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CALCULATION OF THE SEPARATION POINT FOR THE TURBULENT FLOW IN PLANE DIFFUSERS

UDC 532

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Abstract. *This study examines the turbulent flow in plane-wall diffusers. For calculation we use the equations of turbulent boundary layer in integral form, adjusted for internal flow, and for closing the system of equations we use turbulent viscosity model based on the mixing length. Velocity profile in every cross-section of the diffuser is approximated by a sixth-order polynomial, while the coefficients of the polynomial depend on three form parameters. By this transformation system of governing equations is reduced to three ordinary differential equations for form parameters, which is solved numerically. The obtained results show that the performance, position of the separation point and other flow characteristics of diffusers depend on the angle and Reynolds number.*

1. INTRODUCTION

A diffuser, as an element where the stream cross-section changes from inlet to outlet, either plane or axisymetrical, has a great importance in many practical engineering applications. The flow structure in the diffuser transforms into the velocity profile with stream separation (stall), which is defined where the value of shear stress on the wall is equal to zero, then after this, the cross section stream starts to separate from the channel wall. The problem of stall is very old but very important. In the diffuser several regime flows can exist, which depend on the geometry and Reynolds number, and which are defined by Kline's diagram [1], [2] and [3] for turbulent flow (obtained by Kline's work and its Stanford group) and his appendix for laminar flow [4]. In the Kline's diagram (see Fig. 3) for the turbulent flow in the plane, or conical diffusers, which is obtained experimentally, four different regimes exist: (a) no appreciable stall, (b) transitory stall, (c) full stall on one wall with detachment close to the inlet, and (d) jet flow with full stall on all walls. In many practical cases for plane diffusers designed with good geometrical parameters it is shown (see ref. [3]) that the optimal geometry of diffusers corresponds the lengths x_s/δ_o = 5 to 15, and the area ration δ_s/δ_o 2 to 4 (see Fig. 1 and Fig. 3).

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The problem which is examined in this paper is incompressible stationary turbulent flow in a plane diffuser, which is solved by integral method of flow developed in the references [5] and [6]. This method is based in the integral equations of the incompressible turbulent boundary layer adopted for the problem of interior flow in diffusers. In this pattern it is necessary to make an approximation of the velocity profile, and in this paper we use an approximation of the velocity profile by the sixth-order polynomial based in the eddy turbulent viscosity. With this approximation we obtained the system of three nonlinear ordinary differential equations for form parameters, which are solved by classical numerical method of the Runge-Kutte. Finally, we obtained the solutions for the flow in a plane diffuser for different values of the Reynolds number and the diffuser angle, starting with quasi-developed velocity profile in the inlet cross section up to the downstream cross section in which turbulent flow separates from the wall. By using the results of our calculation for the separation flow in diffuser we obtained an addition to the Kline's diagram, which show that the position of separation point depend the angle of diffuser and Reynolds number.

2. PROBLEM STATEMENT AND THE GOVERING EQUATIONS

Two dimensional axisymetric stationary turbulent flow in the diffuser is shown in Fig. 1. Geometry of the diffuser is defined by its half-width $\delta(0) = \delta_o$, half-angle θ and by the downstream changes of half-width $\delta(x)$. The turbulent flow in the diffuser is the one which takes place between the inlet cross section, defined by the maximum velocity in the axis: $u_{eo} = u_e(0)$ and by the average velocity $u_m(0)$, and which is transformed downstream into the velocity profiles $u(x,y)$, the axis velocity $u_e(x)$ and the average velocity $u_m(x)$, and the separation cross section x_s in which the flow separation begins and in which the shear stress on the wall $\tau_w(x_s)$ is zero.

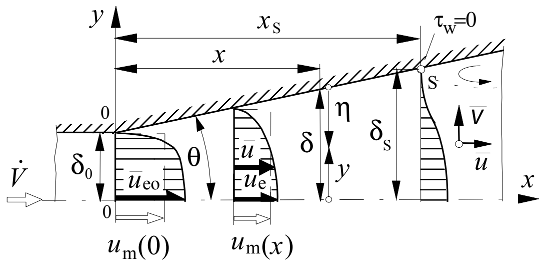

Fig. 1. Flow through a diffuser

Two-dimensional incompressible turbulent flow is described by Reynolds's equations and equation of continuity, which are after transformation (see [5] and [6]) written in the integral form:

