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NUMERICAL INTEGRATION OF THE DIFFERENTIAL EQUATIONS FOR ONE DYNAMIC SYSTEM WITH DRY FRICTION COUPLING

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Liviu Bereteu

"Politehnica" University of Timisoara
Bul. Mihai Viteazul Nr.1, 1900 Timisoara, Romania

Abstract. *The purpose of this paper is to derive an algorithm for numerical integration of the differential equations of motion for one dynamic system with dry-friction coupling based on so called forth-order Runge-Kutta method. There are given some characteristics, which appear in the integration of these equations and are presented the results of dynamic response simulations.*
In order to prevent major failure of the wind turbine structure, by vibratory effects, a solution consist of a vibration absorber, placed inside of the tower structure. The dry friction forces give the main damping
In order to obtain the best parameters of the attached dynamic damper the numerical response of the ensemble structure was simulated.

1. NUMERICAL SOLUTION FOR RESPONSE OF NONLINEAR MULTI-DEGREE-OF-FREEDOM SYSTEMS

Numerical integration procedure such as the forth-order Runge-Kutta method is especially useful in solving for the response of nonlinear systems. The equation of motion for the general case of multi-degree-of-freedom (MDOF) nonlinear systems is:

$$[M]\{\ddot{q}\}+[C]\{\dot{q}\}+[k]\{q\}=\{Q(t)\}+\{R(t)\}\,, \tag{1}$$

where [M], [C] and [K] are nxn mass, damping and stiffness symmetrical matrices, $\{q\}$ is the vector of displacements, $\{Q(t)\}$ is the vector of applied forces, and $\{R(t)\}$ is the vector of dry friction forces.

The first order associated to the system (1) can be written as:

$$\begin{gathered}\{\dot{u}\}=[C_u]\{u\}+[K_u]\{q\}+\{Q_u(t)\}+\{R_u(t)\}\\ \{\dot{q}\}=\{u\},\end{gathered} \tag{2}$$

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where the matrices of the upper equation are:

$$\begin{aligned} [C_u] &= -[M]^{-1}[C], \quad [K_u] = -[M]^{-1}[K] \\ \{Q_u(t)\} &= [M]^{-1}\{Q(t)\}, \quad \{R_u(t)\} = [M]^{-1}\{R(t)\}. \end{aligned} \tag{3}$$

Numerical solution based on the forth order Runge-Kutta algorithm will given by the vector of velocities values:

$$\{u_{i+1}\} = \{u_i\} + \frac{\Delta t}{6}(\{K_1\} + 2\{K_2\} + 2\{K_3\} + \{K_4\}) \tag{4}$$

and the vector of displacements values

$$\{q_{i+1}\} = \{q_i\} + \frac{\Delta t}{6}(\{L_1\} + 2\{L_2\} + 2\{L_3\} + \{L_4\}) \tag{5}$$

which are obtained at time $t_i = (i+1)\Delta t$, and the time step Δt must be small enough that the difference in a linear solution and nonlinear solution over one time step is not great.

The elements of the vectors $\{K_1\}$, $\{K_2\}$, $\{K_3\}$, $\{K_4\}$, $\{L_1\}$, $\{L_2\}$, $\{L_3\}$, $\{L_4\}$ are given as:

$$\begin{aligned} \{K_1\} &= [C_u(t_i)]\{u_i\} + [K_u(t_i)]\{q_i\} + \{Q_u(t_i)\} + \{R_u(t_i)\} \\ \{L_1\} &= u(t_i) \end{aligned} \tag{6}$$

$$\begin{aligned} \{K_2\} = &\left[C_u\left(t_i + \frac{\Delta t}{2}\right)\right]\left(\{u_i\} + \frac{1}{2}\{K_1\}\right) + \left[K_u\left(t_i + \frac{\Delta t}{2}\right)\right]\left(\{q_i\} + \frac{1}{2}\{L_1\}\right) + \\ &+ \left\{Q_u\left(t_i + \frac{\Delta t}{2}\right)\right\} + \left\{R_u\left(t_i + \frac{\Delta t}{2}\right)\right\} \\ \{L_2\} = &\{u_i\} + \frac{1}{2}\{K_1\} \end{aligned} \tag{7}$$

$$\begin{aligned} \{K_3\} = &\left[C_u\left(t_i + \frac{\Delta t}{2}\right)\right]\left(\{u_i\} + \frac{1}{2}\{K_2\}\right) + \left[K_u\left(t_i + \frac{\Delta t}{2}\right)\right]\left(\{q_i\} + \frac{1}{2}\{L_2\}\right) + \\ &+ \left\{Q_u\left(t_i + \frac{\Delta t}{2}\right)\right\} + \left\{R_u\left(t_i + \frac{\Delta t}{2}\right)\right\} \\ \{L_3\} = &\{u_i\} + \frac{1}{2}\{K_2\} \end{aligned} \tag{8}$$

$$\begin{aligned} \{K_4\} = &[C_u(t_i + \Delta t)](\{u_i\} + \{K_3\}) + [K_u(t_i + \Delta t)](\{q_i\} + \{L_3\}) + \\ &+ \{Q_u(t_i + \Delta t)\} + \{R_u(t_i + \Delta t)\} \\ \{L_4\} = &\{u_i\} + \{K_3\} \end{aligned} \tag{9}$$

The equation (6)-(9) will be computed step-by-step and have to be repeated up to the end of simulation time, that is trough t/Δt computing cycles.

