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Letter to Editor

INVARIANCE IN MECHANICS – A CHALLENGE FOR ALL TIMES?

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If the fascination of young Hegelians with the idea that the world is incarnation of an Absolute Mind was one of the main characteristics of the European intellectual youth during the 19th century, then the enthusiasm for the idea on **invariance** (**covariance**, **symmetry**) of the natural laws was certainly a predominant distinction of physical deliberations in the 20th century – the century of natural sciences. Hence, there is nothing strange that, while choosing a theme for a seminar paper (defended in 1973 within the undergraduate course on the *Philosophical Foundations of Natural and Mathematical Sciences*, held by Professor Bogdan Šešić) and under the strong impression of the following Einstein's statement for example:

> " ...a system of coordinates represents only the means of description and has not anything common with the objects to be described. Only the general covariant approach in the formulation of the laws of nature corresponds to this situation, because any other way leads to the interfering of the statements about the means of description with the statements about the described object." [p. 690 in: A. Einstein, Scientific papers, I, Moscow, 1965 (in Russian)],

I decided on a brief survey of the evolution of the idea on invariance of the laws in the physical theories, pointing out some characteristics of these laws and the mathematical apparatus of the General Theory of Relativity[1].

A direct cause for reading such kind of literature was, among other things, the fact that, attending lectures at the Department of Mechanics (Faculty of Natural Sciences and Mathematics, University of Belgrade), I wondered more than once whether the various derivations of equations in three-dimensional Euclidean space, connected to the procedure of integration, unavoidably had to be carried out *in the Cartesian coordinates*. This was usually justified by *"some formal difficulties"* arising in an attempt to derive these same equations in curvilinear coordinates – hence, the equations derived in the Cartesian

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[1] Z. Drašković, *On the natural laws and their formal description*, Seminar paper, Faculty of Natural Sciences and Mathematics, Belgrade, 1973.

coordinates were proclaimed, on the basis of their *tensorial form*, to be valid in the case of arbitrary coordinates.

In 1976 I obtained an answer to these questions, for the first time, from the communications of Professor Veljko Vujičić (and then, by his courtesy, from the original papers!) at the internal sessions of Department of Mechanics in which he *postulated* the absolute integral of a tensor as an integral operator

> *" ... by which it is possible to obtain initial tensor from its absolute differential."* [p. 375 in: V.A. Vujičić, *A contribution to tensor calculus*, Tensor (N. S.), 25, (1972), 375-382].

The doubt with which the audience responded to these communications, concerning the sense of introducing a notion of an absolute, and in essence **invariant** (**covariant**) **integral**, could in my opinion be resolved only by proving that this idea – introduced in an affine *n*-dimensional space – follows in a natural way from the usual notion of a curvilinear integral after the introduction of arbitrary generalized coordinates, at least in three-dimensional Euclidean space. This was done applying Ericksen's concept of addition and integration in Euclidean space:

> *" ... one can form a tensorially invariant integral of a tensor field by shifting the field to an arbitrary fixed point ... , then integrating the shifted components, so obtaining a tensor defined at ... "* [p. 808 in: J.L. Ericksen, *Tensor Fields*, Handbuch der Physik, Bd. III/1, Springer-Verlag, Berlin - Göttingen - Heidelberg, l960],

and the paper, after a critical review by Professor V. Vujičić and following his suggestion, was sent to the *Tensor*[2] journal.

Time passed, and other preoccupations followed ... . Thanks to them, in 1980 I had the presentiment, from Kardestuncer's words[3]:

> *"Since most of all physical entities are invariant under coordinate transformations and those in discrete mechanics are not any exception to this, their treatment as tensors ... may very well be the future trend of the finite element formulation of physical problems."* [pp. 38-39 in: H. Kardestuncer, *Finite Elements Methods via Tensors*, Springer-Verlag, CISM, Udine, 1972],

what in the finite element method (FEM) – although an *approximative* theory! –the idea on invariance, i.e. on **consistent work with tensors** (and not with the matrices) would mean. On the other side, from Truesdell's words – in the paper used in 1983 during the postgraduate course on the *Nonlinear Continuum Mechanics* (held by Professor Jovo Jarić) – concerning the principle of virtual work in *curved spaces*:

> *"However, there are indications that the entire approach through the principle of virtual work ought properly to be regarded in terms of*

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[2] Z. Drašković, *On invariance of integration in Euclidean space*, Tensor (N. S.), 35, (1981), 21-24.

