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SUBSPACE IDENTIFICATION FOR THE MODEL BASED CONTROLLER DESIGN OF A FUNNEL-SHAPED STRUCTURE

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Abstract. *In order to design and apply controller for a funnel-shaped structure with attached piezoelectric actuators and sensors the model of the structure was identified using the subspace-based system identification. Based on the measured input and output signals from the actuators and sensors, the frequency response functions of the appropriate actuator-sensor pairs were identified and compared with the frequency response functions obtained from the model identified using the subspace identification. Results of the system identification as well as of the control based on the identified models are presented in the form of the appropriate time and frequency responses.*

Key words: *subspace based system identification, control of a funnel- shaped structure*

1. INTRODUCTION

System identification represents a meaningful tool for development of models in an appropriate form when the measurement of the input and output variables of the system is possible. Depending on the concrete problem and on the system requirements different identification approaches are proposed. In compliance with the requirements of the concrete problem of the vibration control design for a funnel-shaped piezoelectric structure, this paper treats the nonparametric identification of the frequency response functions between appropriate actuators and sensors as well as a subspace based identification of the state space model.

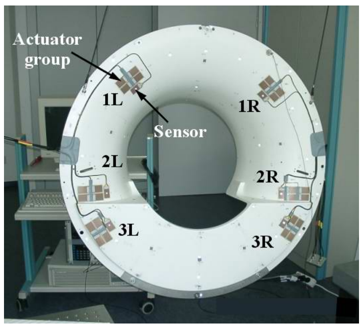

Fig. 1 Funnel-shaped structure

Considered funnel-shaped structure represents an inlet part of the magnetic resonance image (MRI) tomograph used in medical diagnostics. The control

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problem is aimed at the suppression of the secondary vibrations transmitted from the body of the MRI tomograph to the funnel structure, using the piezoelectric actuators and sensors attached with the funnel. The funnel together with the attached actuators and sensors used for the control as well as for the model identification is represented in Fig. 1.

2. SUBSPACE-BASED SYSTEM IDENTIFICATION

Subspace identification problem is considered in its general form, which assumes the identification of combined deterministic-stochastic system. Starting from this general problem and its solution, pure deterministic and pure stochastic identification problems can be derived as special cases depending on the requirements of the particular problem. The system to be identified from the input-output measurements is expressed in its general discrete-time combined deterministic-stochastic form:

$$\mathbf{x}[k+1] = \mathbf{\Phi}\mathbf{x}[k] + \mathbf{\Gamma}\mathbf{u}[k] + \boldsymbol{w}[k], \quad \mathbf{y}[k] = \mathbf{C}\mathbf{x}[k] + \mathbf{D}\mathbf{u}[k] + \boldsymbol{v}[k]. \tag{1}$$

For given s measurements of the input $\mathbf{u} \in \mathrm{R}^m$ and the output $\mathbf{y} \in \mathrm{R}^l$ generated by the unknown combined system (1) of order n, where the process noise and the measurement noise vector sequences $\boldsymbol{w}[k]$ and $\boldsymbol{v}[k]$ are white noise with zero mean and with covariance matrix:

$$\mathcal{E}\left\{\begin{bmatrix} \boldsymbol{w}[i] \\ \boldsymbol{v}[j] \end{bmatrix} \begin{bmatrix} \boldsymbol{w}[i]^{\mathrm{T}} & \boldsymbol{v}[j]^{\mathrm{T}} \end{bmatrix}\right\} = \begin{bmatrix} \mathbf{Q} & \mathbf{S} \\ \mathbf{S}^{\mathrm{T}} & \mathbf{R} \end{bmatrix} \tag{2}$$

the problem of the subspace identification is to determine the order n of the unknown system and the system matrices $\mathbf{\Phi} \in \mathrm{R}^{n\times n}$, $\mathbf{\Gamma} \in \mathrm{R}^{n\times m}$, $\mathbf{C} \in \mathrm{R}^{l\times n}$, $\mathbf{D} \in \mathrm{R}^{l\times m}$ as well as the covariance matrices $\mathbf{Q} \in \mathrm{R}^{n\times n}$, $\mathbf{S} \in \mathrm{R}^{n\times l}$, $\mathbf{R} \in \mathrm{R}^{l\times l}$ of the noise sequences $\boldsymbol{w}[k]$ and $\boldsymbol{v}[k]$.