2. SOME CHARACTERISTICS IN THE INTEGRATION OF THE DIFFERENTIAL EQUATION FOR DYNAMIC SYSTEM WITH DRY FRICTION COUPLING

The damping of wind turbine dynamic system can be improved if at its equivalent mass m_a will be attached a seismic system [2] (figure 1). The seismic system is consisting from seismic mass m_s connected to mass m_a by an elastic-dissipate joint, having, stiffness constant k_s, damping constant c_s and the dry friction of adherent joint force $R(v_r)$. The absolute motion law of the mass m_s will be noted by $y_s(t)$, and the motion entire ensemble will be noted by y(t).

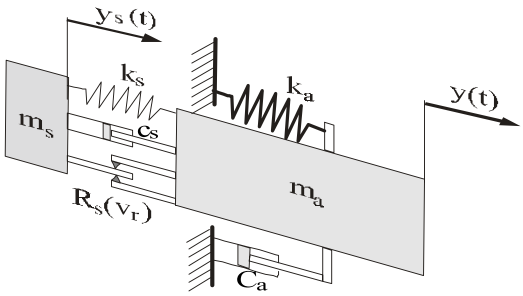

Fig. 1

The differential equation of the relative motion is:

$$m_s(\ddot{y}_s - \ddot{y}) = -m_s\ddot{y} - k_s(y_s - y) - c_s(\dot{y}_s - \dot{y}) + F_l \tag{10}$$

where

$$F_l = -R_s(v_r)sign(\dot{y}_s - \dot{y}) \tag{11}$$

is the force in the coupling dry friction.

The relative motion is blocked when

$$R_s(0) \geq F_a \tag{12}$$

where

$$F_a = \ |-m_s\ddot{y} - k_s(y_s - y)| \tag{13}$$

is the modulus of maximum dry friction.

In the blocked moment the relative equilibrium is:

$$\dot{y}_s - \dot{y} = 0 \tag{14}$$

In these case between two bodies remained one elastic constant force. When the absolute force F_a satisfied the fowling condition:

$$F_a = \ |-m_s\ddot{y} - k_s y_a| \ \geq R(0) \tag{15}$$

the two masses m_a and m_s will have independent motions, and the condition are:

$$\ddot{y} = \ddot{y}_s, \quad y_s = y + y_a \tag{16}$$

3. RESULTS OF NUMERICAL SIMULATION

In order to obtain the best parameters of the attached dynamic damper the digital response of the ensemble structure was simulated, without rotation motion ($\omega = 0$) and with excitation given by the initial conditions: $t = 0$: $y(0) = y_s(0) = y_0 = 40$ mm.

In the figure 2, the diagrams of the tower free vibration, $y(t)$, are presented and, also, of attached dynamic damper, $y_s(t)$, having the mass $m_s = 900$ kg. The energy dissipation is assumed by a force of dry friction, having the maximal value R(0) = 600 N (figure 2a), respectively $R(0) = 1200$ N (figure 2b). The dynamic damper is tuned on natural frequency of the tower, $f_y = 1{,}49$ Hz.

After the realization of the initial condition, at $t = 0$, $y(0) = y_s(0) = 40$ mm the energy of the mass m_a is cyclically transferred to the mass m_s, whose amplitude is increasing. In this time, the friction force dissipates the energy, and the vibration $y(t)$ and $y_s(t)$ amplitudes are decreasing.

When the condition given by the relation (12) is accomplished, the two masses are coupled and they have a harmonic motion, since the damping of the auxiliary system disappears and the damping of the tower is negligible.

By a numerical simulation, for a dry friction force of value $R(0) = 600$ N, a diminution of 94% of the vibration level is obtained (figure 2a), and for a force $R(0) = 1200$ N, a diminution of 82% is obtained.

During the turbine work, at an angular speed ω of the rotor, the vibrating motions are simulated on the basis of the differential equations, using the Runge-Kutta numerical algorithm of the 4-th degree, and the results are graphically represented by the diagrams in the figure 3. The first 20 seconds represent the transient process, when the rotor arrives, from the null angular speed, corresponding to a rotation speed of 50 rot/min.

The excitation is presumed as given by the mass unbalance of the rotor and it is rendered evident by the static moments and by the angular deviation of the blades in relation to the designed value of $2\pi/3$ rad.