[3] My attention to the papers of this author was called by Professor Vedran Žanić (Faculty of Mechanical Engineering and Naval Architecture, Zagreb) during a valuable conversation after the ending of the *2nd Yugoslav Symposium of FEM and CAD* in Maribor, 1979.

> *a principle of invariance."* [p. 15 in: C. Truesdell, *Invariant and complete stress functions for general continua*, Arch. Rational Mech. Anal., 4, (1959), 1-29],

as well from Naghdi's words in the book used in 1985 during the postgraduate course on the *Theory of Surface and Line Supports* (held by Professor Dušan Medić):

> *" ... in some of the literature on the linear shell theory devoted to derivations from the three-dimensional equations, a (two-dimensional) virtual work principle in terms of two-dimensional variables is stated ab initio and is assumed to be valid without any previous appeal to its derivation from the corresponding virtual work principle in the three-dimensional theory. The justification for such an approach (which is not uncommon even in some of the recent or current literature) is of course based on the fact that the two-dimensional principle is postulated to be valid on the middle surface of the shell."* [p. 428 in: P.M. Naghdi, *The Theory of Shells and Plates*, Handbuch der Physik, VIa/2, Springer-Verlag, Berlin, 1972],

we could conclude that the approach using the principle of virtual work − and hence **an integration procedure** − should be considered as **formal** in non-Euclidean spaces!

In the meantime, in the early eighties, two studies of Professor Mladen Berković:

> M. Berković, *Three-field approximations in nonlinear finite element analysis* (unpublished) and
> M. Berković, *Thin shell theory - a three-field approximations approach* (unpublished),

were the basis for the work on the Aeronautical Institute's research project *Three-field model in nonlinear FE analysis of the thin shell*, concerning the applications of the shell theory on the FE analysis of aircraft structures. This model, i.e. the *three-field theory* is a non-classical approah in FEM − it is based on the independent approximations of the displacement, the strain and the stress field. This *mixed* model permitted not only to satisfy locally (in all points of a contour[4]) the stress boundary conditions[5], but also provided the continuity of the stress[6,7] and strain fields, too (in the classical finite element analysis only the continuity of the displacement field is provided!). When the mixed model for the thin shell was in question, the *whole shell was*, in essence, *considered as a finite element*, but only in *ζ-direction*, and the derivation of thin shell field equations from the three-dimensional theory was performed by interpolation of the displacement, the strain and the stress field in this direction[8].

[4] I.e. on the shell faces, if the development of this model in the thin shell theory is in question.
[5] Namely, the discretization of the stress field permits the discretization of the stress boundary conditions, too. On the other side, there is no way to take into account these conditions in the classical FEM, when only the displacement filed approximation is performed!
[6] M. Berković and Z. Drašković, *Stress continuity in the finite element analysis*, Proc. IV World Cong. Finite Element Meth., Interlaken, l984.
[7] Z. Drašković and M. Berković, *Stress continuity in the finite element methods*, Proc. 16th Yugoslav Congress of Theoretical and Applied Mechanics, Bečići, 1984.
[8] It should be noted that the fruitfullness of the idea on independent approximations of these fields was proved in the following dissertations, too: A. Sedmak, *Conservation law of J-integral type for thin shell*, Ph.D. Thesis,

Taking part in the above mentioned project, I had the opportunity to read the paper:

> E.N. Dvorkin and K.-J. Bathe, *A continuum mechanics based four-node shell element for general nonlinear analysis*, J. Eng. Comput., 1, 1, (1984), 77-88,

and its title just approve the well-known opinion that in the approximative theories (like FEM) we should sometimes return to the initial fundamental theory, in this case to the *Continuum Mechanics*. However – knowing that the Tensor Calculus (as a Calculus of Invariants) is still unavoidable in mathematical formulation of contemporary physical theories, and hence the Continuum Mechanics, too – the fact that, instead of the expected **covariant interpolation** of (infinitesimal) strain tensor, the **interpolation** of its **covariant coordinates** was performed must have caused the suspicion!

