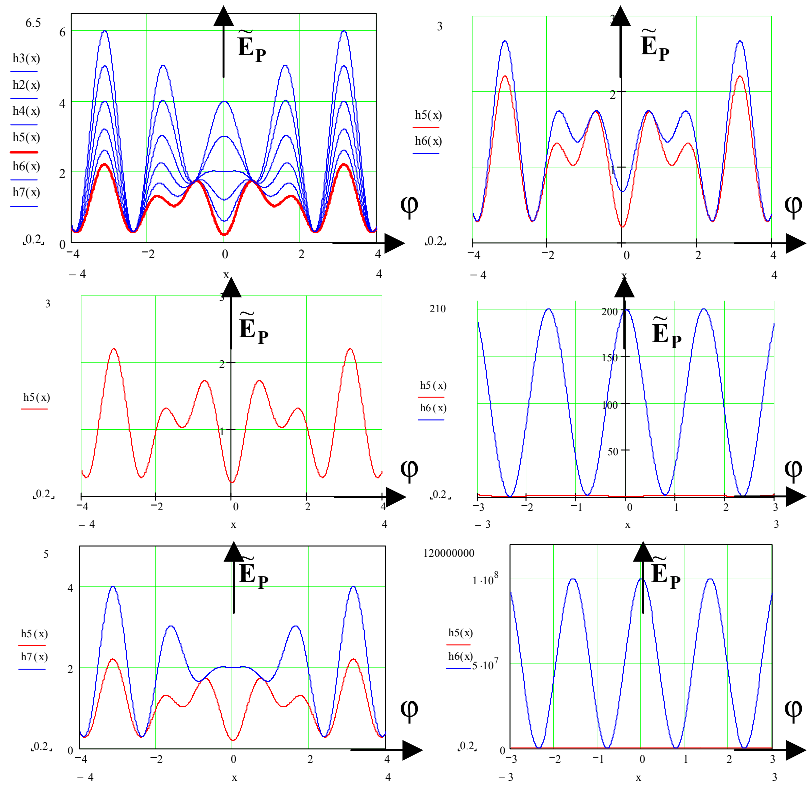

Fig. 2. Qualitative analysis of stationary relative equilibrium positions of rheonomic car dynamical model by using the conservative scleronomic system, which correspond to the rheonomic system of the movable car model.
Potential energy exchange curve for different parameters values of the basic system correspond to the car dynamic model.

Investigation of relative equilibrium positions and conditions of stability of equilibrium it accomplish for particularly construction of mass debalance configuration in form of material particle situated on cog-wheel or rotation parts of car. The configuration of mass debalance of systems parts who rotating coupled with different velocities, shown in figure, for $\varphi = 0$ is the configuration of equilibrium if it is $1 + \frac{\lambda_1 k}{\lambda} \geq 0$ because of $E_{p\,\min} = 0$.

In Figure 2. we can see presentation of the qualitative analysis of stationary relative equilibrium positions of rheonomic car dynamical model by using the conservative scleronomic system which correspond to the rheonomic system of the movable car model. Potential energy exchange curve for different parameters values of the basic system correspond to the car dynamic model are done.

NUMERICAL EXPERIMENTS AND PHASE PORTRAITS

Using MathCad program on accomplished numerical experiment for researching of existence, like as number and character of stationary values of potential energy, as number of configuration of equilibrium positions and character of their stability, and transformations of phase trajectories with exchanging one of the kinematic parameters of system: value of radius and deviational masses of cog-wheel (rotating masses), value of coupled rotating deviation masses their distance from axis of shifts, ratio gear between cog-wheel in catch, like as elementary conditions: elementary angel and angular velocity of shifts.

Graphs of potential energy exchange of corresponding basic system are obtain on numerical way and for characteristic values constructive and kinematic parameters of system shown in Figures 2.

In Figure 3. we can see characteristic potential energy curves, and corresponding homoclinic separatrix phase trajectories for different parameters values of the basic system correspond to the car dynamic model. Examples of the trigger of the coupled singularities and coupled triggers of the coupled singularities are shown. The homoclinic trajectories in the form of the number eight are presented in Figure, as well as in the form of the duplicate number eight.

In Figure 4. we can se characteristic phase trajectories portraits for examples of the potential energy curves from Fig. 3, and corresponding homoclinic separatrix phase trajectories for different parameters values of the basic system correspond to the car dynamic model. Examples of the trigger of the coupled singularities and coupled triggers of the coupled singularities are presented.

In Figure 5. transformations and layering of the homoclinic trajectories with change of the kinetic parameters values of the basic system correspond to the car dynamic model are presented. Examples of the trigger of the coupled singularities and coupled triggers of the coupled singularities and homoclinic trajectories in the form of the duplicate number eight are, also, presented.