Measured input and output data are organized into block Hankel matrices. The Hankel block matrix of measured inputs is defined in accordance with [7] as:

$$U_{0|2i-1} \overset{def}{=} \left[\begin{array}{ccccc} \mathbf{u}_0 & \mathbf{u}_1 & \mathbf{u}_2 & \cdots & \mathbf{u}_{j-1} \\ \mathbf{u}_1 & \mathbf{u}_2 & \mathbf{u}_3 & \cdots & \mathbf{u}_j \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \mathbf{u}_{i-1} & \mathbf{u}_i & \mathbf{u}_{i+1} & \cdots & \mathbf{u}_{i+j-2} \\ \hline \mathbf{u}_i & \mathbf{u}_{i+1} & \mathbf{u}_{i+2} & \cdots & \mathbf{u}_{i+j-1} \\ \mathbf{u}_{i+1} & \mathbf{u}_{i+2} & \mathbf{u}_{i+3} & \cdots & \mathbf{u}_{i+j} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \mathbf{u}_{2i-1} & \mathbf{u}_{2i} & \mathbf{u}_{2i+1} & \cdots & \mathbf{u}_{2i+j-2} \end{array}\right] \overset{def}{=} \left[\begin{array}{c} U_{0|i-1} \\ \hline U_{i|2i-1} \end{array}\right] \overset{def}{=} \left[\begin{array}{c} U_p \\ \hline U_f \end{array}\right] \overset{def}{=} \left[\begin{array}{c} U_{0|i} \\ \hline U_{i+1|2i-1} \end{array}\right] \overset{def}{=} \left[\begin{array}{c} U_p^+ \\ \hline U_f^- \end{array}\right]. \tag{3}$$

The output block Hankel matrices $Y_{0|2i-1}, Y_p, Y_f, Y_p^+, Y_f^-$ are defined in a similar way. The block Hankel matrix consisting of inputs and outputs is defined as:

$$W_{0|i-1} \overset{def}{=} \begin{bmatrix} U_{0|i-1} \\ Y_{0|i-1} \end{bmatrix} = \begin{bmatrix} U_p \\ Y_p \end{bmatrix} = W_p \quad \text{and} \quad W_p^+ = \begin{bmatrix} U_p^+ \\ Y_p^+ \end{bmatrix}. \tag{4}$$

Subspace identification algorithms make a wide use of the controllability and observability matrices. Extended controllability and observability matrices for the system (1) are used in the subspace identification algorithm in the following form:

$$\mathbf{G}_i^{co} = \begin{bmatrix} \mathbf{\Phi}^{i-1}\mathbf{\Gamma} & \mathbf{\Phi}^{i-2}\mathbf{\Gamma} & \cdots & \mathbf{\Phi\Gamma} & \mathbf{\Gamma} \end{bmatrix} \in \mathrm{R}^{n \times mi}, \quad \mathbf{G}_i^{ob} = \begin{bmatrix} \mathbf{C} \\ \mathbf{C\Phi} \\ \mathbf{C\Phi}^2 \\ \vdots \\ \mathbf{C\Phi}^{i-1} \end{bmatrix} \in \mathrm{R}^{li \times n}. \tag{5}$$

Index i represents the number of the block rows which appears also in the definition of the Hankel matrix. The number of block rows i is a user-defined index which should be at least larger than the state dimension of the system to be identified and smaller than the data length.

Following the unbiased combined deterministic-stochastic identification algorithm presented in [5] and [7] the following oblique and orthogonal projections are calculated:

$$\mathrm{O}_i = Y_f /_{U_f} W_p, \qquad Z_i = Y_f / \begin{bmatrix} W_p \\ U_f \end{bmatrix}, \qquad Z_{i+1} = Y_f^- / \begin{bmatrix} W_p^+ \\ U_f^- \end{bmatrix}. \tag{6}$$

By inspecting the singular values **S** obtained from the singular value decomposition of the weighted oblique projection

$$\mathbf{W}_1 \mathrm{O}_i \mathbf{W}_2 = \mathbf{USV}^{\mathrm{T}} = \begin{bmatrix} \mathbf{U}_1 & \mathbf{U}_2 \end{bmatrix} \begin{bmatrix} \mathbf{S}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1^{\mathrm{T}} \\ \mathbf{V}_2^{\mathrm{T}} \end{bmatrix} = \mathbf{U}_1 \mathbf{S}_1 \mathbf{V}_1^{\mathrm{T}} \tag{7}$$

the order of the system can be determined as the number of singular values different from zero. The extended observability matrix $\mathbf{G}_i^{ob}$ is determined as:

$$\mathbf{G}_i^{ob} = \mathbf{W}_1^{-1}\mathbf{U}_1\mathbf{S}_1^{1/2}\,\mathbf{T} \tag{8}$$

where $\mathbf{T} \in \mathrm{R}^{n \times n}$ is an arbitrary non-singular matrix of the similarity transformation. Introducing the Toeplitz matrix $\mathbf{H}_i^d$:

$$\mathbf{H}_i^d \overset{def}{=} \begin{bmatrix} \mathbf{D} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{C\Gamma} & \mathbf{D} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{C\Phi\Gamma} & \mathbf{C\Gamma} & \mathbf{D} & \cdots & \mathbf{0} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \mathbf{C\Phi}^{i-2}\mathbf{\Gamma} & \mathbf{C\Phi}^{i-3}\mathbf{\Gamma} & \mathbf{C\Phi}^{i-4}\mathbf{\Gamma} & \cdots & \mathbf{D} \end{bmatrix} \in \mathrm{R}^{li \times mi} \tag{9}$$

the following system can be solved for **Φ**, **C** and $\mathcal{K}$:

$$\begin{bmatrix} (\mathbf{G}_{i-1}^{ob})^+ Z_{i+1} \\ \hline Y_{i|i} \end{bmatrix} = \begin{bmatrix} \mathbf{\Phi} \\ \hline \mathbf{C} \end{bmatrix} (\mathbf{G}_i^{ob})^+ Z_i + \mathcal{K} U_f + \begin{bmatrix} \rho_w \\ \rho_v \end{bmatrix}, \tag{10}$$

where
$$\mathcal{K} \overset{def}{=} \begin{bmatrix} \left[\mathbf{\Gamma} \mid (\mathbf{G}_{i-1}^{ob})^{+}\mathbf{H}_{i-1}^{d}\right] - \mathbf{\Phi}(\mathbf{G}_{i}^{ob})^{+}\mathbf{H}_{i}^{d} \\ \left[\mathbf{D} \mid \mathbf{0}\right] - \mathbf{C}(\mathbf{G}_{i}^{ob})^{+}\mathbf{H}_{i}^{d} \end{bmatrix}, \tag{11}$$

$\mathbf{G}_{i-1}^{ob}$ is the extended observability matrix $\mathbf{G}_{i}^{ob}$ without the last l rows and ρ_w and ρ_v are the Kalman filter residuals orthogonal to the row space of $W_p, U_f, \hat{X}_i$. Matrices $\mathbf{\Gamma}$ and $\mathbf{D}$ are determined from $\mathcal{K}$ using the least squares. The covariances **Q**, **S**, **R** are approximated from the residuals ρ_w and ρ_v in the following way, where for larger i the approximation error is smaller:

$$\begin{bmatrix} \mathbf{Q} & \mathbf{S} \\ \mathbf{S}^{\mathrm{T}} & \mathbf{R} \end{bmatrix} \approx \mathcal{E}_j\left\{\begin{bmatrix} \rho_w \\ \rho_v \end{bmatrix}\begin{bmatrix} \rho_w^{\mathrm{T}} & \rho_v^{\mathrm{T}} \end{bmatrix}\right\}. \tag{12}$$

3. Results of the Subspace Identification and Control of the Funnel

Described procedure for the subspace identification was applied in order to identify the state space model of the funnel-shaped structure in Fig. 1 and subsequently, based on the obtained model, to design an optimal controller used for the vibration suppression of the structure. This section presents the main results. The piezoelectric patches attached to the funnel were used as actuators and sensors. The actuators were excited by random signals and the sensor responses were measured as the output data of the system to be identified. Data acquisition for both the actuator and sensor signals was performed using a Hardware-in-the-Loop realization with the dSPACE system. Random excitation signals were generated by the computer and transmitted to the piezoelectric patches via DAC board of the dSPACE system. The acquisition of the sensor signals was performed using the ACD board of the dSPACE. In the subsequent presentation of the results the following notation is used. The notation A and S (which stand for *actuator* and *sensor* respectively) is followed by a number (1 – 6), which determines the position of the actuator/sensor. The notation L and R stands for the left and right hand side of the funnel (see Fig. 1). Applying the described subspace identification procedure, the state space models were obtained from the measured input/output data.