$$\frac{d\delta_2}{dx} + (2\delta_2 + \delta_1)\frac{\bar{u}'_e}{\bar{u}_e} = \frac{\tau_w}{\rho\bar{u}_e^2} + \frac{\nu\,\delta}{\bar{u}_e^2}\left(\frac{\partial^2\bar{u}}{\partial y^2}\right)_e \ , \tag{1}$$

$$\frac{d\delta_3}{dx} + 3\delta_3\frac{\bar{u}'_e}{\bar{u}_e} = 2\int_0^{\delta}\frac{\bar{u}}{\bar{u}_e}\left[\frac{\nu\,\delta}{\bar{u}_e^2}\left(\frac{\partial^2\bar{u}}{\partial y^2}\right)_e - \frac{\partial}{\partial y}\left(\frac{\tau}{\rho\bar{u}_e^2}\right)\right]dy \tag{2}$$

where (1) represents momentum equation and (2) represents mechanical energy equation, subscript e represents the value in the axis of diffuser, and where total sheer stress τ is identified as:

$$\tau = \rho(\nu + \nu_t)\frac{\partial \bar{u}}{\partial y} , \tag{3}$$

variable $\bar{u}'_e$ represents the derivative $\bar{u}'_e = \mathrm{d}\bar{u}_e / \mathrm{d}x$, and the variables: δ_1, δ_2 and δ_3 - displacement thickness, momentum thickness and energy thickness respectively, are defined in the classical way:

$$\delta_1 = \int_0^{\delta}(1 - \frac{\bar{u}}{\bar{u}_e})\,\mathrm{d}y \ , \ \delta_2 = \int_0^{\delta}\frac{\bar{u}}{\bar{u}_e}(1 - \frac{\bar{u}}{\bar{u}_e})\,\mathrm{d}y \ , \ \delta_3 = \int_0^{\delta}\frac{\bar{u}}{\bar{u}_e}\left[1 - (\frac{\bar{u}}{\bar{u}_e})^2\right]\mathrm{d}y \tag{4}$$

For closing the physical-mathematical model which describes turbulent flow we use for eddy viscosity the mixing length model where $\nu_t = l^2 \mathrm{d}\bar{u} / \mathrm{d}\eta$, where mixing length $l(\eta)$ is defined by expression [6]:

$$\frac{l}{\delta} = 0{,}472\frac{\eta}{\delta} - 0.98\left(\frac{\eta}{\delta}\right)^2 + 0.894\left(\frac{\eta}{\delta}\right)^3 - 0.301\left(\frac{\eta}{\delta}\right)^4 . \tag{5}$$

where $\kappa = 0{,}4$ is the Prandtl's constant.

With the aim of solving equations (1)-(2) we predicted that velocity profile is the sixth-order polynomial:

$$u^+(x, y) = a(x) + b(x){y^+}^2 + c(x){y^+}^4 + d(x){y^+}^6 . \tag{6}$$

The velocity profile has to be symmetrical with relation to the axis of the channel and for this reason we only applied even numbers in the power ratios, and $a(x)$, $b(x)$, $c(x)$ and $d(x)$ denote the coefficient of the polynomial. Analysis of turbulent flow shows that it is useful to introduce: a coordinate measured positive from the wall: $\eta = \delta - y$, friction velocity $u^*(x) = \sqrt{\tau_w(x)/\rho}$ and dimensionless variables:

$$u^+ = \frac{\bar{u}}{u^*} \ , \ y^+ = \frac{y u^*}{\nu} = \delta^+ - \eta^+ \ , \ \eta^+ = \frac{\eta u^*}{\nu} \ , \ \delta^+ = \frac{\delta u^*}{\nu} . \tag{7}$$

If we use velocity profile (6) and satisfy boundary conditions as:

$$y = 0,\ \bar{u}(x,0) = \bar{u}_e(x) \ , \ \partial\bar{u}/\partial y = 0 \ ; \ y = \delta \ , \ \bar{u}(x,\delta) = 0 \ , \ \bar{v}(x,y) = 0 \ ;$$

$$\dot{V} = 2\int_0^{\delta}\bar{u}\,dy = \text{const.} \ ;$$

we will determine polynomial coefficients as:

$$a(x) = u_e^+ \ , \ b(x) = \frac{-8q + 5.25\,\mathrm{Re} - 0.0333\,\lambda\delta^{+3}}{\delta^{+3}} \ , \ c(x) = \frac{70q - 52.5\,\mathrm{Re} + 0.667\,\lambda\delta^{+3}}{6\delta^{+5}} \ ,$$

