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NONLINEAR DYNAMICS OF THE HEAVY GYRO-ROTOR WITH TWO ROTATING AXES *

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Abstract. *By using an example of the rotor system which rotates around two axes with the section, the scalar equation of the rotor dynamics is derived, as well as the expressions for the kinetic pressure on the rotor system bearings. For the case when the scewlly eccentrical disc rotates around the shaft support axis with constant angular velocity, the nonlinear dynamics around the moveable axis of the proper own rotation is studied. Nonlinear rotor system dynamics is presented by the phase portrait in the phase plane, with the trigger of the singularities as well as with the homoclinic orbits and homoclinic points of the nonstable saddle and that is done for the different values of eccentricity of the heavy disc as well as of the angle of skewlly disc.*

1. INTRODUCTION

In numerous of machines the shaft appears as the most common basic element. By using the example of the heavy rotor as well as a disc, eccentrically positioned on the light, mass neglected shaft with bearings on the light, mass neglected support which rotates around two axes with the section, nonlinear dynamic analysis of the free rotor dynamic, in this paper, is presented.

In one of classical monographs [1] by A. A. Andronov, A. A. Witt and S. E. Hajkin, which has a great number of editions, some classical examples of nonlinear systems with one degree of freedom of oscillatory motion and their phase portraits except general theory of nonlinear oscillations are presented, and such similar examples can also be found in books by J. J. Stoker [3] as well as in the text book by D.P. Rašković [14].

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Especially in monograph by Guckenheimer, J. and Holmes, Ph. [3], the results of research on nonlinear systems and properties of various kinds of bifurcations are pointed out, and also some kind of bifurcations in book by Gerard, I. and Daniel, J. [2]. A series of monographs by *Mitropolyskiy, Yu. A.* [4] deals with theory, methods and problems of asymptotic theory of non-stationary nonlinear processes of nonlinear system oscillations.

Nonlinear dynamics of gyro-rotors is a very old engineering problem with many different research results and discoveries of new nonlinear phenomena (see Refs. [1] and [3]), and of stationary and nonstationary vibrations regimes (see Ref. [4]) with different kinetic parameters of the dynamical system. However, even nowadays many researchers pay attention to this problem again, and again arose interest in researching the nonlinear dynamics of coupled rotors and gyro-rotors (see Refs. [5], [6], [7], [8], [9] and [10]) by using new analytical, numerical and experimental methods to discover the properties of nonlinear dynamics and for finer possibilities for controlling nonlinear phenomena, instabilities and non stationary regimes and the appearance of chaotic-like and stochastic-like processes.

2. THE MODEL OF THE GYROROTOR SYSTEM AND BASIC EQUATIONS

The rotation of rigid body which is rotating around two intersecting axes is known as rotating around fixed point. In this case when the support shaft axis is vertical and the rotor shaft axis is horizontal, the angular velocity is: $\vec{\omega}_1 = \omega_1\vec{n}_1 + \omega_2\vec{n}_2 = \dot{\varphi}_1\,\vec{n}_1 + \dot{\varphi}_2\,\vec{n}_2$.

The angles φ_1 (the angle of own rotation around the moveable axis oriented by the unit vector $\vec{n}_1$) and φ_2 (the angle of rotation around the shaft support axis oriented by the unit vector $\vec{n}_2$) are generalized coordinates in case when we investigate system with two degree of the freedom, but in our case we choose φ_1 as a generalized coordinate, and φ_2 as a rheonomic coordinate defined by time function. Their derivatives to time $\dot{\varphi}_1$ and $\dot{\varphi}_2$ are angular velocities.

The position vector of the mass center in relation to the intersection point of the axes is: $\overline{oc} = \vec{n}_1 e\cos\beta - \vec{v}_1 e\sin\beta$, $\overline{OC} = e$. The expression for the linear momentum is: $\vec{\mathrm{K}} = M\vec{V}_C = M\vec{\omega}\times\overline{oc}$, where M is the mass of the rotor, and the expression for the angular momentum is: $\vec{L}_o = \Im\vec{\omega}$, where $\Im$ is tensor matrix of inertia in the following form:

$$\Im = \begin{pmatrix} J_u & -J_{uv} & -J_{un} \\ -J_{vu} & J_v & -J_{vn} \\ -J_{nu} & -J_{nv} & J_n \end{pmatrix}$$

On the system beside the own weight $\vec{G} = -G\vec{n}_2$, bearing reactions: the reaction of spherical bear A $(\vec{F}_A)$ and the reaction of cylindrical bear B $(\vec{F}_B)$ apear.

Using the theorems of linear momentum and angular momentum derivatives in the following forms: $\frac{d\vec{K}}{dt} = \sum\vec{F}_i$ and $\frac{d\vec{L}_o}{dt} = \sum\vec{M}_o(\vec{F}_i) + \sum\vec{M}_i$ we can obtain the system of six scalar equations. By solving this system with respect to the known bearing reactions we can obtain the system containing one differential equation of motion and corresponding expressions of the bear reaction forces.

The moving coordinate system defined by three unit vectors of the coordinate axes orientation, $\vec{u}_1$, $\vec{v}_1$ and $\vec{n}_1$ is rigidly connected with the moving shaft of the gyro-rotor. The unit vector $\vec{n}_1$ is vector orientation of the moving shaft axis. The shaft axis (1) of its own rotation axis, with the spherical bear A and cylindrical B, on the length 2a are on the support (2). The rotation of the support is determined by the coordinate φ_2.

