**FACTA UNIVERSITATIS**
Series: **Mechanics, Automatic Control and Robotics** Vol.3, N° 15, 2003, pp. 1147 - 1150

# CONTACT STRESS BETWEEN THE RUBBER BALL AND THE HEAD

*UDC 531.353:611.91(045)*

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**Abstract**. *Mechanical injuries of the head occur because of the following: 1. Contact stresses between the head and some other solid bodies, when mechanical stresses exceed endurance of the skin and skull bones 2. The great accelerations (frequently decelerations) which cause pressure oscillation and consequent pulsing in brain mass. This can occur even if foreign body is not in touch with injury. This paper represents biomechanical model for the calculation of impact force F and contact stress σ between a ball and a head. This work contributes 2 absolutely new formulas for force F, and stress σ. These formulas are first step in biomechanical analysis of further consequences (of brain injuries) to the scull bones and skin.*

## INTRODUCTION

Scientists show great interest in mechanical analysis of the head injuries (neurosurgeons, engineers, biomechanics and others). This interest is presented through a number of scientific works which analyze that kind of problem worldwide. This paper shows only certain works [1,2,...,6], wherein generally considered the great practical importance of the problem is emphasized. The protection equipment industry (helmets and special clothes for dangerous sports, e.g. boxing, football, auto and motor racing, bicycling, skiing, etc.) reflects this special interest. Also, there are many head injuries caused by traffic accidences and war conflicts. The total number of the mechanical head injuries in the USA is approximately 1,000,000 a year [1], of which 250,000 are caused exercising dangerous contact sports. Treatment for only 20% (of the total 1,000,000) severely injured is very expensive and time consuming. Direct and indirect costs cause serious problem for every country so it is obvious why this problem is thoroughly analyzed in some world organization like: World Medical Association, American Medical Association, National Medical Association of the United Kingdom, etc. [1,2]. Also, many countries keep statistics of the number and types of head injuries and their

Received June 10, 2003

medical, economical, social, and other consequences. Besides, well-known world sport equipment companies, motor industries [1,2], experiment with the helmets for protection of mechanical injuries of the head. These experiments enable better solutions for constructional and mechanical properties of the helmet. This paper shows biomechanical model which provides theoretical analysis for more successful experimenting, since the experimenting itself is both very expensive and does not give all the answer to important questions. This paper shows that analysis should take into account (10 - 20) biomechanical parameters for evaluation of impact force $\vec{F}$ and contact stress σ.

## METODOLOGY

We have analyzed the impact of the ball ($m_1$, $R_1$) and the head ($m_2$, $R_2$) according to fig. 1. The ball is homogeneous, with the modulus of elasticity $E_1$. The head is substituted with the $m_2 \sim 5$kg ball, and $2R_2 = 0{,}2$m diameter, with the modulus of elasticity $E_2$. Head rotation around the fourth neck vertebra $Ox$, with angular velocity $\omega_2$, causes two strike forces: joint reaction $\vec{R}_0$ and contact force $\vec{F}$ at the point of contact $A$. This problem neglects friction. The head has a known moment of inertia around axis $Ox$, $J_{Ox}$. If $t = 0$ is a start of impact, and $t = \tau$ is a impact ending, then: $v_1(0) = v_1 -$ is velocity of the contact point A at the beginning of a impact. Further on If point A is treated as if it belonged to the second body, then $v_2(0) = v_2$, $v_2(\tau) = v'_2$. The following formula defines coefficient of restitution $k$:

$$k = -\left.\frac{v'_1 \cos\alpha'_1 - v'_2 \cos\alpha'_2}{v_1 \cos\alpha_1 - v_2 \cos\alpha_2}\right|_A = -\frac{v'_1 - a\omega'_2}{v_1 - a\omega_2} \tag{1}$$

Annotation: this is the direct central impact.

$$\alpha_1 = \alpha_2 = 0, \quad \omega_1(0) = 0, \qquad \omega_2(0) = \omega_2, \qquad \omega_2(\tau) = \omega'_2$$