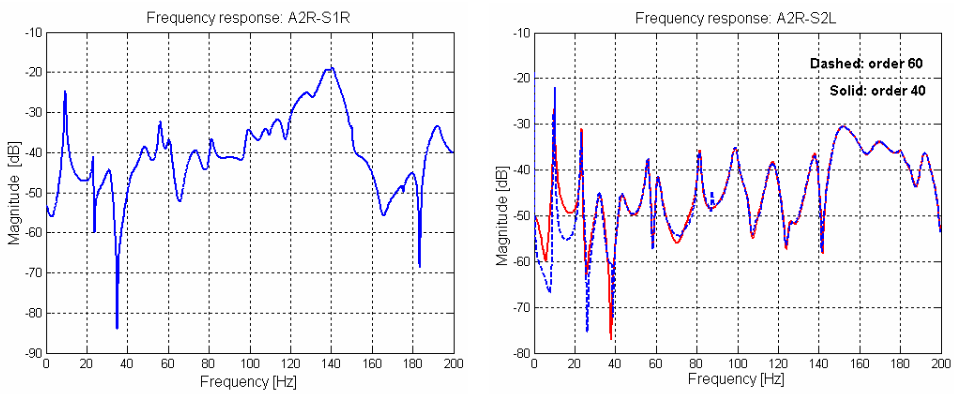

Fig. 2 Frequency responses obtained using the subspace identification

Frequency response functions based on the state space models obtained using the subspace identification in a SISO case are shown in Fig. 2. For the actuator/sensor pair A2R–S1R a good approximation of the measured FRF is obtained with the state space model of the order 60, while for the actuator/sensor pair A2R–S2L the comparison of the frequency response functions for orders 40 and 60 is represented in the right-hand portion of the Fig. 2.

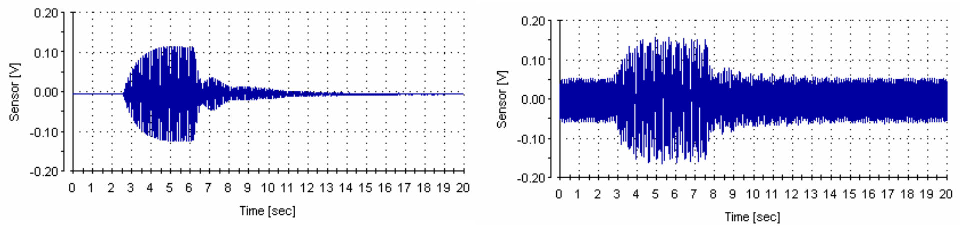

Fig. 3 Response of the sensor A2R with the excitation on the sensor S1R

Vibration suppression in terms of the oscillation amplitudes reduction using an optimal LQ controller for the actuator/sensor pair A2R–S1R is represented in Fig. 3. Left-hand portion of the figure represents time response when the excitation is sinusoidal with the frequency equal to one selected lower eigenfrequency of the funnel. The period without control is approximately between 2.5 and 6 seconds. The right-hand portion of the figure represents time response in the case when the excitation is obtained as a sum of the three sinusoidal signals with frequencies corresponding to the eigenfrequencies of the funnel. The period without control corresponding to bigger vibration amplitudes is between the 3rd and the 8th second.

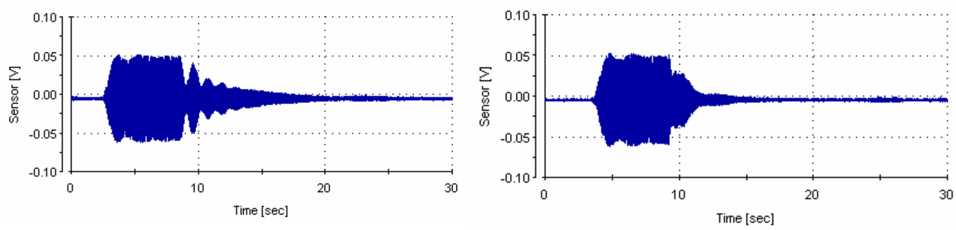

Fig. 4 Response of the sensor A2R with the excitation on the sensor S2L
left: order 40, right: order 60

For the actuator/sensor pair A2R–S2L two models of different orders (40 and 60) are developed using the subspace identification. Controlled responses of the sensor S2L, when the actuator A2R is excited by the sinus excitation, for both model orders are shown in Fig. 4. Comparison of the frequency response functions obtained on the basis of the identified model and from the measurements shows good agreement, especially in the higher frequency region. From the settling-time point of view the model of the higher order is more convenient. On the other hand it does not require much additional computational effort for the controller design in comparison with the lower order model.

For the multiple-input multiple-output model identification case, the sensors S1R, S2L and the actuator A2R were used. Comparison of the identified and measured frequency response functions is represented in Fig. 5. The identified model is of order 50.