So, the momentary angular velocity of the rotor is:

$$\vec{\omega} = \dot{\varphi}_2 \sin\varphi_1 \vec{u}_1 + \dot{\varphi}_2 \cos\varphi_1 \vec{v}_1 + \dot{\varphi}_1 \vec{n}_1 \tag{1}$$

By using the theorems of linear momentum and angular momentum derivatives in the listed previous forms, we obtain the differential equation of the rotation around the rotor shaft in the form

$$\dot{\omega}_1 + \dot{\omega}_2 \frac{J_{vn}}{J_n}\cos\varphi_1 + \omega_2^2 \frac{J_v - J_u}{J_n}\sin 2\varphi_1 + \frac{Mge\sin\beta}{J_n}\sin\varphi_1 = \frac{M_{n_1}}{J_n} \tag{2}$$

3. THE DIFFERENTIAL EQUATION OF THE GYROROTOR SYSTEM NONLINEAR DYNAMICS

When the angular velocity $\vec{\omega}_2$ is with constant intensity $\omega_2 = const$, and when the system is without a couple M_{n_1}, this equation is in the form:

$$\dot{\omega}_1 + \omega_2^2 \frac{1}{2}\frac{J_v - J_u}{J_n}\sin 2\varphi_1 + \frac{Mge\sin\beta}{J_n}\sin\varphi_1 = 0 \tag{3}$$

We can transform it so that it can be written in a well known form (see Ref. [1], [6], [8]):

$$\dot{\omega}_1 + \Omega^2(\lambda - \cos\varphi_1)\sin\varphi_1 = 0 \tag{4}$$

Here we use the following notation:

$$\Omega^2 = \omega_2^2 \frac{J_u - J_v}{J_n} \qquad \text{and} \qquad \lambda = \frac{Mge\sin\beta}{\omega_2^2 (J_u - J_v)} \tag{5}$$

In case when the rotor is eccentrically positioned disc, we have:

$$\lambda = \frac{g(\varepsilon - 1)\sin\beta}{e\omega_2^2(\varepsilon\sin^2\beta - 1)}, \qquad \Omega^2 = \omega_2^2\frac{\varepsilon\sin^2\beta - 1}{\varepsilon\sin^2\beta = 1}, \qquad \varepsilon = 1 + 4\frac{e^2}{r^2}\cdot \tag{6}$$

Where r is radius of disc. One can see that λ and Ω^2 are in function of eccentricity e and angle β. So, we decide to analyze their influence on nonlinear dynamic behaviour of system.

4. THE EQUATIONS OF THE PHASE TRAJECTORIES AND CONSTANT ENERGY CURVES. ANALYSIS OF THE DYNAMICAL RELATIVE EQUILIBRIUM POSITIONS

The solution of differential equation (4) is:

$$\dot{\varphi}_1 = \pm\sqrt{\dot{\varphi}_{10}^2 + 2\Omega^2\left[\lambda(\cos\varphi_1 - \cos\varphi_{10}) + \frac{1}{2}(\cos^2\varphi_1 - \cos^2\varphi_{10})\right]} \qquad (7)$$

The dynamical equilibrium positions (the relative rest equilibrium positions) exist for: $(\lambda - \cos\varphi_1)\sin\varphi_1 = 0$, and that is for: $\varphi_1 = 0$ and $\cos\varphi_1 = \lambda$, $|\lambda| \le 1$.

We show graphical presentation of $\lambda(\nu,\beta)$ in function of angle β, as a family curves, depending of ν. It can be seen in the diagrams shown in the Figure 1. a*, and b* by using different scales and for ν = 0,1; 0,2; 0,5; 1.

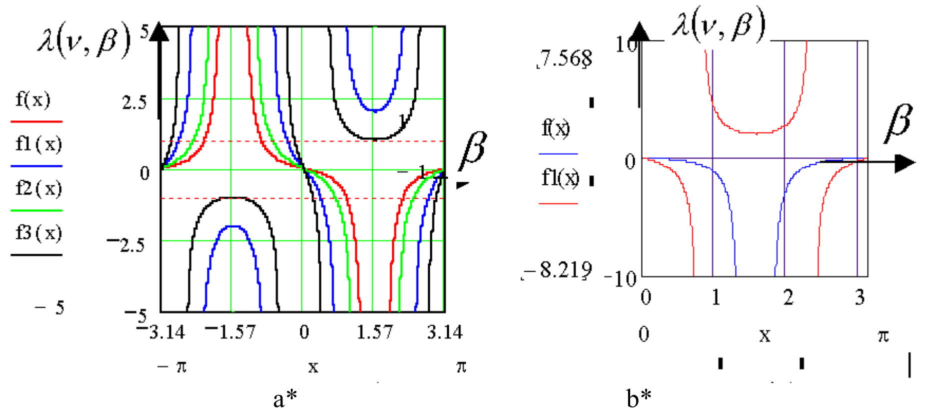


Fig. 1. Parameter $\lambda(\nu,\beta)$ in function of the angle β. [in Figure $\beta = x$, $\nu(\beta) = f(x)$ and family depending of ν = 0,1; 0,2; 0,5; 1;].

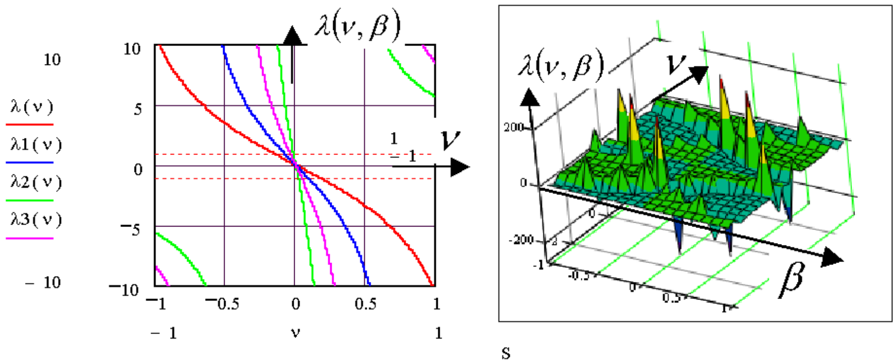
If we use the coefficient of eccentricity $\nu = \frac{e}{r}$, we can analyze $\lambda(\nu,\beta)$ in function of it. We can see the diagram as a family curves depending of β, and for different $\beta = \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}$, shown in the Figure 2.a*.