In this problem impact force $\vec{R}_0$ can cause the fourth neck vertebra injuries (or adjacent vertebras). Conservation of angular momentum for the system (the ball and the head) around axis $Ox$ is defined by the following equation:

$$L'_{Ox}(\tau) = L_{Ox}(o) = const$$

$$m_1 v'_1 a + J_{Ox}\omega'_2 = m_1 v_1 a + J_{Ox}\,\omega_2(0) \tag{2}$$

Impact force $\vec{F}$ does not affect the angular momentum, because it is the inner force, and angular momentum of outer force $\vec{R}_0$ equals zero around axis $Ox$. Combination of (1) and (2) yields following solution:

$$\omega'_2(\tau) = \omega_0 \frac{J_{Ox} - m_1 a^2 k}{J_{Ox} + m_1 a^2} + \frac{m_1 a(1+k)}{J_{Ox} + m_1 a^2} v_1 \tag{3}$$

$$\omega_o = 0 \quad \Rightarrow \quad \omega'_2 = \frac{m_1 a(1+k)}{J_{Ox}(m_2) + m_1 a^2} v_1$$

$\omega'_2(\tau) = \omega'_2$ – Head angular velocity after the termination of impact.

Impact momentum is:

$$J_{Ox}(\omega'_2 - \omega_0) = a\int_0^\tau F(t)dt = a\,F_{sr}\,\tau \approx \frac{aF\tau}{2} \tag{4}$$

H. Hertz (1881, German physics scientist) has determined:

$$\Delta = 0{,}83\left[F^2\cdot\frac{R_1+R_2}{R_1R_2}\left(X_1+X_2\right)^2\right]^{1/3} \tag{5}$$

Using force F (in elastic limits) to press the two balls together, produces their centers $\tilde{C}_1$ and $\tilde{C}_2$ to draw closer for the value of Δ. Contact surface $A = b^2\pi$, $b << R_i$

$$b = 0{,}91\left[F\cdot\frac{R_1R_2}{R_1+R_2}\left(X_1+X_2\right)\right]^{1/3} \tag{6}$$

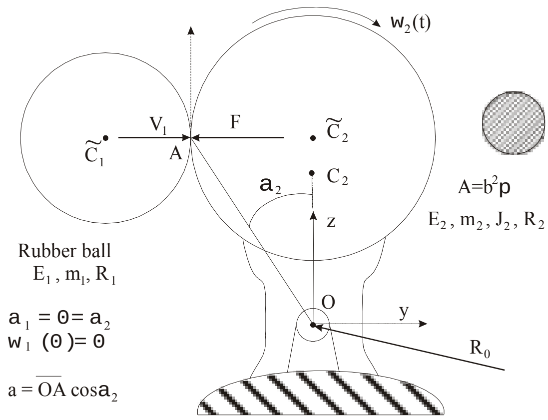


Fig. 1. Biomechanical model – the ball strikes the head

$$X_i = \frac{1-\mu_i^2}{E_i}, \qquad i = 1{,}2\ ,\ \mu = \text{Poisson's ratio} \tag{7}$$

$$\frac{\tau}{2} = \frac{\Delta}{v_1/2} = \frac{2\Delta}{v_1} \tag{8}$$

From equations 3°, 4°, 5°, 6°, 8° $\Rightarrow$ F, ω'$_2$, Δ, b, τ

$$F = \left\{\frac{\left[0{,}22\frac{m_1(1+k)}{J_{Ox}+m_1a^2}J_{Ox}(m_2)\right]^3}{\frac{R_1+R_2}{R_1R_2}(X_1+X_2)^2}\right\}^{0,2} v_1^{1,2} \tag{9}$$

$F= F(m_1, J_2, k, a, h, R_1, R_2, \mu_1, \mu_2, E_1, E_2, v_1)$

$\omega_1(0) = 0$, $\omega_2(0) = \omega_o = 0$, smoothly (friction is neglected), $\alpha_1 = \alpha'_1 = 0$,

$$\left|\sigma_{\max}\right| = 1{,}5\frac{F}{b^2\pi} = 0{,}6\left[F\frac{\left(\frac{R_1+R_2}{R_1R_2}\right)^2}{\left(X_1+X_2\right)^2}\right]^{1/3} \tag{10}$$