In the figure 3a the diagrams of the vibrating motions $y(t)$ and $y_s(t)$ are represented, with the blocked dynamic damper, the seismic mass being coupled with the mass of the whole ensemble.

If the dynamic damper is unblocked, it can notice (figure 3b) that the amplitudes of the wind turbine decrease with 55%, even for small value of the attached system: m_s = 900 kg (2,96% from the mass of the whole ensemble), $k_s = 7{,}716\times10^5$ N/m and $R(0)$ = 600 N. This thing is possible since the dynamic system has the periodic coefficients, as functions of the angular speed ω, so that the solution has components, tuned on the natural frequency, even if $f_v \neq \omega/2\pi$.

This effect is verified by the numerical simulation of the vibrating motions y(t) and $y_s(t)$, calculated and graphically represented in the figure 3, together with a third signal of the angular speed.

The time step pays a key role in determining the stability and accuracy of numerical integration of the equation of motion of a MDOF system. According to authentic reference [3] the time steps of computing Δt should satisfy

$$\Delta t \leq \frac{T_{\min}}{10} = \frac{2\pi}{10\omega_{\max}} \tag{17}$$

were $\omega_{\max}$ is the highest frequency of free vibrations of the system and $T_{\min}$ is the smallest period time.

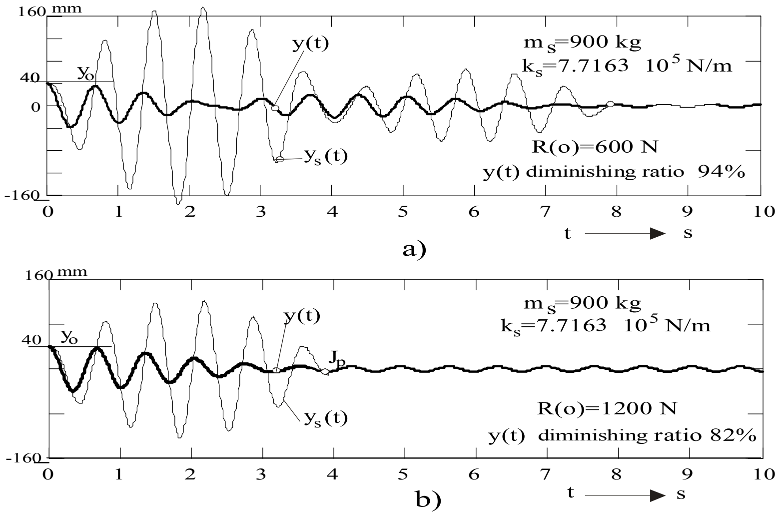

Fig. 2.

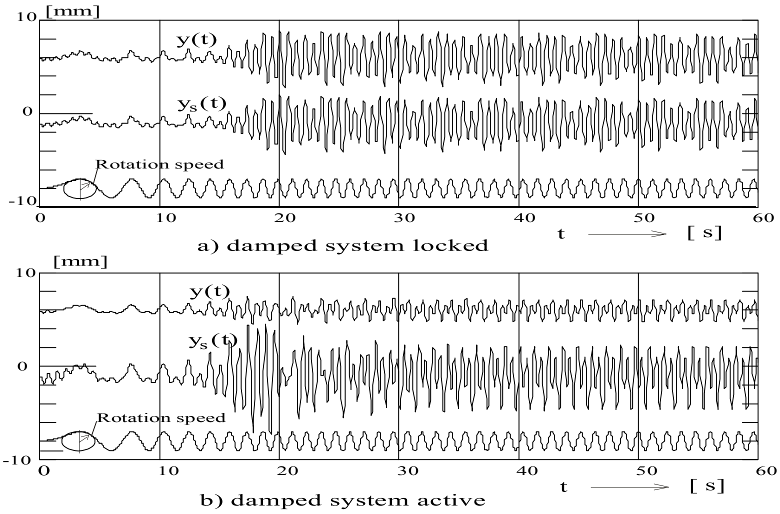

Fig. 3.

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NUMERIČKA INTEGRACIJA DIFERENCIJALNIH JEDNAČINA ZA JEDAN DINAMIČKI SISTEM SA SPREGOM SUVIM TRENJEM

Liviu Bereteu

Cilj ovog rada je da se izvede algoritam za numeričku integraciju diferencijalnih jednačina kretanja za dinamički sistem sa spregom suvim trenjem, koji se zasniva na Runge-Kutta metodi četvrtog reda. Ovde su dobijene neke karatkteristike koje se javljaju pri integraciji tih jednačina i predstavljeni su rezultati simulacije odziva dinamike.

Da bi se predupredili glavna oštećenja lopatica turbinske strukture usled oscilatornih efekata, jedno rešenje sadrži prigušivač vibracija unutar strukture. Sila suvog trenja dejstvuje kao glavni prigušivač.

Da bi se dobili najbolji parametri izučavanog dinamičkog prigušivača numerički odziv pridružene strukture je simuliran.