We, also, show graphical presentation of $\lambda(\nu,\beta)$ in function the coefficient of eccentricity $\nu = \frac{e}{r}$ and $\lambda(\nu,\beta)$ in function of the angle β, as the surface in parameter space $(\lambda,(\nu,\beta),\nu,\beta)$. We can see that by the surface shown in system coordinate (λ,ν,β) in the Figure 2. b*.

The dynamic relative equilibrium positions of the gyro-rotor are for: $\varphi_1 = \pm\arccos\lambda$, $|\lambda| \le 1$, then the relative equilibrium positions determined by $\varphi_1 = \pm\arccos\lambda$ are stable - the stable center types, and for $\varphi_1 = k\pi, k = 0, \pm 1, \pm 2, ...$ then the relative equilibrium positions are unstable saddle types.

We saw that $\lambda(\nu,\beta)$ is in function of coefficient of eccentricity $\nu = \frac{e}{r}$ and angle β, so we have that is:

$$\varphi_1 = \pm \arccos \frac{g}{r\omega^2} \frac{4\nu \sin\beta}{4\nu^2 \sin^2\beta - \cos^2\beta} \quad \text{and} \quad -1 \le \frac{g}{r\omega^2} \frac{4\nu \sin\beta}{4\nu^2 \sin^2\beta - \cos^2\beta} \le 1 \tag{8}$$



a* b*

Fig. 2. a* Parameter $\lambda(\nu,\beta)$ in function of the coefficient of eccentricity $\nu = \frac{e}{r}$ for different $\beta = \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}$. b* Parameter $\lambda(\nu,\beta)$ in function of the coefficient of eccentricity $\nu = \frac{e}{r}$ and $\lambda(\nu,\beta)$ in function of the angle β.

The dynamic relative equilibrium positioned are determined by the following positions: 1* stable $\dot{\varphi}_1 = 0, \varphi_1 = 2k\pi, k = 0, \pm 1, \pm 2, \ldots$;

and unstable $\dot{\varphi}_1 = 0, \varphi_1 = \pi(2k+1), k = 0, \pm 1, \pm 2, \ldots$ for $|\lambda| \ge 1$

2* ustable $\dot{\varphi}_1 = 0$; $\varphi_1 = k\pi, k = 0, \pm 1, \pm 2, \ldots$;

and stable $\varphi_1 = \pm \arccos \lambda$, for $|\lambda| \le 1$.

The singular point for which is $\varphi_1 = 0$ is an unstable saddle point when $|\lambda| < 1$, and is a stable center when $|\lambda| > 1$. The singular point for which is $\varphi_1 = \pm\pi$ is the unstable saddle point for $|\lambda| > -1$, and is the stable center when $|\lambda| < -1$.

The equation of the homoclinic orbit passing through the saddle point ($\dot{\varphi}_1 = 0, \varphi_1 = 0$) has the following form:

$$\dot{\varphi}_1 = \pm \Omega \sqrt{2\lambda(\cos\varphi_1 - 1) + \sin^2\varphi_1} \tag{9}$$

The equation of the homoclinic orbit passing through the saddle point ($\dot{\varphi}_1 = 0, \varphi_1 = \pi$) is in the following form:

$$\dot{\varphi}_1 = \pm \Omega \sqrt{2\lambda(\cos\varphi_1 + 1) + \sin^2\varphi_1} \tag{10}$$

These separatrices are shown in the Figure 4. for different values of parameters: the coefficient of eccentricity $\nu = \frac{e}{r}$ and the angle β.

For $\dot{\varphi}_1 = 0$ and $\varphi_1 = \pm \arccos \lambda$, $|\lambda| \le 1$ we have:

$$\dot{\varphi}_1 = \pm \Omega \sqrt{\lambda^2 + 2\lambda \cos \varphi_1 - \cos^2 \varphi_1} \tag{11}$$

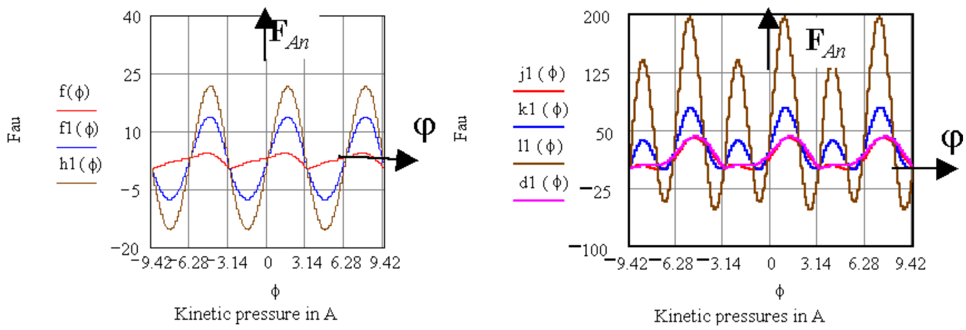
5. Expressions of the kinetic bearing pressures. Numerical analysis.