Characteristic phase trajectories of stationary regimes of nonlinear dynamic are obtained by using the conservative scleronomic system which correspond to the rheonomic system of the movable car model. Potential energy exchange curve for different parameters values of the basic system correspond to the car dynamic model are done also.

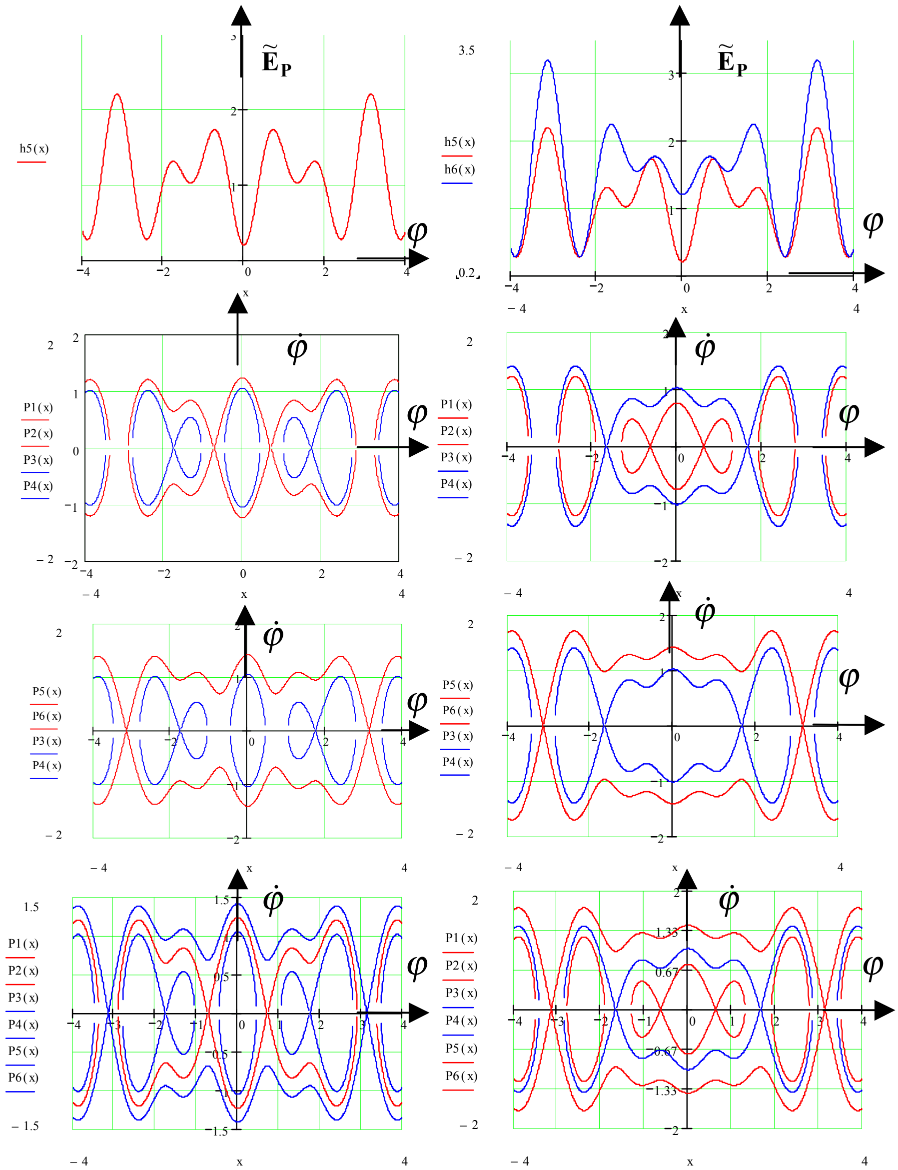

Fig. 3. Characteristic potential energy curves, and corresponding homoclinic separatrix phase trajectories for different parameters values of the basic system correspond to the car dynamic model. Examples of the trigger of the coupled singularities and coupled triggers of the coupled singularities and homoclinic trajectories in the form of the number eight and also in the form of the duplicate number eight.

We can see more than five types of characteristic phase portraits which contains two types of singular points: by type stable center and unstable saddle, which correspond stability apropos instability relative equilibrium positions or relative rest positions, which appear respectively. By comparison of the homoclinic orbits and phase portraits in Figures 3 and 4. for different system parameters, we see that structures of phase portraits are different by types of phase trajectories and homoclinic orbits (phase trajectories of separatrix).

