FACTA UNIVERSITATIS
Series: **Mechanics, Automatic Control and Robotics** Vol.3, N° 15, 2003, pp. 1033 - 1038

CONTROLLER DESIGN FOR A FUNNEL-SHAPED SMART SHELL STRUCTURE

UDC 681.5.01+62-52

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Abstract. *The paper is aimed at controller design for the vibration suppression of a funnel-shaped shell structure, which is a part of a complex medical device – magnetic resonance tomograph. Control and sensing are achieved with piezoelectric actuators and sensors attached to the surface of the funnel. State space model of the structure is developed applying the modal analysis and reduction of the finite element model order. Optimal LQ controller combined with tracking system based on additional dynamics is designed. As an alternative for the controller design a direct robust model reference adaptive control is proposed. Both controllers perform considerable vibration suppression in comparison with uncontrolled case.*

1. INTRODUCTION

Vibration control plays an important role in the design procedure of smart structures. Active elements embedded in smart structures together with appropriate control laws applied, enable the structure to change its behavior in accordance with changing environmental conditions, providing thus the adaptivity of its response. Various examples of the vibration suppression in smart structures presented in the literature (for example [5], [7], [12], [13], [14]) confirm the need for further development and application of smart structures. The Finite Element Method (FEM) approach can be viewed as an efficient method for modeling specific smart mechanical structures from the control point of view. Furthermore it can take into account dynamics of active materials (like piezoelectrics) and offers the possibility to represent the model of the structure in the form appropriate for the controller design using some standard FEM software. In this paper the model-based controller design for the vibration suppression of the funnel shaped smart structure is presented.

Received June 20, 2003

2. DESCRIPTION AND MODEL OF THE CONTROLLED STRUCTURE

The structure under investigation is a funnel shaped inlet of the magnetic resonance tomograph* (Fig. 1) used in medical diagnostics. One major problem in magnetic resonance image (MRI) equipment is the high-level noise that a patient must undergo during the medical treatment. Some papers like [15], [16] treated the problems of the vibration control of a cylindrical shell in the magnetic resonance tomograph, as a source of the noise. This paper deals with the controller design for the vibration suppression of the funnel inlet in order to reduce the vibrations transmitted from the copper coil around the cylindrical body of the tomograph. The vibrations transmitted to the funnel are considered to be a sec-ondary source of the noise and therefore their suppression plays an important role in the noise reduction.

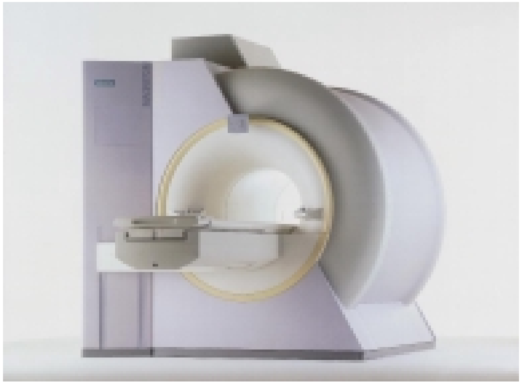

Fig. 1. Magnetic resonance tomograph

The state space model of the funnel used as a starting point for the controller design is obtained using the finite element approach. Fig. 3 represents the finite element mesh of the funnel created by the finite element software COSAR [2]. It shows the shape of the funnel (a) and placement of the sensors and actuators (b).