In the case when we have two intersecting axes and one of which is vertical and second horizontal, and when the angular velocity of shaft support is constant, we determined that the kinetic bearing pressures of the defined gyro-rotor system are:

$$\begin{aligned}\vec{\mathrm{F}}_{\mathrm{A}} = &\frac{1}{2a}\vec{u}_1(Mg\sin\varphi_1(a - e\cos\beta) + Mea(\dot{\omega}_1\sin\beta - \omega_2{}^2\sin\beta\sin\varphi_1\cos\varphi_1) - \varepsilon J\,\dot{\omega}_1\sin\beta\cos\beta + \\ &+ \omega_2^2\varepsilon J\sin\beta\cos\beta\sin\varphi_1\cos\varphi_1 + \omega_1\omega_2\varepsilon J\cos 2\beta\sin\varphi_1) + \\ &+ \frac{1}{2a}\vec{v}_1(Mg\cos\varphi_1(a - e\cos\beta) + Mea(\omega_1^2\sin\beta + \omega_2{}^2\sin\beta\sin^2\varphi_1) + \varepsilon J\omega_1^2\sin\beta\cos\beta + \\ &+ \omega_2^2\varepsilon J\sin\beta\cos\beta\cos^2\varphi_1 + \omega_1\omega_2\varepsilon J(\cos\varphi_1(1 + \sin^2\beta) - \frac{1}{2}\sin 2\beta\sin\varphi_1) + \\ &+ \vec{n}_1(-2Me\omega_1\omega_2\sin\beta\cos\varphi_1 - Me\omega_2^2\cos\beta)\end{aligned} \tag{12}$$

$$\begin{aligned}\vec{\mathrm{F}}_{\mathrm{B}} = &\frac{1}{2a}\vec{u}_1(Mg\sin\varphi_1(a + e\cos\beta) + \varepsilon J\omega_2^2\sin\beta\cos\beta\cos\varphi_1\sin\varphi_1 + \dot{\omega}_1\,\varepsilon J\sin\beta\cos\beta + \\ &+ 2\omega_1\omega_2\varepsilon J\sin^2\beta\sin\varphi_1) + \\ &+ \frac{1}{2a}\vec{v}_1(Mg(a + e\cos\beta)\cos\varphi_1 + \varepsilon J\omega_2^2\sin\beta\cos\beta\cos\varphi_1 + \varepsilon J\omega_1^2\sin\beta\cos\beta \\ &+ 2\omega_1\omega_2 J\cos\varphi_1 + \varepsilon J\sin\beta\cos\beta\omega_1\omega_2\sin\varphi_1 + Mea(\omega_1^2\sin\beta - \dot{\omega}_1\cos\beta\sin\varphi_1))\end{aligned} \tag{13}$$

We give graphically presentation of kinetic bearing pressures $\mathbf{F}_A$ for different values of system parameters: the eccentricity coefficient $\nu = \frac{e}{r}$ and the angle β (see the Figures 3. a* and b*, 4. a* and b*, 5.a*, b*, c* and d* and 6.a* and b*).



a* **b***

Fig. 3. a* and b* Graphical presentation of kinetic bearing pressures $\mathbf{F}_A$ for different values of system parameters: the eccentricity coefficient $\nu = \frac{e}{r}$ and the angle β.

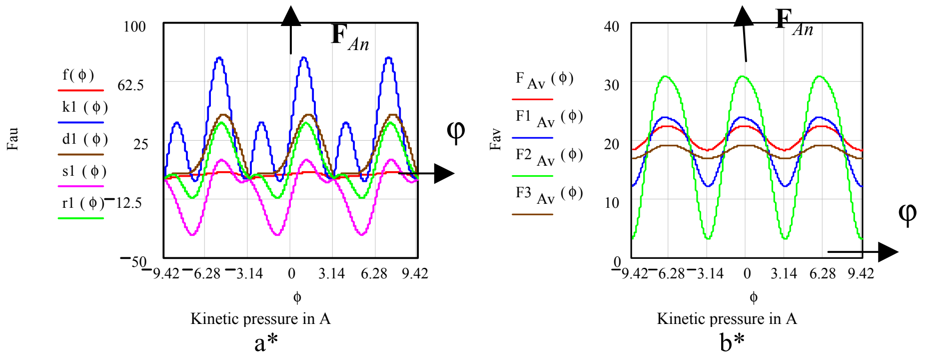


Fig. 4. a* and b* Graphical presentation of kinetic bearing pressures $\mathbf{F}_A$ for different values of system parameters: the eccentricity coefficient $v = \frac{e}{r}$ and the angle β.