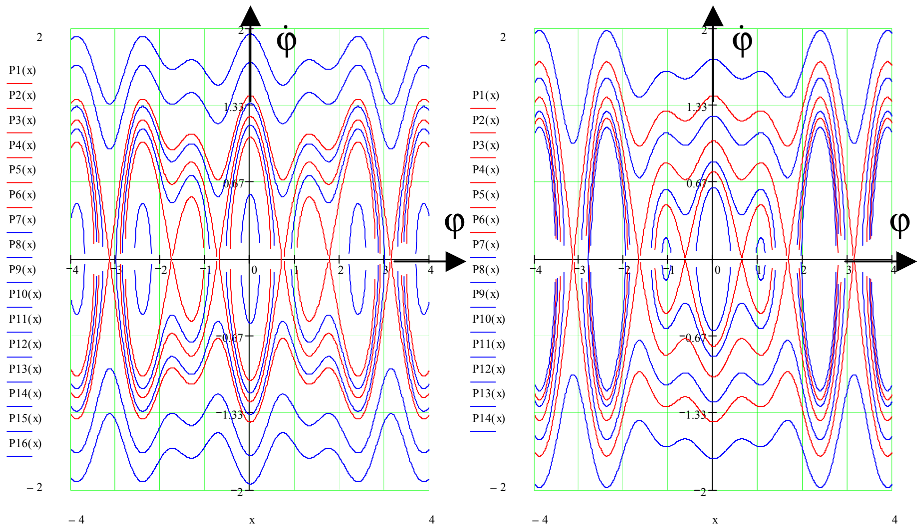

Fig. 4. Characteristic phase trajectories portraits for examples of the potential energy curves from Fig. 3, and corresponding homoclinic separatrix phase trajectories for different parameters values of the basic system correspond to the car dynamic model. Examples of the trigger of the coupled singularities and coupled triggers of the coupled singularities and homoclinic trajectories in the form of the number eight and also in the form of the duplicate number eight.

We can see on Figure 4, one-sided separatrix, which are "prolating", and we see also open phase trajectories, which are comprising enclosed phase trajectories which are matching to the periodical oscillator motion-rotations system round stability configurations of equilibrium positions for specific initial conditions when initial angular velocity are small and small angles elongation of rotations, and when that condition are satisfying for any time.

In Figure 4. on phase portrait we notice augmentation of singular points, and we deduce by researching that for some kinetic parameters of system one stable equilibrium position loses stability and that positions now on phase portrait response to homoclinic point by type unstable saddle, but in symmetrical neighborhood appear two near-by stable equilibrium positions (configuration of masses), which on phase portrait response two singular points by center type. We can see also that all of three points are coupled in one "trigger" (trigger of coupled singularities, see reference [6] or [8]). Two stable singular points by type centers enclose one, and the new, closed homoclinic orbit which goes around three singularities, and passing trough one homoclinic point by type saddle in which it self-cross, that it is shaped like form of the number eight or in the form of duplicate of number eight or multiplication. Inside that new separatrix trajectory-homoclinical orbit we notice a series of common closed phase trajectories which correspond to periodic oscillatory motion for certain initial conditions, apropos oscillations around new stable position of equilibrium. We notice that homoclinic orbit shaped by number eight and multiplicate of number eight self-cross in points type by saddles which are issue from stable points type by saddle which is lose stability exchanging parameters of system an it is “disintegrate” on three, or even number which are trigger of coupled singularities or coupled triggers of coupled singularities. That point(s) is (are) also bifurcation point(s), because types of bifurcation, and define triple point.

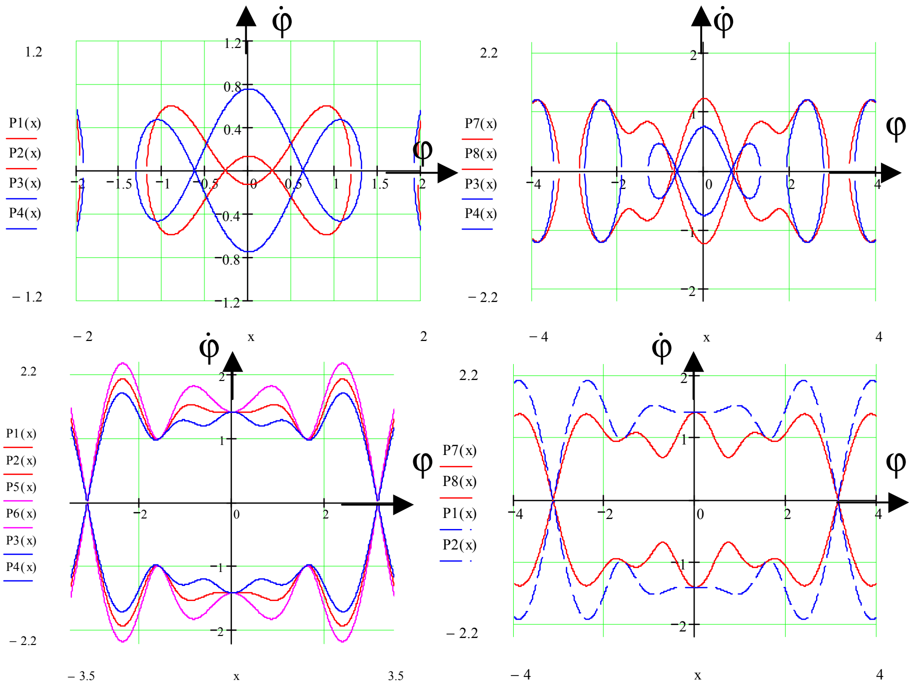

Fig. 5. Transformations and layering of the homoclinic trajectories with change of the kinetic parameters values of the basic system correspond to the car dynamic model. Examples of the trigger of the coupled singularities and coupled triggers of the coupled singularities and homoclinic trajectories in the form of the duplicate number eight.