Almost at the same time, having read – in the book used in 1984 during the post-graduate course on the *Numerical Methods in Continuum Mechanics* (held by Professor Mladen Berković) – that:

> *" ... a less accurate but considerably simpler form of the equations of motion in general coordinates is obtained if, instead of approximating the components ... , we introduce a vector-valued representation ... "* [p. 191 in: J.T. Oden, *Finite Elements of Nonlinear Continua*, McGraw-Hill, New York, 1972]

and having noticed that in there obtained equations (immediately rejected as *"less accurate"* than the usual ones!) of motion in arbitrary curvilinear coordinates *do not appear the shifting operators* (*"Euclidean shifters"*), it was logical to wonder about the consistency of the performed approximation of the corresponding vector (tensor) fields.

And although in the literature was present an opinion expressing a doubt that the true laws of nature must necessarily be tensorial ones:

> *" ... it is not even clear that exact laws of nature must necessarily be expressible in tensor form ... "* [p. 130 in: B. Budiansky and J.L. Sanders, *On the "best" first-order linear shell theory*, Progress in Applied Mechanics, The Prager Anniv. Vol., Macmillan, New York, (1963), 129-140],

– hence the insisting on the *tensorial representation of approximative theories*[9] would be more unacceptable! – in 1987 I asserted, through a communication[10] on **invariant FE approximations** (in essence on **invariant approximation of tensor fields**) in Euclidean space, that:

> *"After all, what we call 'the natural laws' are only the approximative forms of the true laws of the nature, and nevertheless we request their*

Faculty of Mathematics, Belgrade, 1988 and A. Janković, S*ome problems of nonlinear FE analysis of shells*, Ph.D. Thesis, Faculty of Mathematics, Belgrade, 1989, advised by Professor M. Berković.

[9] However, as a rule no one desists from the use of the Tensor Calculus in these theories; for example, in the shell theory this is motivated by the tensorial notation elegance!

[10] Z. Drašković, *On invariance of finite element approximations*, Yugoslav - Polish Conference on New Trends in Mechanics of Solids and Structures, Dubrovnik, l987.

> *invariance! This request, if we stay on the natural laws described by the tensor equations, would mean that the approximations of tensor fields which take part in these equations, must be invariant under coordinate transformations."* [11];

in favour of this speak Krätzig's words concerning the approximative character of shell theories:

> *" ... this approximate character of any shell theory sometimes has been used to apologize for the large variety of different shell equations ... ...But aren't all other mechanical theories approximations too? Models, which portray only certain aspects of the physical reality."* [p. 353 in: W.B. Krätzig, *On the structure of consistent linear shell theories*, Proc. 3rd IUTAM Symp. on Shell Theory, North-Holland, (1980), 353-368].

Such conviction in necessity of the invariant character of approximations is used in some papers concerning the shell theory[12,13,14] – the mixed model[15] for the thin shell was in question once again and the shell is considered as a finite element in ζ-direction, but the new was the **invariant** interpolations of the displacement, the strain and the stress field in this direction during the derivation of the corresponding equations from the three-dimensional theory. It should be noted, even at the price to be immodest, that only in these papers Rutten's words concerning the role of shifting operators in the shell theory:

> *" ... the determination of the resultant actions and moments of force vector fields which are referred to general curvilinear coordinates is one of the most important fields of application of the finite shifters ... "* [p. 502 in: H.S. Rutten, *Theory and Design of Shells on the Basis of Asymptotic Analysis*, Rutten+Kruisman, Consulting Engineers, Voorburg, 1973]

have received their full *geometrical* meaning; namely, in spite on the insistence on a *geometrical exactness* of the shell theory in the paper:

> J.C. Simo and D.D. Fox, *On a stress resultant geometrically exact shell model. Part I: Formulation and optimal parametrization*, Computer Methods in Applied Mechanics and Engineering, 72, (1989), 267-304,

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[11] At that time the derivation (based on *invariant approximations*) of the finite element equations of motion in arbitrary curvilinear coordinates was announced, as well as their (numerical) comparison with the usual ones.

[12] Z. Drašković, *Contribution to the invariant introduction of stress resultants in shell theory*, Proc. 18th Yugoslav Congress of Theoretical and Applied Mechanics, Vrnjačka Banja, 1988.

[13] Z. Drašković, *Thin shell constitutive equations - an invariant three-field approximations approach*, M.Sc. Thesis, Faculty of Mathematics, Belgrade, 1988.

[14] Z. Drašković, *Thin shell field equations - an invariant approach*, Ph.D. Thesis, Faculty of Mathematics, Belgrade, 1990.

[15] It should be noted that the idea on invariant FE approximations obtained its application in a few papers concerning to the two-field theory: M. Berković and Z. Drašković, *On the essential mechanical boundary conditions in two-field finite element approximations*, Computer Methods in Applied Mechanics and Engineering, 91, (1991), 1339-1355 and M. Berković and Z. Drašković, *A two-field finite element model related to the Reissner's principle*, Teorijska i primenjena MEHANIKA, 20, (1994), 17-35.

this does not provide its *geometrical consistence*[16]. The *"laboriousity"* of the consistent work in curvilinear coordinates was pointed out by the following words of the very recognized authors, which – when the integration in the section on shells was in question – decided on the Cartesian rectangular coordinates, and then the obtained relations, on the basis of their *tensorial form*, were proclaimed to be valid in arbitrary curvilinear coordinates, too:

> *"According to the convention of Sect. App. 23, these vector integrals are understood to be written in rectangular Cartesian co-ordinates. ... while ... we employed rectangular Cartesian co-ordinates, the results are tensorial equations ... and hence are valid in all co-ordinate systems."* [p. 557 in: C. Truesdell and R.A. Toupin, *The Classical Field Theories*, Handbuch der Physik, Bd. III/1, Springer-Verlag, Berlin - Göttingen - Heidelberg, l960],
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> *"**23. Conventions for integrals.** While the operation of shifting ... permits integration of tensors in curvilinear co-ordinate systems in Euclidean space, it is laborious. For the purpose of this treatise it suffices when integrating tensors of order greater than 0 to consider rectangular Cartesian co-ordinates only."* [p. 813 in: J.L. Ericksen, *Tensor Fields*, Handbuch der Physik, Bd. III/1, Springer-Verlag, Berlin - Göttingen - Heidelberg, l960].

In the meantime – as an answer to a question concerning the possibility of applying an absolute integral to determine the displacement field from the strain field, but *in curvilinear coordinates* – Cesàro's formula in these coordinates[17] was derived[18] 1991, in connection with some considerations in the shell theory. And these considerations, according to Golab's statement (but, truth to say, almost thirty years ago) concerning the Green, Stokes and Gauss formulae:

> *"The essential nature of these theorems did not become clear until they were written in vector or tensor form, which revealed the invariant, and, hence, geometric character of these formulae. ... These theorems are still waiting for a suitable monograph to be written presenting all aspects ... of theorems in a way which is both up-to-date and of a satisfactory standard as regards mathematical rigour."* [p. 288 in: S. Golab, *Tensor Calculus*, Elsevier, Amsterdam - London - New York, 1974],

were the occasion to point out another forms of these theorems[19]. Subsequently, bearing in mind Flügge's warning concerning the precaution needed in the use of the Tensor Calculus[20]:

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[16] Z. Drašković, *Stress-resultants in the shell theory - asymmetric or symmetric?*, Teorijska i primenjena MEHANIKA, 21, (1995), 19-28.