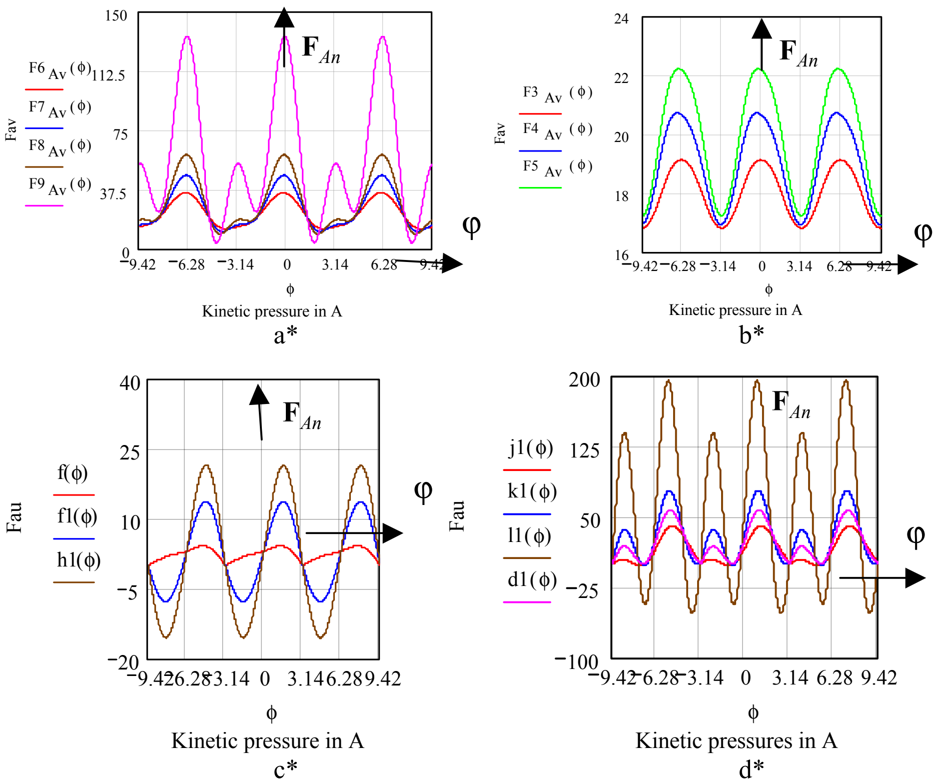


Fig. 5. a*, b*, c* and d* Graphical presentation of kinetic bearing pressures $\mathbf{F}_A$ for different values of system parameters: the eccentricity coefficient $v = \frac{e}{r}$ and the angle β.

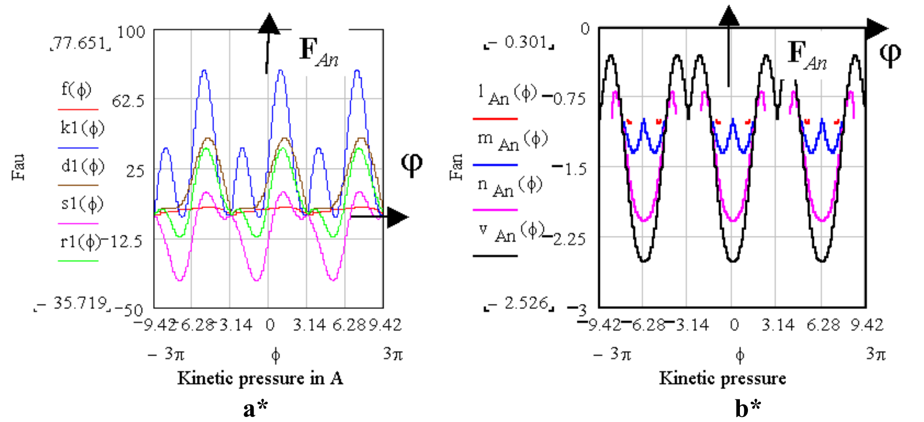


Fig. 6. a* and b* Graphical presentation of kinetic bearing pressures $\mathbf{F}_A$ for different values of system parameters: the eccentricity coefficient $v = \frac{e}{r}$ and the angle β.

We give presentation of kinetic bearing pressures $\mathbf{F}_B$ for different values of system parameters graphically: the eccentricity coefficient $v = \frac{e}{r}$ and the angle β (see the Figures 7.a*, b*, c* and d*, and 8.a*, b*, c* and d* .).

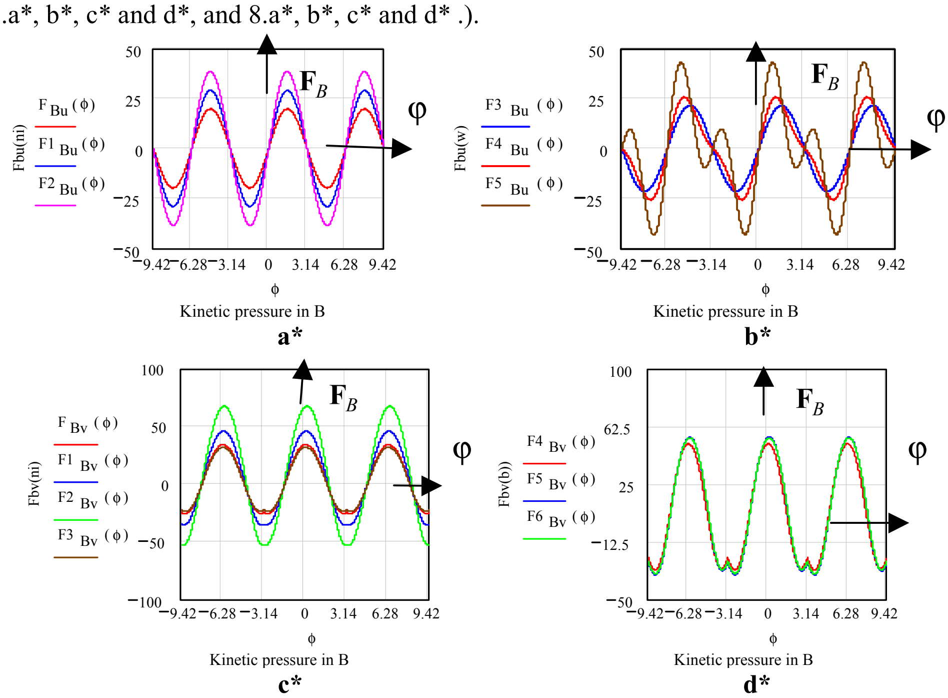


Fig. 7. a*, b*, c* and d* Graphical presentation of kinetic bearing pressures $\mathbf{F}_B$ for different values of system parameters: the eccentricity coefficient $v = \frac{e}{r}$ and the angle β.