EFFECT OF BUMPY–WAY ON OSCILLATORY MOTION OF CAR MODEL

Now we observe nonhomogenious differential equation (4) which we obtained like reasearch resulatat of very simple car model of dynamics on bumpy-way and of the influences, of masses debalances of car system and of rough spot (prominence) in the way on which the car is in move by constant horizontal velocity. By respect that this equation is nonlinear, nonhomogenious differential equation and with member depending explicitly of time, we appropriate for numerical experiment in MathCad for analize same solutions of this equation and for abstract characteristics regimes of dynamic.

Numerical experimenting with this nonlinear equation we obtain after visualization some diagrams of elongacy-time and phases trajectories, which are presented on following Figures for different parameters of system.

On Figure 6. a*, b*, c*, d*, e*, f*, g* and h* it is shown characteristics phase trajectories of nonstatonary, forced regimes of nonlinear oscillatory motion. These characteristic phase trajectories, depending of time, in Figure 6 e*, f*, g*, h* are done for the initial conditions with elongations correspond to the saddle points and with zero values of the velocities. In Figures 6 a*, b*, c*, d* we can see graphical presentation of the numerical experiment over the nonhomogeneous nonlinear differential equation for the initial conditions with elongations correspond to the saddle points and with nonzero values of the velocities. Different characteristic phase trajectories correspond to the different value of the ratio between parameters of the system and of the rough spot in the way. For example, the phase trajectories in Figure 6 e* and 6. f* correspond to the same system parameters, but for different initial elongations of the material debalance (initial positions from relative rest).

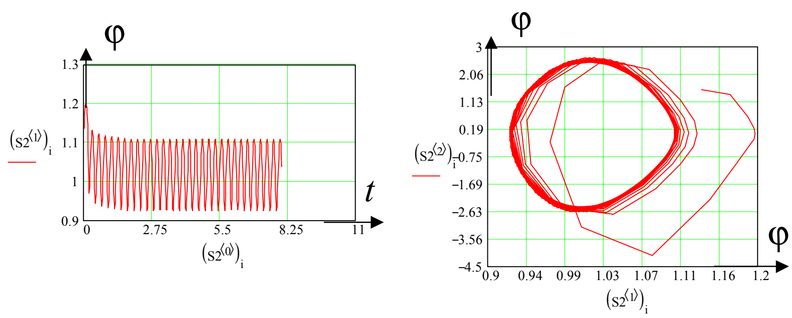

Fig. 6. a* Diagrams of elongacy-time and phase trajectory

($\frac{l\Omega}{\pi r}$=2.35 and $2h\Omega$=3, Ω=12).

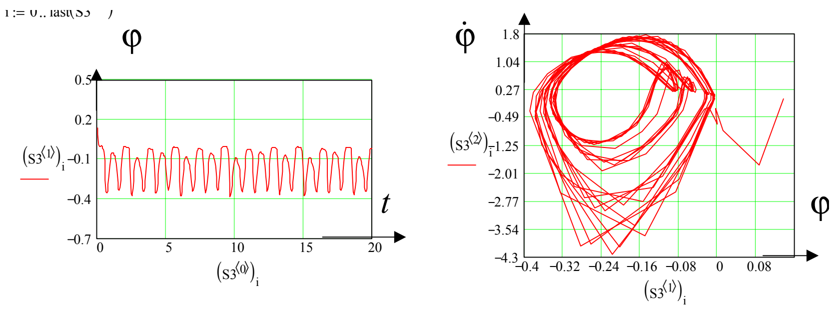

Figure 6. b* Diagrams of elongacy-time and phase trajectory

($\frac{l\Omega}{\pi r}$=2.65 and $2h\Omega$=3, Ω=3.5).

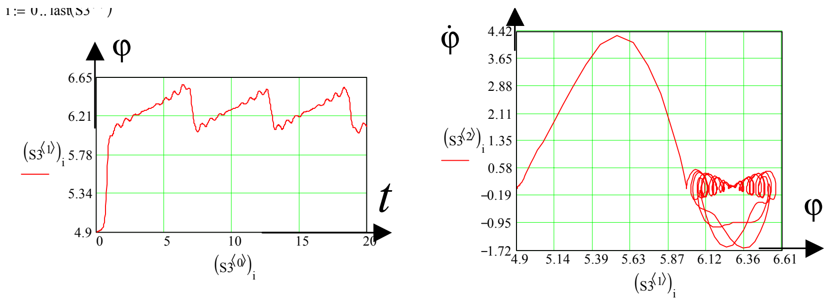

Fig. 3. c* Diagrams of elongacy-time and phase trajectory ($\frac{l\Omega}{\pi r}$ =2.1 and $2h\Omega$=0.81, Ω=5).