Variants:

$R_1 \rightarrow \infty$, $R_1 << R_2$, $R_1 >> R_2$,

$m_1 \rightarrow \infty$, $m_1 << m_2$, $m_1 >> m_2$

Formulas (3), (9) and (10) are original contribution of this work. Dimensional check shows equality of the left and right side. Besides, in special singular cases these formulas produce logical results:

a) For $v_1 = \omega_2 \cdot a$ from (3) $\Rightarrow$ $\omega'_2(\tau) = \omega_2(0)$
This is total strike amortization

b) For $k = -1$ from (3) $\Rightarrow$ $\omega'_2 = \omega_2(0)$
This is the case of bullet (mass $m_1$) passing through the snow ball (mass $m_2$), whereby bullet keep its velocity after the strike $v'_1 = v_1$. In this case, equations (9) and (10) are not true, because contact stresses exceed elastic limit.

c) Equations (9) and (10) show dependence of force $F$ and stress $\sigma$. Varying these biomechanical parameters one can influence to the geometrical and mechanical properties of the helmet in order to reduce stress $\sigma$.

## REFERENCES


1. Michael L.J. Apuzzo, Michael Y. Wang "*The Neurosurgeon in the Arena of Sport*", Clinical Neurosurgery, chapter 17, pp(333-370), The Legacy of Galen of Pergamon, 2001.
2. Vincent J. Miele, M.D., Julian E. Bailes, MD. and Joseph L. Volker, M.D "*Boxing and the Neurosurgeon*" Clinical Neurosurgery, Vol. 49, Procedings of the Congress of Neurological Surgeons, pp(395-406), chapter 19, San Diego, California 2001.
3. D. Mikičić, V. Savanović "*Mechanical Injury of the head caused by exterior force*", Facta Universitatis, Series Mechanics, Vol 3. N°11.2001.pp(295-300), Niš 2001.
4. H. J. Proctor, M.D., F.A.C.S., G.W.Palladino, M.S., and D. Fillipo, M.S., "*Failure of Autoregulation After closed Head Injury: An Experimental Model*", The Jorunal of Trauma, Vol.28, No. 3, march 1998.
5. J.J. Broz, S.J.Simske, A.R.Greenberg, and L.J.Bond, "*The use of scanning acoustic microscopy for assessing material difference in cortical bone*", Proceeding of the Twelth Southern Biomedical Engineering Conference, New Orleans, Lousiana, april 1993.
6. D. Mikičić, V. Savanović, "*The influence of Bio-mechanical Parameters on the Distribution of Contusion Damages of the Brain* ", Engineering Simulation, 1998, Vol.15, pp.(351-357).
7. G.S. Pisarenko, A.P.Jakovlev, V.V.Matvejev, "*Handbook Strenght of Materials*", Kiev 1975.
8. S.M.Targ, "*Theoretical Mechanics*", Moskow, 1980.
9. Falanga V. Bucalo B. Use of a durometer to assess skin hardness, Journal Amer. Acad. Dermatol. 29: pp(47-51), 1993.


# KONTAKTNI NAPON IZMEĐU GUMENE LOPTE I GLAVE

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*Mehaničke povrede glave nastaju zbog: 1. Kontaktnih naprezanja izmedju glave i drugih čvrstih predmeta, kada mehanički naponi prevazilaze izdržljivost kože i kostiju lobanje. 2. Drugi razlog za mehaničke povrede glave su velika ubrzanja (češće usporenja) koja uzrokuju pulzaciju pritiska u moždanoj masi. Ovo se dešava čak i u slučajevima kada strano telo ne dopre do povređenog mesta. U ovome radu biće prikazan biomehanički model za proračun udarne sile i kontaktnog naprezanja između lopte i glave čoveka. Pri tome su korišćene Hertz-ove formule za kontaktna naprezanja dve kugle. Proračun sile F i kontaktnog naprezanja σ je prvi korak u biomehaničkoj analizi daljih posledica na kožu i kosti lobanje. Nakon toga sledi procena posledica na unutrašnju strukturu mozga.*