[17] The proposed approach to formula's derivation and its ensuing form was new, judging from the literature accessible to me.

[18] Z. Drašković, *On the derivation of E. Cesàro's formula in curvilinear coordinates*, Teorijska i primenjena MEHANIKA, 17, (1991), 53-58.

[19] Z. Drašković, *A note on the invariant formulation of Gauss' theorem in curvilinear coordinates in Euclidean space*, Facta Universitatis, Series "Mechanics, Automatic Control and Robotics" 1, 4, (1994), 511-517.

[20] Even if some of its approximations are in question (e.g. in FEM, as it was already mentioned above).

> *"The general, noncartesian tensor is a much sharper thinking tool and, like other sharp tools, can be very beneficial and very dangerous, depending on how it is used. Much nonsense can be hidden behind a cloud of tensor symbols and much light can be shed upon a difficult subject."* [p. iv in: W. Flügge, *Tensor Analysis and Continuum Mechanics*, Springer-Verlag, Berlin - Heidelberg - New York, 1972],

some *inconsistencies* in the shell theory[21,22] was pointed out. This – in accordance with the statements (their actuality was also confirmed by Professor Milan Mićunović, in a discussion at the 21st Yugoslav Congress on Theoretical and Applied Mechanics) on the *strain measures* role in the shell theory:

> *"One of the difficulties encountered in the development of a satisfactory theory of shells, especially for finite strains, lies in the choice of suitable strain measures. ... The choice of ... measures for finite deformation of shells has not been assessed or sufficiently explored. At any rate, the choice depends also on the constitutive equations as well as the point of view that may be adopted in seeking the complete formulation of the theory."* [p. 25 and p. 32 in: P.M. Naghdi, *Foundations in elastic shell theory*, Progress in Solid Mechanics 6, North-Holland, (1963), 1-90]

– should be used for some further *stipulations* in the shell theory, with the **recapitulation of thin shell field equations derivation**. It should be remarked that one of the reasons to return to the foundations of the shell theory was the statement read in the *BENCHmark* journal a long time ago:

> *"A perfect thin shell element is still the 'holy grail', but shells in the meantime have still to be analysed and there are a wide variety of shell elements in common use."* [p. 10 in: G.A.O. Davies, *Results for selected benchmarks*, BENCHmark (Oct), (1987), 8-12],

as well as the belief that the situation can hardly be improved without discussing the very premises of the shell theory. A contribution to this conviction represents, it seems, the following very distinctive title – *"Efficient finite elements for shells – do they exist?"* – in Proceedings of a relatively recent international conference:

> *"We demonstrate that 'shell problem' as a mathematical concept is of very complex nature. This helps to understand why the shell modeling by finite elements is so hard."* [J. Pitkranta, *Efficient finite elements for shells – do they exist?*, International Conference on Numerical Methods and Computational Mechanics in Science and Engineering, Miskolc, 1996].

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[21] Z. Drašković, *On noninvariance of the usual approaches to the introduction of stress-resultants in shell theory*, Proc. 21st Yugoslav Congress of Theoretical and Applied Mechanics, Niš, 1995.

[22] Z. Drašković, *On a stipulation of the relationships between the covariant derivatives of space and surface tensors in shell theory*, Facta Universitatis, Series "Mechanics, Automatic Control and Robotics", 1, 5, (1995), 561-566.

However, the application of the idea of invariant FE approximations (although announced in 1987!) was not continued until 1993 through the consistent, i.e. **invariant derivation** of FE equations of motion in curvilinear coordinates[23], with **invariant numerical** (i.e. approximative) **integration**. The comparison of their numerical efficiency with the one of the usual equations remained for some other time.