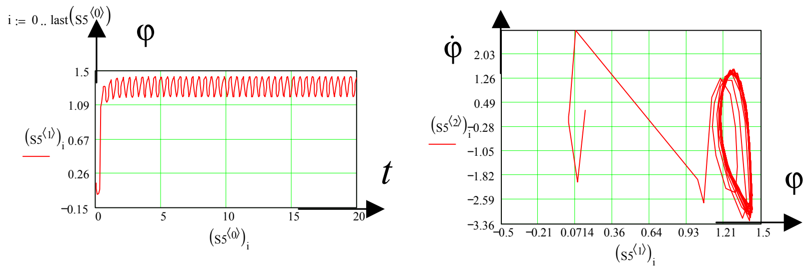

Fig. 6. d* Diagrams of elongacy-time and phase trajectory ($\frac{l\Omega}{\pi r}$ =2.65 and $2h\Omega$=3, Ω=6.5).

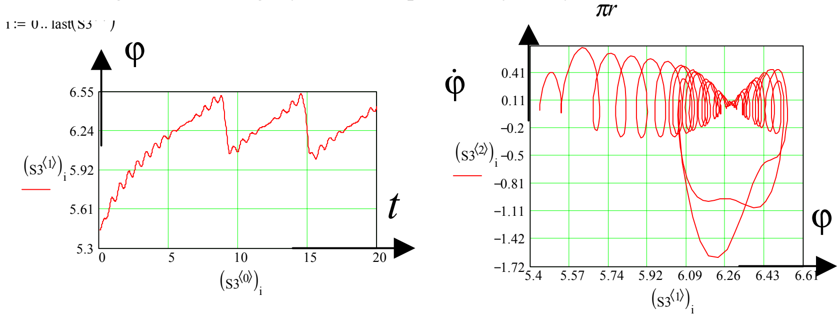

Fig. 6. e* Diagrams of elongacy-time and phase trajectory ($\frac{l\Omega}{\pi r}$ =0.81 and $2h\Omega$=2.1, Ω=5).

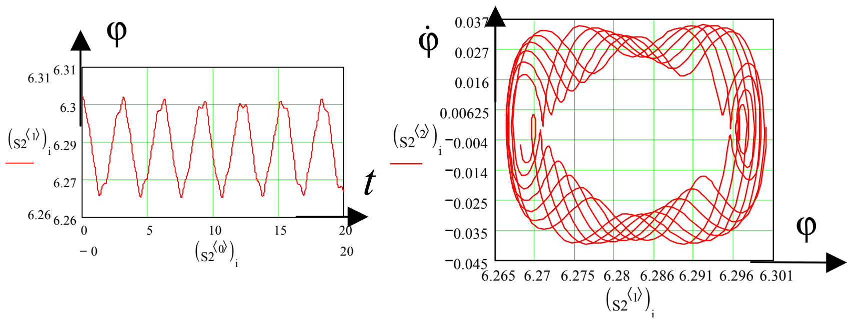

Fig. 6. f* Diagrams of elongacy-time and phase trajectory ($\frac{l\Omega}{\pi r}$ =0.5and $2h\Omega$=2.5, Ω=5).

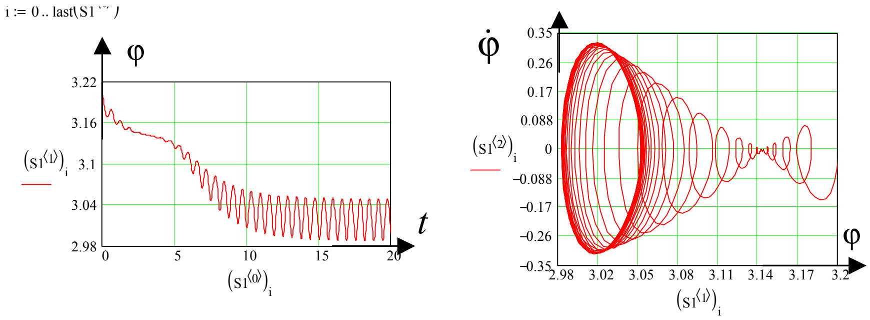

Fig. 6. g* Diagrams of elongacy-time and phase trajectory ($\frac{l\Omega}{\pi r}$=2and $2h\Omega$=3, Ω=5).