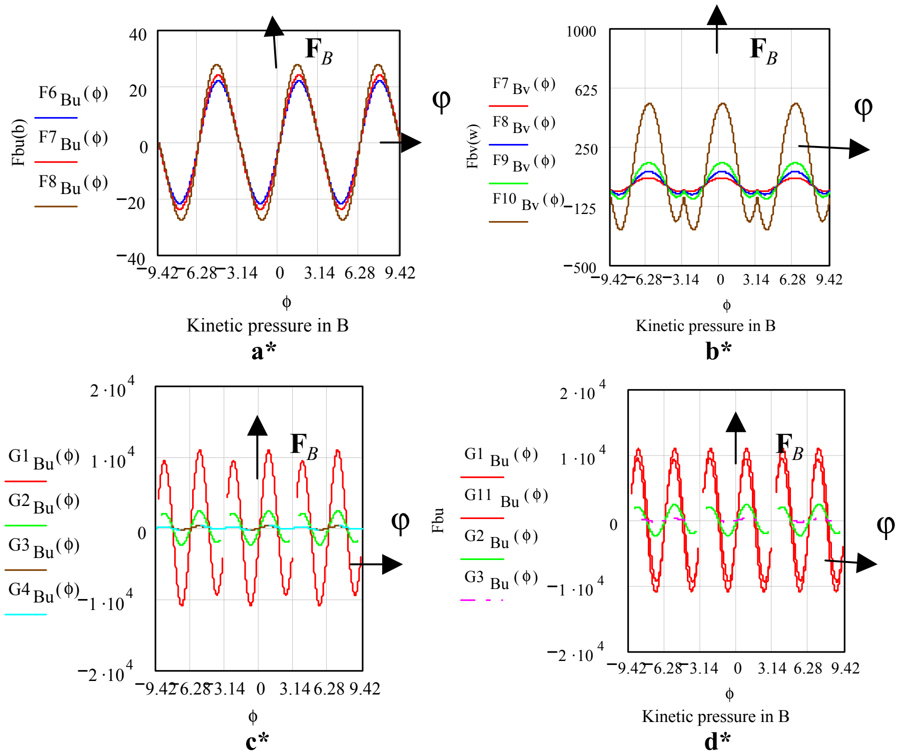


Fig. 8. a*, b*, c* and d* Graphical presentation of kinetic bearing pressures $\mathbf{F}_B$ for different values of system parameters: the eccentricity coefficient $\nu = \frac{e}{r}$ and the angle β.

6. The numerical analysis and graphical presentation in phase plane

By using the numerical experiment and graphical presentation of the numerical results we can build a qualitative analysis of the nonlinear dynamics properties of relative gyro-rotor rotation. A qualitative analysis of stationary relative equilibrium positions of rheonomic dynamical model by using the equivalent conservative scleronomic system, which correspond to the rheonomic system of the gyro-rotor model is pointed out (see Ref. [7]). The potential energy exchange curve for different parameters value of the basic system which corresponds to the gyro-rotor dynamic model is presented in Figure 9. a*, b* and c*. The graphical presentation of potential energy portraits for different values of system parameters: the eccentricity coefficient $\nu = \frac{e}{r}$ and the angle β is shown in Figure 9. a*, b* and c*.

Using MathCad program on the accomplished numerical experiment for researching the existence, like the number and character of stationary values of potential energy of the equivalent conservative scleronomic system, as number of configuration of dynamic relative equilibrium positions and character of their stability, and transformations of phase trajectories with exchanging one of the kinematic parameters of system: the eccentricity coefficient $\nu = \frac{e}{r}$ or the angle β.

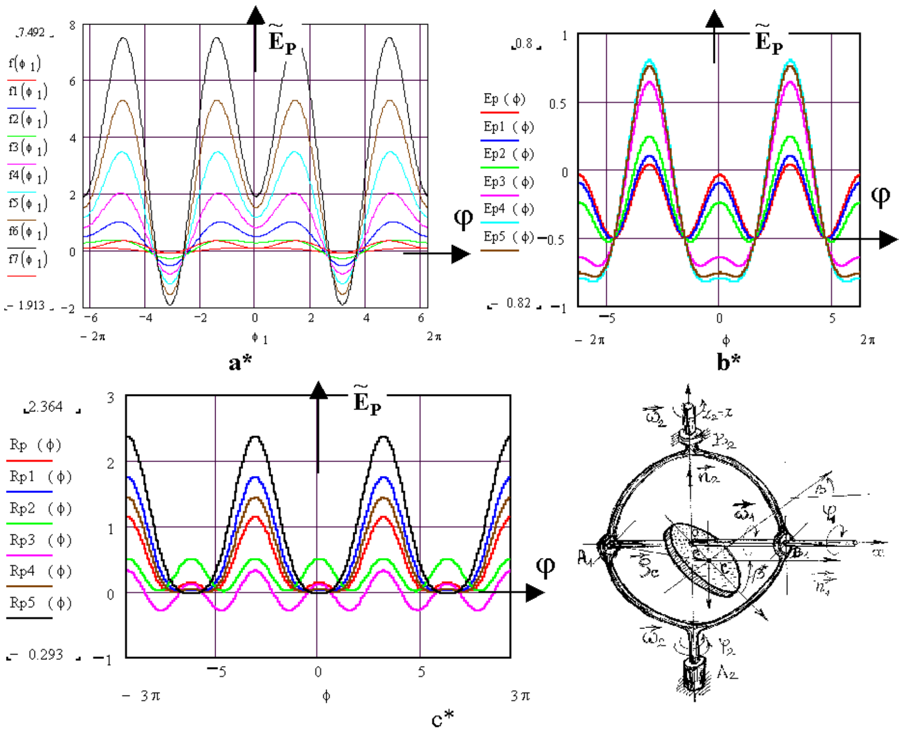


Fig. 9. a*, b* and c* Qualitative analysis of stationary relative equilibrium positions of rheonomic dynamical model by using the conservative scleronomic system, which correspond to the rheonomic system of the gyro-rotor model. The potential energy exchange curve for different parameters value of the basic system which corresponds to the gyro-rotor dynamic model. Graphical presentation of potential energy portraits for different values of system parameters: the eccentricity coefficient $\nu = \frac{e}{r}$ and the angle β.