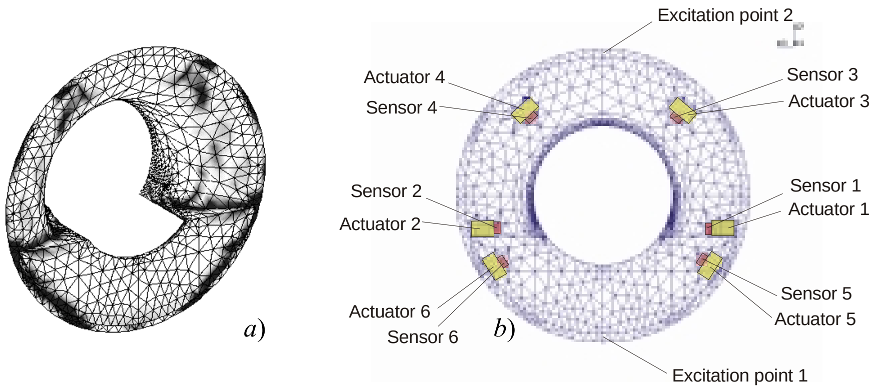

Fig. 2. FEM mesh of the funnel: a) shape and b) actuator/sensor placement

The basis for the finite element description of the coupled electro mechanical behavior of a piezoelectric smart material is a set of linearized constitutive equations [1]:

$$\boldsymbol{\sigma} = \mathbf{C}\boldsymbol{\varepsilon} - \mathbf{e}\mathbf{E}, \quad \mathbf{D} = \mathbf{e}^{T}\boldsymbol{\varepsilon} + \boldsymbol{\kappa}\mathbf{E} \tag{1}$$

* Product of the Siemens company

where $\boldsymbol{\sigma}$ represents the stress vector, $\mathbf{C}$ is the symmetric elasticity matrix, $\boldsymbol{\varepsilon}$ is the strain vector, $\mathbf{e}$ is the piezoelectric matrix, $\mathbf{E}$ is the electric field vector, $\mathbf{D}$ is the vector of electric displacements and $\boldsymbol{\kappa}$ is the symmetric dielectric matrix. The constitutive equations together with the mechanical and electric balance equations as well as the mechanical and electric boundary conditions represent a unique set of equations for the coupled electromechanical problem. Assembled equation of motion of the structure approximated by finite elements takes the form:

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{D}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \overline{\mathbf{E}}\mathbf{w}(t) + \overline{\mathbf{B}}\mathbf{u}(t) \tag{2}$$

where $\mathbf{q}$ is the vector of generalized displacements (mechanical displacements and electric potentials) and $\mathbf{M}$, $\mathbf{D}$, $\mathbf{K}$ are mass, damping and stiffness matrices, respectively; the total load on the right-hand side of the equation is represented as the sum of external forces (excitations/disturbances) and the controller contribution. Through the procedures of modal analysis and truncation, introducing modal coordinates $\mathbf{z}$, $\mathbf{q} = \boldsymbol{\Phi}\mathbf{z}$ and the state space vector $\mathbf{x}^{\mathrm{T}} = [\mathbf{z} \quad \dot{\mathbf{z}}]^{\mathrm{T}}$, the reduced state-space model is obatained in the form:

$$\dot{\mathbf{x}} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\boldsymbol{\Lambda} & -\boldsymbol{\Delta} \end{bmatrix}\mathbf{x} + \begin{bmatrix} \mathbf{0} \\ \boldsymbol{\Phi}^{\mathrm{T}}\overline{\mathbf{B}} \end{bmatrix}\mathbf{u}(t) + \begin{bmatrix} \mathbf{0} \\ \boldsymbol{\Phi}^{\mathrm{T}}\overline{\mathbf{E}} \end{bmatrix}\mathbf{w}(t) \qquad \text{or} \qquad \begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} + \mathbf{E}\mathbf{w} \\ \mathbf{y} &= \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u} + \mathbf{F}\mathbf{w} \end{aligned} \tag{3}$$

where $\boldsymbol{\Phi}$ is the modal matrix, $\boldsymbol{\Lambda}$ is the spectral matrix, and $\boldsymbol{\Phi}$ is ortho-normalized with $\boldsymbol{\Phi}^{\mathrm{T}}\mathbf{M}\boldsymbol{\Phi} = \mathbf{I}$ and $\boldsymbol{\Phi}^{\mathrm{T}}\mathbf{K}\boldsymbol{\Phi} = \boldsymbol{\Lambda}$. The modal form of the measurement equation is expressed by $\mathbf{y}$.