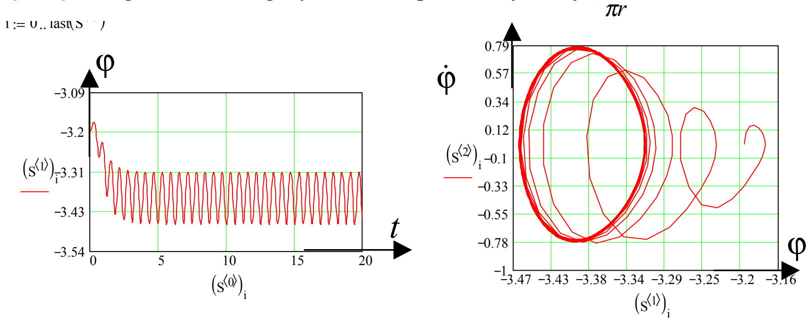

Fig. 6. h* Diagrams of elongacy-time and phase trajectory ($\frac{l\Omega}{\pi r}$=3 and $2h\Omega$=2, Ω=5).

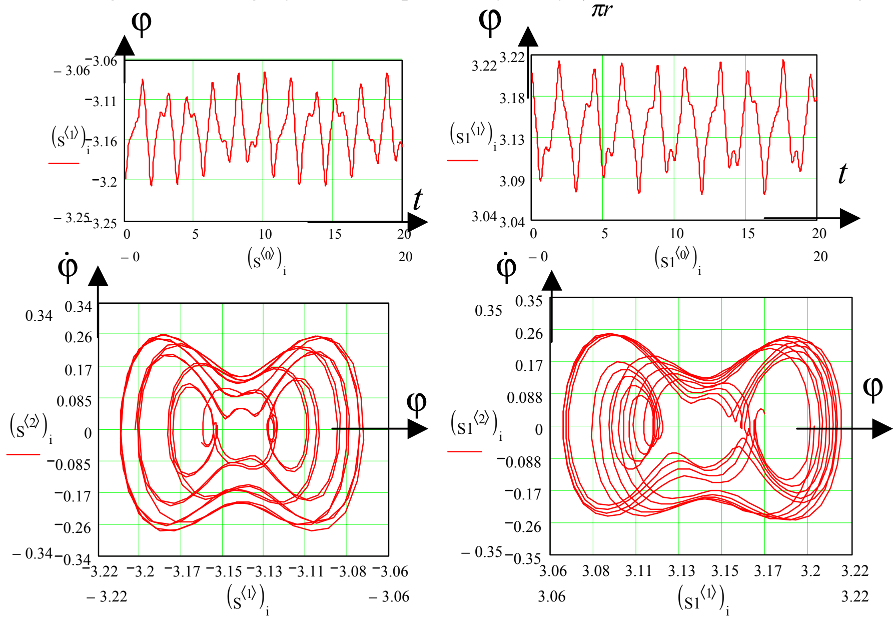

Fig. 7. Diagrams of elongacy-time and phase trajectory for characteristic parameters of the rheonomic system correspond to the car dynamic model and for two different nonstationary regime, which follows homoclinic orbit trajectory of the corresponding basic, scleronomic conservative system.

In Figure 7. diagrams of elongacy-time and phase trajectory for characteristic parameters of the rheonomic system correspond to the car dynamic model and for two different nonstationary regime which follows homoclinic orbit trajectory of the corresponding basic, scleronomic conservative system are presented.

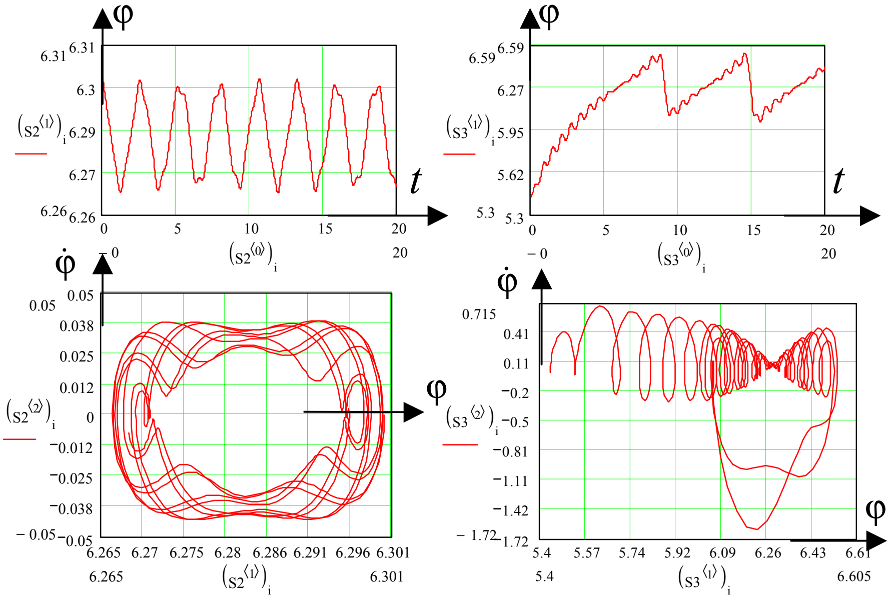

Fig. 8. Diagrams of elongacy-time and phase trajectory for characteristic parameters of the rheonomic system correspond to the car dynamic model and for two different nonstationary regime one of which follows homoclinic orbit trajectory of the corresponding basic, scleronomic conservative system.