In Figure 10. we can see characteristic phase trajectories portraits for examples of the potential energy curves from Fig. 9, and corresponding homoclinic separatrix phase trajectories for different parameters values of the basic system correspond to the gyro-rotor nonlinear dynamic model. The examples of the trigger of the coupled singularities are presented. Graphical presentation of constant energy curves and portraits for different values of system parameters: the eccentricity coefficient $\nu = \frac{e}{r}$ and the angle β are presented in Figure 10. a*, b*, c*, d*, e* and f*.

In Figure 11.a*, b*, c* and d* The transformations and layering of the homoclinic trajectories with change of the kinetic parameters values of the basic system correspond to the gyro-rotor dynamic model, the eccentricity coefficient $\nu = \frac{e}{r}$ and the angle β, are presented. The examples of the trigger of the coupled singularities and homoclinic trajectories in the form of the number eight are, also, presented.

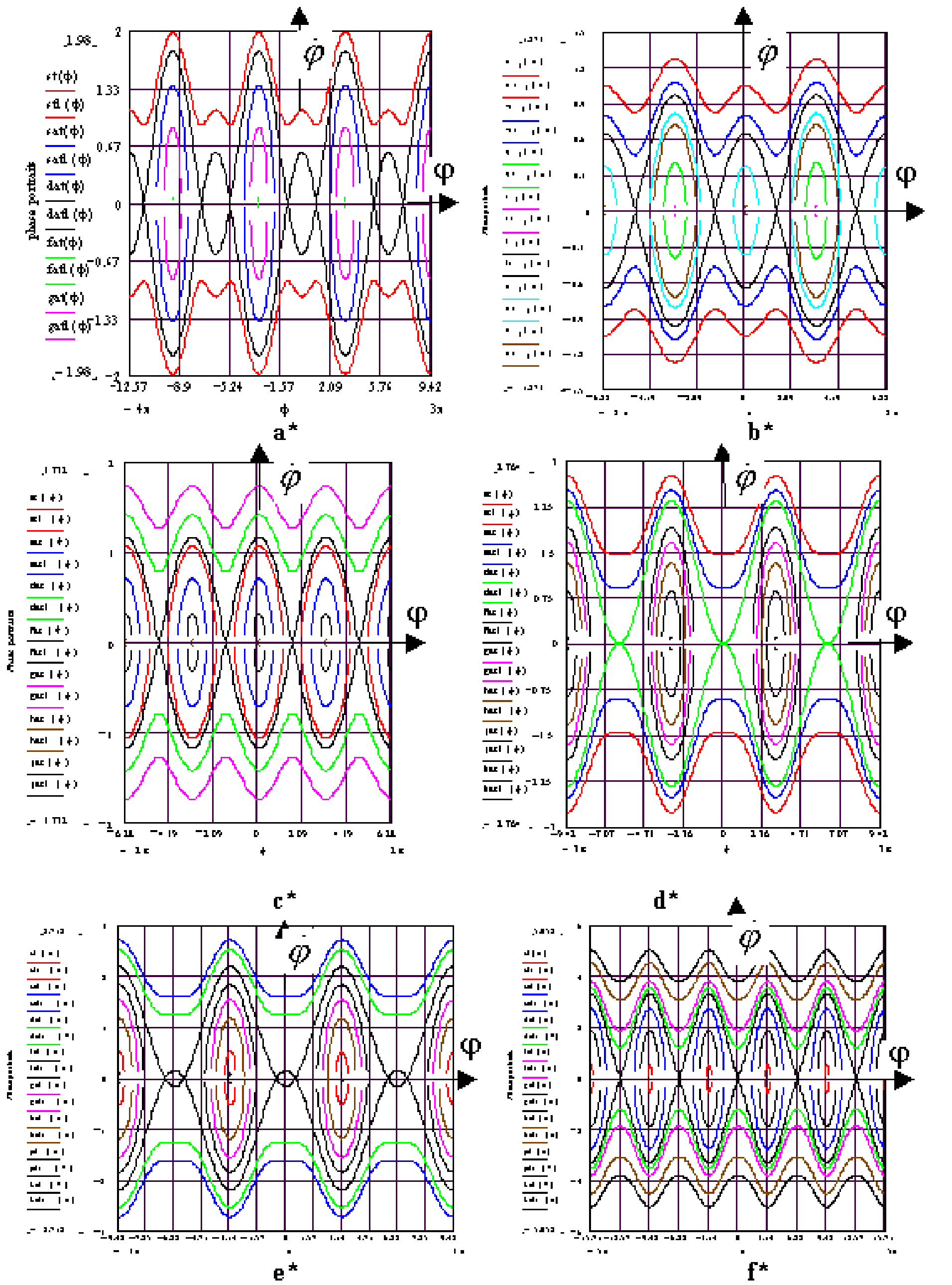


Fig. 10.a*, b*, c*, d*, e* and f*. Characteristic phase trajectories portraits for examples of the potential energy curves from Fig. 9, and corresponding homoclinic separatrix phase trajectories for different parameters values of the basic system correspond to the gyro-rotor nonlinear dynamic model. Examples of the trigger of the coupled singularities.