In the meantime 1994 Professor Vujičić himself obtained the paper:

Z. Horák, *Sur le problème fondamental du calcul intégral absolu*, C. R. Ac. Sci., 189, (1929), 19-21,

so that, once again thanks to him I had the opportunity to return to some of my "wonders" now nearly two decades old[24]. And, lo and behold – **absolute integral** a'priori declared to be *nonsens*, was the subject of a communication on one of the sessions of the French Academy of Sciences back in the distant year of 1929! So a remark of Professor V. Vujičić that:

> *" ... in the integral calculus and its application to mechanics almost no attention seems to be paid to the question of invariance of the differential expression's integration, namely the differential equations among which the differential equations of motion are most frequent."*

and courageous effort to overcome the fact that:

> *" ... ordinary integration destroys the tensor character of geometrical and mechanical objects ...."* [p. 183 in: V.A. Vujičić, *Preprinciples of Mechanics*, Mathematical Institute of Serbian Academy of Sciences and Arts, Belgrade, 1999]

obtained their "historical" justification.

Of course, the idea of *invariant FE approximations* is not left aside and in 1995 – through a communication[25] on invariant stress extrapolation – it received a *numerical confirmation*, as well a *graphical*[26] one. Finally, the testing (although announced in 1993!) of the **numerical efficiency** of the invariant approach was performed in 1999 in the case of determining the nodal displacements in some typical FE problems in curvilinear coordinates, using the **invariant** FE equations of motion[27]. However, without hurrying to proclaim several numerical examples as crucial evidence to the superiority of the proposed *invariant* (*covariant*) approach in the finite element method, something undisputable should be emphasized – the least that this approach deserves is to be fully reconsidered once again, especially bearing in mind that it can be successfully applied not only

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[23] Z. Drašković, *Contribution to the invariant derivation of finite element equations of motion in curvilinear coordinates*, Facta Universitatis, Series "Mechanics, Automatic Control and Robotics", 2, 6, (1995), 25-32.

[24] Z. Drašković, *Again on the absolute integral*, Proc. 21st Yugoslav Congress of Theoretical and Applied Mechanics, Niš, 1995.

[25] Z. Drašković, *Contribution to a more accurate nodal stresses determination in the classical finite element method*, Naučno-tehnički PREGLED, XLV, 9, (1995), 3-8.

[26] Z. Drašković, *Visualization as a criterion of invariant finite element approximation naturalness*, Facta Universitatis, Series "Physics, Chemistry and Technology", 1, 3, (1996), 237-239.

[27] Z. Drašković, *Numerical comparison of the scalar, pseudoinvariant and invariant approach in the derivation of finite element equations of motion in curvilinear coordinates*, Facta Universitatis, Series "Mechanics, Automatic Control and Robotics", 3, 12, (2002), 351-357.

in the local, but in the global *"stress recovery"* procedures[28], too; besides, in view of the fact that the paper[29], pleading for an **invariant tensor fields approximation**, in the meantime was cited several times[30,31,32,33,34], as well the fact that this approach has been recently[35] used in *three-dimensional* FE analysis too, it seems that its applicability to the approximation of laws in any physical theory is being more and more approved.

Finishing the chronology of my acceptance of the idea on the **invariance of fields** and the **invariance of operations** (for example **integral** ones) performed on these fields in a physical theory, as well the chronology of my own enduring on the **invariant approximation** of these fields (either,for example, FE approximations or the numerical integration being in question) – I dare to express the following conviction: all above mentioned give the hope for a, perhaps immodest, expectation that these few research directions – **absolute integration** (as a part of *Theory of Invariants*), **shell theory** (as an *invariant* approximation of Solid Mechanics) and corresponding applications in **finite element method** (as an *invariant* approximative theory) – might together, in the time to come, lead to the improvement both of theoretical and applied aspects of the contemporary Mechanics.