In Figure 8. diagrams of elongacy-time and phase trajectory for characteristic parameters of the rheonomic system correspond to the car dynamic model and for two different nonstationary regime one of which follows homoclinic orbit trajectory of the corresponding basic, scleronomic conservative system.

CONCLUDING REMARKS

Configuration of debalance masses exchange while system is in motion and subsystem in coupled rotations, which exchange deviation characteristics and properties of system and that brings in system apart based nonlinearity called first order also nonlinearity of second order with periodical character less or more periodical then first nonlinearity depend if it is reduction or multiplying of rotation number of second mass particle around corresponding axis in relation to first material particle rotation, also of anther constructive parameter of model, so with that on can explain phenomena of bifurcation of equilibrium position and phenomena of trigger of coupled singularities (see ref. [6]), like as existence or nonentity on phase portrait homoclinic orbits shaped by number eight or multiple of number eight. Here we have also two coupled rotations with deviation mass particle properties, which are sources of rebalance of masses distribution of cog-wheel which rotate with different velocities, so we can conclude that deviation properties of cog-wheel with coupled rotations with different velocities in dynamic of system moving car brings, under certain constructive parameters of system, bifurcation phenomena of nonlinear dynamic second order which is property of system which when on his effects external periodical forces or kinematical excitations can bring a dynamic similarly to chaoticlike and stochasticlike dynamic process, what we see on figures number 6, 7 and 8, which shows phases trajectories for nonstationary regimes of motion. At such time on can appear regime of double period of compulsive oscillation. That conclusion indicate a need to route research of compulsive dynamic of that car model and different stationary and nonstationary processes which are consequence of mentioned structure of nonlinearity of first and second order in this system, which are consequence of the coupled rotation motions of debalance masses.

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REFERENCES

1. Andronov, A.A., Vitt, A.A., Haykin, S.E., (1981), Teoriya kolebaniy, Nauka, Moskva., pp. 568
2. Gerard I. and Daniel J., Elementary Stability and BIfurcation Theory, Springer Verlag, 1980 (Russ. Ed Moscow, 1983, pp. 300).
3. Guckenheimer, J. and Holmes, Ph., (1983), Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Fields, Springer-Verlag, pp. 461.
4. Hedrih (Stevanović), K.: Teorija nelinearnih oscilacija, Niš (1975), str. 500. (in Serbian, Preprint).
5. Katica (Stevanović) Hedrih: Vector Method of the Heavy Rotor Kinetic Parameter Analysis and Nonlinear Dynamics, University of Niš 2001,Monograph, p. 252. (in English), (Reviewers: Academician NANU Ju. A. Mitropolskiy, Ukraine, academician ANN Veljko Vujčić, academician UHEAS Oleg Aleksandrovich Goroshko, Ukraine, and prof. dr Vlatko Brčič), YU ISBN 86-7181-046-1.
6. Hedrih (Stevanović) K., A Trigger of Coupled Singularities, MECCANICA, 330-12: 1-21, 2002 (First Proof, to appear), International Journal of the Italian Association of Theoretical and Applied mechanics, 0,230; CODEN MECC B9, ISSN 025-6455, Kluwer Academic Publisher. (Journal Citation Reports – JRC- Science Edition Journal Rankings Sorted by Journal Abbreviation; Rank 4001; Impact Factor IF=0.230).
7. Hedrih (Stevanović) K., Homokliničke orbite nelinearne dinamike višestepenog reduktora (Homoclinic Orbits of Nonlinear Dynamics of Multi-step Reductor), Nauka Tehnika Bezbednost – NTB, Godina XII, No. 1(2002) 45-65., UDC 531.3, Interdisciplinarni časopis, Institut Bezbednsti, YU ISSN 0353-5517, UDC 62+66:54(05).
8. Katica (Stevanovic) Hedrih (Nis, Yugoslavia): Trigger of Coupled Singularities (invited plenary lecture), Dynamical Systems-Theory and Applications, Edited By J. Awrejcewicz, J. Grabski And J. Nowakowski, Lodz 2001, pp. 51-78.
9. Hedrih (Stevanović), K., Nonlinear Dynamics of a Gyrorotor, and Sensitive Dependence on initial Conditions of a Heavy Gyrorotor Forced Vibration/Rotation Motion, Semi-Plenary Invited Lecture, Proceednings: 2000 2nd International Conference - Control of Oscillations and Chaos - COC 2000, Edited by F.L. Chernousko and A.I. Fradkov, IEEE, CSS, IUTAM, SPICS, St. Petersburg, Institute for Problems of Mechanical Engineering of Russian Academy of Sciences and St. Petersburg State University, 2000., Vol. 2 of 3, pp. 259-266. IEEE, CSS, IUTAM, SPICS.
10. Katica (Stevanović) Hedrih, On Rheonomic Systems in the Field with Turbulent Damping with Equivalent Holonomic Nonconservative System, EUROMECH Forth European Conference on European Nonlinear Oscillations Conference (ENOC), August 19-23, 2002, Moscow, Russia. Topics: Analysis of Bifurcations and Chaos.