3. Optimal LQ and adaptive controllers

For the purpose of the vibration suppression in the funnel structure, discrete-time optimal LQ tracking system with additional dynamics is designed. Additional dynamics is introduced in the form of the matrices (formed from the coefficients of the polynomial obtained by mapping the disturbance/excitation s-poles into z-domain). The matrices of the additional dynamics, ($\boldsymbol{\Phi}_a$, $\boldsymbol{\Gamma}_a$) or $(\overline{\boldsymbol{\Phi}}, \overline{\boldsymbol{\Gamma}})$, in a cascade combination with the discrete-time equivalent ($\boldsymbol{\Phi}$, $\boldsymbol{\Gamma}$) of the state space-model of the structure, form an augmented state-space model, so-called design model:

$$\mathbf{x}_d[k+1] = \boldsymbol{\Phi}_d\mathbf{x}_d[k] + \boldsymbol{\Gamma}_d\mathbf{u}[k], \quad \text{where} \quad \boldsymbol{\Phi}_d = \begin{bmatrix} \boldsymbol{\Phi} & \mathbf{0} \\ \boldsymbol{\Gamma}^{*}\mathbf{C} & \boldsymbol{\Phi}^{*} \end{bmatrix}, \boldsymbol{\Gamma}_d = \begin{bmatrix} \boldsymbol{\Gamma} \\ \mathbf{0} \end{bmatrix}, \mathbf{x}_d = \begin{bmatrix} \mathbf{x}[k] \\ \mathbf{x}_a[k] \end{bmatrix} \tag{4}$$

Realization $(\boldsymbol{\Phi}^{*}, \boldsymbol{\Gamma}^{*})$ denotes ($\boldsymbol{\Phi}_a$, $\boldsymbol{\Gamma}_a$) or $(\overline{\boldsymbol{\Phi}}, \overline{\boldsymbol{\Gamma}})$, depending on whether the controlled structure is modeled as a single-input single-output or a multiple-input multiple-output system, respectively. Feedback gain matrix $\mathbf{L}$ of the optimal LQ regulator is calculated on the basis of the design model (4) in such a way that the feedback law $\mathbf{u}[k] = -\mathbf{L}\mathbf{x}_d[k]$ minimizes the performance index (5) subject to the constraint (4), where $\mathbf{Q}$ and $\mathbf{R}$ are symmetric, positive-definite matrices [3], [17].

$$J = \frac{1}{2}\sum_{k=0}^{\infty}(\mathbf{x}_d[k]^T\mathbf{Q}\mathbf{x}_d[k] + \mathbf{u}[k]^T\mathbf{R}\mathbf{u}[k]) \tag{5}$$

For the estimation of the state variables Kalman estimator is used. In case when **D**=0 and **E**=0 in the output equation of the state-space model (3), the Kalman estimator equations are given by [6]:

$$\hat{\mathbf{x}}[k] = \bar{\mathbf{x}}[k] + \mathbf{L}_k[k](\mathbf{y}[k] - \mathbf{C}\,\bar{\mathbf{x}}[k]) , \qquad \bar{\mathbf{x}}[k] = \mathbf{\Phi}\hat{x}[k-1] + \mathbf{\Gamma}\mathbf{u}[k-1] , \tag{6}$$

where the Kalman gain matrix is:

$$\mathbf{L}_k[k] = \mathbf{P}[k]\mathbf{C}^{\mathrm{T}}\mathbf{R}_v^{-1} \tag{7}$$

and:

$$\mathbf{P}[k] = \mathbf{M}_k[k] - \mathbf{M}_k[k]\mathbf{C}^{\mathrm{T}}(\mathbf{C}\mathbf{M}_k[k]\mathbf{C}^{\mathrm{T}} + \mathbf{R}_v)^{-1}\mathbf{C}\mathbf{M}_k[k] , \tag{8}$$

$$\mathbf{M}_k[k+1] = \mathbf{\Phi}\mathbf{P}[k]\mathbf{\Phi}^{\mathrm{T}} + \boldsymbol{\varepsilon}\mathbf{Q}_w\boldsymbol{\varepsilon}^{\mathrm{T}} . \tag{9}$$

Matrices **P** and $\mathbf{M}_k$ are determined by solving equations (8) and (9).