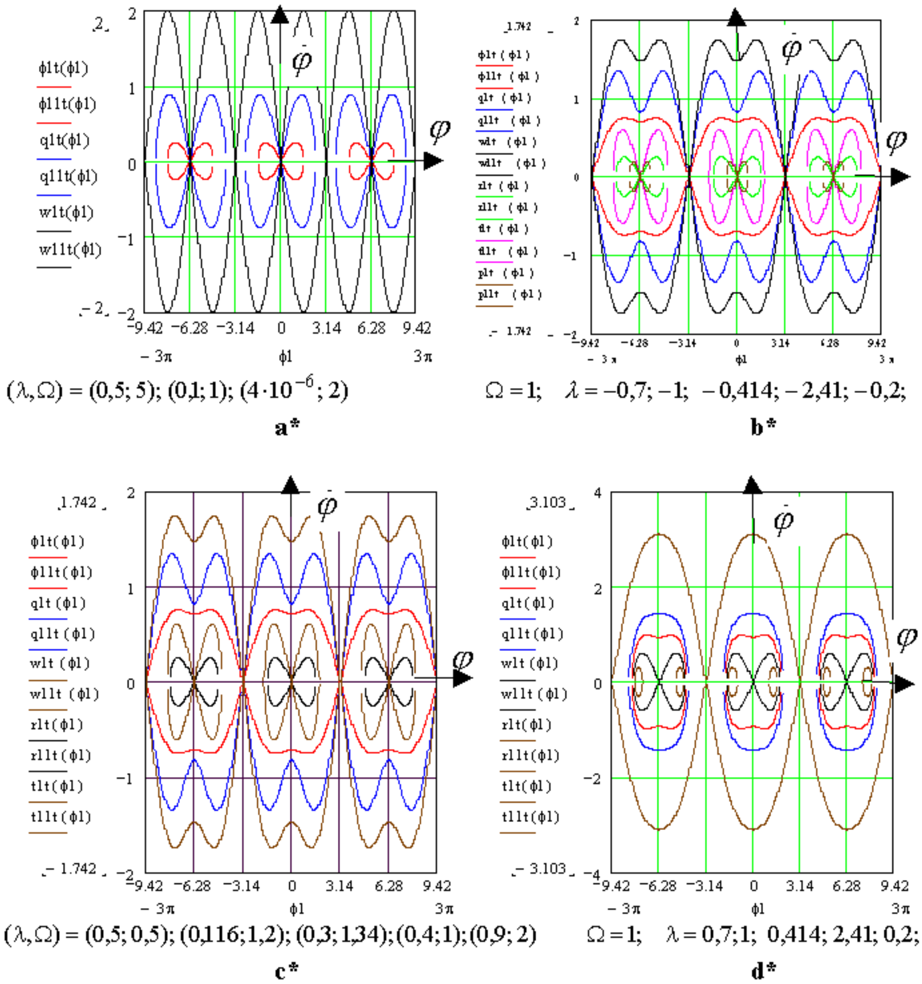


Fig. 11. a*, b*, c* and d*. Graphical presentation of the transformations and layering of the homoclinic trajectories with change of the kinetic parameters values of the basic system correspond to the gyro-rotor dynamic model. Examples of the trigger of the coupled singularities and homoclinic trajectories in the form of the number eight.

The characteristic homoclinic phase trajectories of stationary regimes of nonlinear dynamic are obtained by using the conservative scleronomic system which correspond to the rheonomic system of the gyrorotor model own rotation.

CONCLUDING REMARKS

By using the example of the heavy rotor as well as a disc, eccentrically positioned on the light, mass neglected shaft with bearings on the light, mass neglected support which rotates around two axes with the section, nonlinear dynamic analysis of the free rotor dynamic, in this paper, is presented. By using the integral of equations of the phase trajectories family, the kinetic pressures of the shaft bearing are determined. For the different cases of eccentricity of the heavy disc, as well as of the angle of skewlly positioned disc, the phase trajectories are graphically presented. Here the kinetic pressures of the shaft bearings are also graphically presented.

It can be seen that for small values of angle β there exist a wide region where parameter $\lambda(\nu,\beta)$ may be less than 1 and that is the region where the dynamical stable equilibrium positions exist. For greater values of an angle β the region of $|\lambda| < 1$ is very narrow. In that case $\lambda(\nu,\beta)$ becomes greater than 1 in a space where the coefficient of eccentricity ν is smaller.

First the author defines a trigger of coupled singularities theorem (see Refs. [5] and [12]) and the existence of homoclinic orbit and their transformation shaped by number eight, as their application on systems relevant for technical practice in her articles [6]-[11], she also constructs phase portraits and particularly considers phenomena of homoclinical orbits transformations and their disintegration, appearance and disappearance of homoclinical orbits shaped by number eight, as the trigger of coupled singularities.

In this paper we can also see a good example of practical application theorem of the trigger of coupled singularities and homoclinic orbits shaped by number eight. We used the researched results in the form of phase portraits or the family of layering homoclinic orbits of nonlinear dynamic of skewly positioned discs relative rotation as visualization of the phase portraits transformations with respect to the variation of the system parameters.

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NELINEARNA DINAMIKA TEŠKOG GIROROTORA OKO DVE OSE KOJE SE SEKU

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Za rotor, kao i za disk, koji rotira oko dve ose koje se seku u nepomičnoj tački, dobijena je diferencijalna jednačina kretanja, kao i izrazi za kinetičke pritiske u ležištima. U slučaju kada ekscentrični okvir-suport diks oko ose konstantnom ugaonom brzinom, proučavaju se nelinearna dinamika obrtanja oko sopstvenoe ose. Svojstva nelinearne dinamike se prikazuju pomo]u faznog portreta u faznoj ravni, homokliničkih trajektorija i singularnih tačaka, a za razne vrednosti koeficijenta ekscentričnosti diska kao i ugla zakošenja istog u odnosu na osu sopstvene rotacije.