$$d(x) = \frac{-4.67q + 3.5\,\mathrm{Re} - 0.0778\,\lambda\delta^{+3}}{\delta^{+7}} \ , \tag{8}$$

in which: $\mathrm{Re} = 2\delta\bar{u}_e / \nu$ is the Reynolds number, and parametrical forms:

$$\lambda(x) = \frac{\overline{u}_e \overline{u}_e' \nu}{u^{*3}} = \frac{\overline{u}_e' \delta^2}{\nu} \frac{u_e^+}{\delta^{+2}} \ , \ q(x) = \delta^+ \overline{u}_e^+ \ . \tag{9}$$

If we use formula (9) we find a relationship between the parametrical forms:

$$q' = \frac{1}{\delta q}(\lambda \delta^{+3} + q^2 \delta') \ . \tag{10}$$

Velocity profile (6) is defined by formula (8) as a function of parametrical forms, as $\overline{u} = \overline{u}(\lambda, q, \delta^+)$. After a huge mathematical process, which is dictated by equations (3) and (4), and by formula (7), a system of three simple differential equations are obtained:

$$\frac{\mathrm{d}\lambda}{\mathrm{d}x} = f(x, \lambda, q, \delta^+) \ ; \ \frac{\mathrm{d}q}{\mathrm{d}x} = g(x, \lambda, q, \delta^+) \ ; \ \frac{\mathrm{d}\delta^+}{\mathrm{d}x} = h(x, \lambda, q, \delta^+) \tag{11}$$

The functions which depend on the longitudinal coordinate and parametrical forms: $f(x, \lambda, q, \delta^+)$, $g(x, \lambda, q, \delta^+)$ and $h(x, \lambda, q, \delta^+)$, and which are given in the reference [5]. Initial conditions of the parametrical forms according to ref. [5] are:

$$\lambda(0) \neq 0 \ , \tag{12}$$

but which is very near zero, and:

$$q(0) = \frac{\mathrm{Re}}{2}(1 + 3{,}75\sqrt{C_f(0)/2}) \ , \tag{13}$$

$$\delta^+(0) = \frac{\mathrm{Re}}{2}\sqrt{C_f(0)/2} \ . \tag{14}$$

Finally, the system of differential equations (11) and initial conditions (12) to (14) is solved by the application of the Runge-Kutte numerical method. Numerical calculations are stopped in the downstream cross section in which the flow separates from the wall of the diffuser, that is when on a distance $x = x_s$ shear stress becomes $\tau_w(x_s) = 0$.

3. Numerical results and discussion

In order to have concrete numerical results we have to define the geometry of the diffuser and the value of the Reynolds number at the diffuser inlet. In this paper the geometry of the diffuser (see Fig. 1) is defined as a straight walled slope of half-angle θ, and the cross section change is defined by the linear function: $\delta(x) = \delta_0 + tg\theta \cdot x$.

In Fig. 2 velocity profiles are shown in a diffuser with the half-angle of 15° and the Reynolds number: Re=6000, in the inlet cross section ($x/x_s = 0$) and in the downstream cross section ($x/x_s = 1$) in which the flow separation occurs. In this diagram velocity is normalized with the maximum velocity in the corresponding cross section, and the transversed coordinate is normalized with the corresponding half-width of the channel. The inlet velocity profile (for $x = 0$) is different from the fully developed turbulent velocity profile taking place between parallel plates upstream from the diffuser. This difference originates in the slight transformation of the fully developed flow immediately in front of the diffuser. Velocity profiles in the separation cross section ($x/x_s = 1$) for conditions stated in Fig. 1, as well as for other conditions, agrees well with the experimental date stated in [7] and [8].

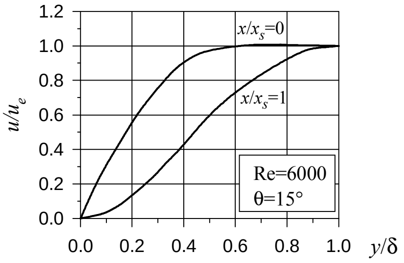

Fig. 2. Velocity profile

For a possible application of the obtained results on the flow separation the velocity profile in the cross section $x/x_s = 1$ is important only. By variations of the Reynolds number and the different angle, the whole spectrum of different separation regimes of the flow is obtained, shown in Fig. 3, and compared with Kline's diagram, where $n = x_s/\delta_o$ is the characteristic parameter. It is seen from this figure that most of the calculated flow regimes belong to the region of transitional flow in the Kline's diagram, as well as that the position of separation point depends on the Reynolds number and the diffuser angle in such a way that for a fixed Reynolds number the separation is enhanced with the increase of the angle. Also, it is seen that for a fixed diffuser angle the separation is enhanced with the increase of the Reynolds number.