In these endeavours – although a long time ago it was stated that the finite elements can be used in Euclidean as well as in non-Euclidean spaces:

> *" ... the general concept of finite element is applicable to ... tensor field, defined on Euclidean or non-Euclidean spaces ... . General finite-element representations of covariant and contravariant components of vectors defined on non-Euclidean spaces ... were used ... in the analysis of thin shells."* [p. 46 in: J.T. Oden, *Finite Elements of Nonlinear Continua*, McGraw-Hill, New York, 1972],

– the true chalenge will represent wrestling with the consistent **invariant finite element approximations in non-Euclidean spaces**. Some contributions[36,37,38] – where a heretical

[28] D. Mijuca, Z. Drašković and M. Berković, *Stress recovery procedure based on the known displacements*, Facta Universitatis, Series "Mechanics, Automatic Control and Robotics", 2 (7/2), (1997), 513-523.

[29] Z. Drašković, *On invariance of finite element approximations*, Mechanika teoretyczna i stosowana, 26 (4), (1988), 597-601.

[30] S. Vujić, M. Berković, D. Kuzmanović, P. Milanović, A. Sedmak and M. Mićić, *The Application of the Finite Element Method to Geostatic Analysis in Mining*, Faculty of Mining and Geology, Belgrade, 1991.

[31] D. Mijuca, *Continual interpretation of the solid body FE stress state*, M.Sc. Thesis, Faculty of Mathematics, Belgrade, 1995.

[32] D. Mijuca and M. Berković, *Some stress recovery procedures in the classical finite element analysis*, Proc. 21st Yugoslav Congress of Theoretical and Applied Mechanics, Niš, 1995.

[33] D. Mijuca, *Primal-mixed finite element approach in solid mechanics*, Ph.D. Thesis, Faculty of Mathematics, Belgrade, 1999.

[34] M. Berković and D. Mijuca, *On the main properties of the primal-mixed finite element formulation*, Facta Universitatis, Series "Mechanics, Automatic Control and Robotics", 2, 9, (1999), 903-920.

[35] D. Mijuca, *Higher tests for a new reliable 3D finite element in the linear elasticityi*, Communications of Department of Mechanics, Mathematical Institute of Serbian Academy of Sciences and Arts, Belgrade, 2001.

[36] Z. Drašković, *Again on the absolute integral*, Proc. 21st Yugoslav Congress of Theoretical and Applied Mechanics, Niš, 1995.

[37] Z. Drašković, *Contribution to the discussion on absolute integration of differential equations of geodesics in non-Euclidean space*, Facta Universitatis, Series "Mechanics, Automatic Control and Robotics", 3, 11, (2001), 55-70.

idea concerning the necessity for a *different definition* of the invariant operations of differentiation and integration in non-Euclidean spaces (the middle shell surface is an example of these spaces!) was declared – represent in essence the searching for an appropriate foothold. Such a forward coming should be the subject of future activities, and Professor Vujičić 's words:

> *"There is no one single general configurational ordering in mechanics ... motion problems are not solved in one single way, i.e. uniformly, but in many equivalent ways, that is, in polifold or manifold ways. Therefore, the statement 'differentiation and integration of tensor on manifolds' is meaningful so long as it is clearly stated what particular manifolds are referred to or if valid proofs are given about invariance of differentiation and integration upon manifolds. ... The required integral can be determined only to the degree of knowledge about manifolds ...."* [pp. 183-184 in: V.A. Vujičić, *Preprinciples of Mechanics*, Mathematical Institute of Serbian Academy of Sciences and Arts, Belgrade, 1999]

look like a prediction of the variety of approaches which then will appear, but impose a question, too: which path is the right one?

Acknowledgement. *The author wishes to express his gratitude to Professor Veljko Vujičić (Mathematical Institute, Belgrade) for a critical review of this essay*[*].

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[38] Z. Drašković, *On the geometrical sense of covariant differentiation in non-Euclidean space*, Recent advances in Analytical dynamics – Control, stability and differential geometry, Mathematical Institute of Serbian Academy of Sciences and Arts, Belgrade, 2001.

[*] The word *essay* denotes a short treatise, but in its original sense (fr. *essai*) this is in fact an *attempt*!