As an alternative a direct robust model reference adaptive controller is proposed. Desired output behavior is specified by the output $\mathbf{y}_m$ of the reference model defined by its state and output equations:

$$\dot{\mathbf{x}}_m(t) = \mathbf{A}_m(t)\mathbf{x}_m(t) + \mathbf{B}_m(t)\mathbf{u}_m(t) , \qquad \mathbf{y}_m = \mathbf{C}_m\mathbf{x}_m(t) . \tag{10}$$

The advantage of the approach can be viewed through the fact that the order of the reference model can be much less than the order of the controlled structure. Basic form of the model reference adaptive control law, in which the adaptive gain is represented with its proportional and integral parts, is modified by extending the integral part [11] in order to achieve the robustness with respect to disturbances. The control is calculated as:

$$\mathbf{u}(t) = \mathbf{K}_r(t)\mathbf{r}(t) = [\,\mathbf{K}_{\mathbf{e}_y}(t) \quad \mathbf{K}_{\mathbf{x}_m}(t) \quad \mathbf{K}_{\mathbf{u}_m}(t)\,] \begin{bmatrix} \mathbf{y}_m(t) - \mathbf{y}(t) \\ \mathbf{x}_m(t) \\ \mathbf{u}_m(t) \end{bmatrix} . \tag{11}$$

The adaptive gains $\mathbf{K}_r(t)$ are represented as a sum of a proportional (13) and integral part (14):

$$\mathbf{K}_r(t) = \mathbf{K}_p(t) + \mathbf{K}_I(t) \tag{12}$$

where:

$$\mathbf{K}_p(t) = [\mathbf{e}_\mathbf{y}(t)\mathbf{e}_\mathbf{y}^{\mathrm{T}}(t)\overline{\mathbf{T}}_{\mathbf{e}_\mathbf{y}} \quad \mathbf{e}_\mathbf{y}(t)\mathbf{x}_m^{\mathrm{T}}(t)\overline{\mathbf{T}}_{\mathbf{x}_m} \quad \mathbf{e}_\mathbf{y}(t)\mathbf{u}_m^{\mathrm{T}}(t)\overline{\mathbf{T}}_{\mathbf{u}_m}] = \mathbf{e}_\mathbf{y}(t)\mathbf{r}^{\mathrm{T}}(t)\overline{\mathbf{T}} \tag{13}$$

$$\dot{\mathbf{K}}_I(t) = \mathbf{e}_\mathbf{y}(t)\mathbf{r}^{\mathrm{T}}\mathbf{T} - \sigma\mathbf{K}_I(t) \tag{14}$$

Matrices **T** and $\overline{\mathbf{T}}$ are specified to be positive definite scaling matrices, and the σ-term of the integral adaptive gain is introduced in order to avoid divergence of integral gains in the presence of disturbances.

4. RESULTS OF THE CONTROL LAWS APPLICATION

For the control of the funnel the state-space model was developed by modal reduction in the frequency range of interest, in which the calculated eigenfrequencies correspond to the measured ones. The eigenfrequencies from the range of interest are: f_1=18.598 Hz, f_2=32.685 Hz, f_3=47.016 Hz, f_4=62.199 Hz. Optimal LQ tracking system with additional dynamics was designed to suppress vibrations caused by periodic excitations the frequencies of which correspond to the eigenfrequencies of the funnel. Time and frequency responses of the first sensor are represented in Fig. 3 (the excitation is sinusoidal with amplitude 1 and frequency which corresponds to the first eigenfrequency of the funnel). Sampling time for the discretization of the continuous state-space model is T_s=0.0001s and the weighting matrices **Q** and **R** for the optimal controller are selected to be identity matrices of the appropriate order. The state-variables are estimated using the current full-order estimator.