11. Katica (Stevanović) Hedrih, On Rheonomic Systems with Equivalent Holonomic Conservative System, Forth International Conference on Nonlinear Mechanics (ICNM-IV) August 14-17, 2002, Shanghai, P.R. China. Topic: Nonlinear dynamics. pp. 1046-1054. ISBN 7-81058-077-9/0-007.
12. Hedrih (Stevanović), K. and Simonović, J., Phase Portraits and Homoclinic Orbits – Visualization of Nonlinear Dynamics of Reductor, Journal of Politechnica University Timisoara, Romonia, Transaction on Mechanical Engineering, Tom 47(61), Suplement, May 2002., Editura Politehnika., pp.76-86.
13. Stoker, J. J., (1950), Nonlinear Vibrations, Interscience Publishers, New Yor, pp.273.
14. Rašković, D. Teorija oscilacija (Theory of Oscillations) Naučna knjiga, Beograd, 1965., pp. 504.

NELINEARNI FENOMENI U DINAMICI MODELA VOZILA

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U radu su predstavljeni rezultati proučenih uticaja debalansa masa vozila i neravnine puta po kojoj se posmatrano vozilo kreće na osobenosti njegove nelinearne dinamike. Izvedene su jednačine faznih trajektorija relativne dinamike i proučena svojstva i struktura faznih portreta nelinearne dinamike baznpg scleronomnog modela koji odgovara reonomnom modelu takvog modela vozila. Posebno su analizirani oblici homokliničkih orbita i transformacija homokliničkih orbita oblika broja osam, čije postojanje i nepostojanje je vezano za odredjenu promenu parametara sistema. Pomoću MathCad programa sastavljene su familije faznih portreta baznog sistema, i faznih trajektorija izučavanog sistema, tako da je pomocu njih data vizuelizacija nelinearnih fenomena u dinamici modela vozila i slikovito je prikazan uticaj parametara debalansa rotacionih masa, kao i neravnine puta na svojstva nelinearne dinamike modela vozila.

Posmatran je sistem sa tri stepeni pokretljivosti i jednim stepenom slobode kretanja i zadatak se sveo na izučavanje sledeće nelinearne diferencijalne jednačine

$$\ddot{\varphi}\left(1+k\frac{\lambda_1^2}{\lambda}+\frac{J}{mr^2}+\frac{J_1}{\lambda^2 mr^2}\right)+\frac{g}{r}\left(\sin\varphi+\frac{\lambda_1}{\lambda}k\sin\frac{\varphi}{\lambda}\right)=-\ddot{x}\left(-\cos\varphi+\frac{\lambda_1}{\lambda}k\cos\frac{\varphi}{\lambda}\right)\frac{1}{r}-$$
$$-\ddot{z}\left(\sin\varphi+\frac{\lambda_1}{\lambda}k\sin\frac{\varphi}{\lambda}\right)\frac{1}{r}+\dot{\varphi}\,\dot{x}\left(\sin\varphi+\frac{\lambda_1}{\lambda^2}k\sin\frac{\varphi}{\lambda}\right)\frac{1}{r}-\dot{\varphi}\,\dot{z}\left(\cos\varphi+\frac{\lambda_1}{\lambda^2}k\cos\frac{\varphi}{\lambda}\right)\frac{1}{r}$$

kao i njoj odgovorajuće homogene oblika:

$$\ddot{\varphi}\left(1+k\frac{\lambda_1^2}{\lambda}+\frac{J}{mr^2}+\frac{J_1}{\lambda^2 mr^2}\right)+\frac{g}{r}\left(\sin\varphi+\frac{\lambda_1}{\lambda}k\sin\frac{\varphi}{\lambda}\right)=0$$

Sa karakterističnih vizualizacija dinamike baznog sistema uočava se pojava trigera spregnutih singulariteta i homokliničkih orbita u obliku broja osam, kao i udvojenih brojeva osam. Analizom svojstava osnovnog nelinearnog sistema dolazi se do zaključka da se sa promenom parametara sistema javlja raslojavanje jedne homokliničke orbite u više, kao i da dolazi do bifurkacije položaja relativnog mirovanja u reonomnom sistemu, odnosno položaja ravnoteže u ekvivalentnom skleronomnom sistemu koji mu odgovara. U tome se objašnjava pojava sličnih haotičnim i stohastičnim kao odziv na sasvim periodične pobude.