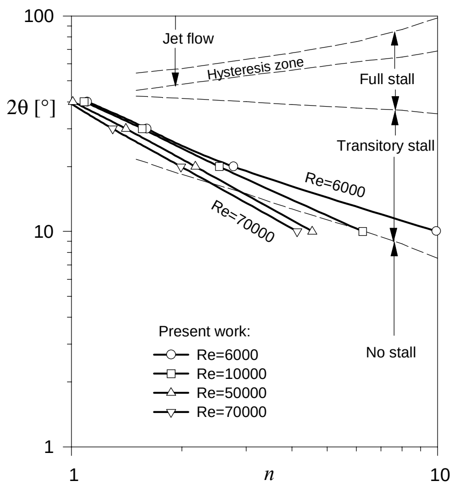

Fig. 3. Dependence of the separation point on the parameters: Re, θ and n. Dash lines correspond a Kline's diagram and full lines correspond at our results.

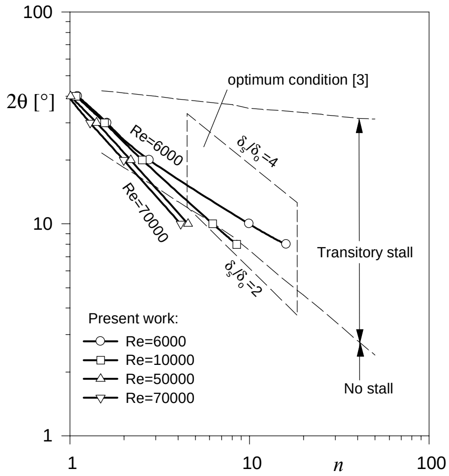

Fig. 4. Comparison of our results and the optimal regime flow in plane diffuser [3].

In Fig. 4 we show the position of our results within the optimal region [3], which is defined by $\delta_s/\delta_o = 2$ up to $\delta_s/\delta_o = 4$, and n between 5 and 15. It is seen that optimal parameters can be achieved for relatively small values of both the Reynolds number and the diffuser angle. The presented results can be used for the choice of optimal dimensions of plane diffusers in the case in which no flow separation occurs.

4. CONCLUSION

For the calculation of turbulent flow in plane diffusers in this paper we use the method of integral equations of the boundary layer theory, adopted for the problem of interior flow. The results obtained by our calculations contain the development of velocity profiles from the inlet cross section of the diffuser up to the separation cross-section. The results obtained show that the position of the separation point is a function of the Reynolds number and the diffuser angle, and their changes are more intense in the diffusers with greater angles. For the fixed value of the Reynolds number the position of the separation cross section is postponed for smaller diffuser angles. Our results of calculations can by use for design or choice of the optimal geometrical and flow characteristics of plane diffusers.

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PRORAČUN TAČKE ODVAJANJA PRI TURBULENTNOM STRUJANJU U RAVANSKIM DIFUZORIMA

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U radu se proučava turbulentno strujanje u ravanskim difuzorima. Za proračun se koriste jednačine turbulentnog graničnog sloja prilagodjene za unutrašnja strujanja, i to njihov integralni oblik. Za zatvaranje sistema jednačina koristi se model turbulentne viskoznosti baziran na putanji mešanja. Profil brzina u poprečnom preseku je aproksimiran polinomom šestog stepena, pri čemu se pokazuje da koeficijenti polinoma zavise od tri parametra oblika. Sa ovom pretpostavkom polazni sistem jednačina se transformiše na tri obične diferencijalne jednačine koje su rešene numerički. Dobijeni rezultati proračuna pokazuju da performanse, položaj tačke odvajanja i druge strujne karakteristike difuzora zavise od Rejnoldsovog broja i ugla širenja difuzora. Rezultatima proračuna koji se odnose na tačku odvajanja struje izvršena je dopuna Kline-ovog dijagram mogućih režima strujanja u ravanskim difuzorima.