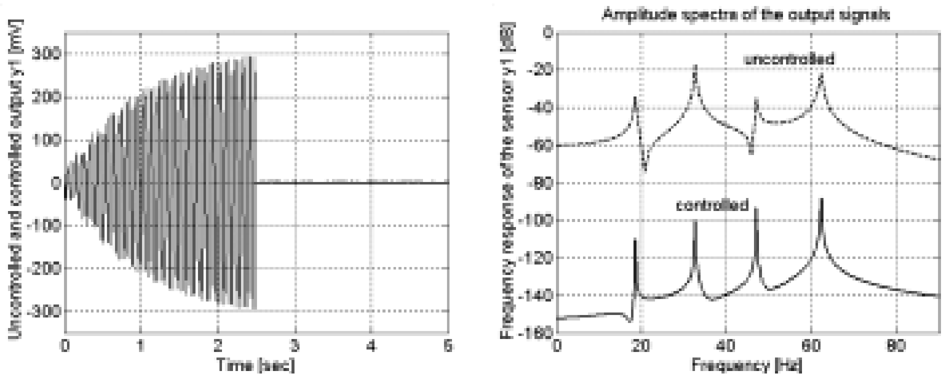

Fig. 3. Time and frequency responses of the optimal LQ tracking system

For the adaptive controller design reduced model based on the first two eigenfrequencies was used. Values of the adaptive gains **T** and $\overline{\mathbf{T}}$ are 10. Excitation is sinusoidal (frequency equal to the first eigenfrequency, amplitude $1.5 \cdot 10^{-3}$). Uncontrolled and controlled outputs of the first sensor are represented in Fig. 4a and 4b, respectively. Both the optimal LQ and adaptive controller perform considerable vibration suppression in comparison with uncontrolled case.

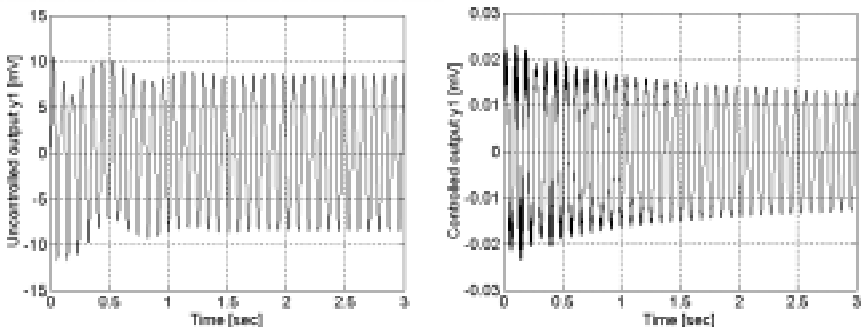

Fig. 4. Uncontrolled and controlled outputs of the first sensor

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PROJEKTOVANJE UPRAVLJANJA ZA AKTIVNU KONSTRUKCIJU TIPA LJUSKE U OBLIKU LEVKA

Tamara Nestorović Trajkov, Ulrich Gabbert, Heinz Köppe

U radu je prikazano projektovanje kontrolera za redukciju oscilacija aktivne konstrukcije tipa ljuske u obliku levka, koja predstavlja deo složenog medicinskog uređaja, magneto-rezonantnog tomografa. Upravljanje je realizovano pomoću piezoelektričnih aktuatora i senzora pričvršćenih za površinu levka. Model u prostoru stanja dobijen je na osnovu modela konačnih elemenata primenom modalne analize i redukcije reda modela. Projektovani kontroler je optimalni LQ sistem praćenja sa dodatnom dinamikom. Kao alternativa za projektovanje upravljanja predložen je direktni robustni adaptivi algoritam sa referentnim modelom. Primenom oba projetovana upravljačka zakona uočava se značajna redukcija oscilacija levka u poređenju sa slučajem bez upravljanja.