Time responses of the sensors and the controller signal are represented in Fig. 6 (left for the sine excitation with the frequency corresponding to an eigenfrequency of the funnel, right for the excitation obtained as a sum of three sinusoids with frequencies corresponding to eigenfrequencies of the funnel).

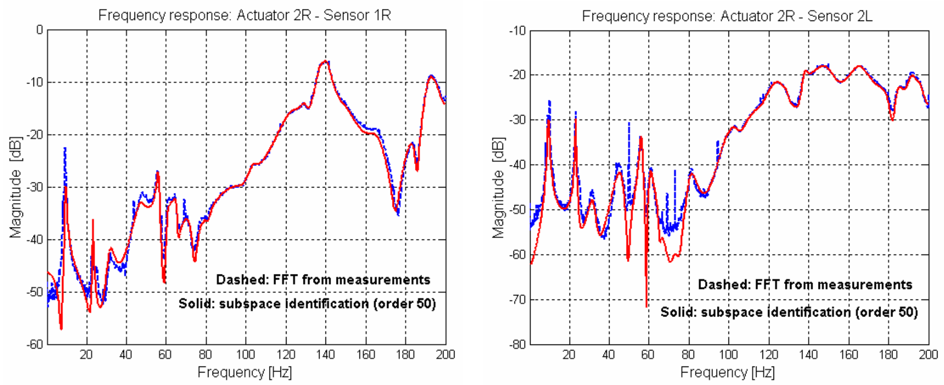

Fig. 5 Frequency responses functions obtained from the identified model and from the measurements using the Fast Fourier Transform (FFT)

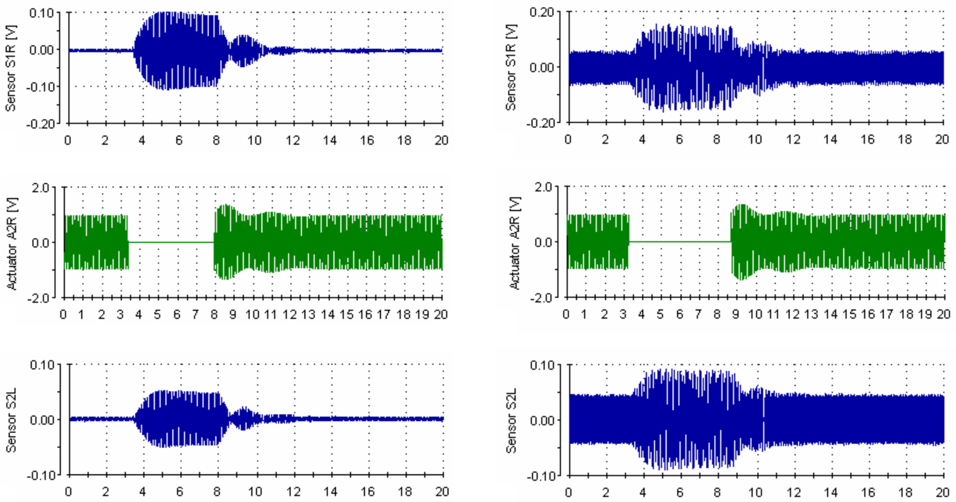

Fig. 6 Actuator and sensor signals for the controlled system designed on the basis of the identified model (two excitation cases)

4. Conclusion

In this paper the subspace based identification method is used for the model development of the funnel-shaped piezoelectric structure and applied in the SISO and

MIMO control cases. The method is characterized by efficient model development based on the measured input and output signals resulting in a state-space realization which is convenient for the controller design. This parametric identification method enables thus a multiple-input multiple output control with an arbitrary number of actuators and sensors and with an appropriate model order, depending on the frequency range of interest.

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IDENTIFIKACIJA MODELA NA BAZI PODPROSTORA MATRICA U CILJU PROJEKTOVANJA UPRAVLJANJA ZA STRUKTURU OBLIKA LEVKA

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U cilju projektovanja i primene upravljanja za strukturu u obliku levka sa piezoelektričnim aktuatorima i senzorima, identifikovan je model primenom identifikacije na bazi podprostora matrica (subspace-based identification). Na osnovu merenih ulaznih i izlaznih signala sa aktuatora i senzora, identifikovani su frekventni odzivi za odgovarajuće parove aktuatora i senzora i uporedjeni sa frekventnim odzivima dobijenim na osnovu modela identifikovanog primenom identifikacije na bazi podprostora odgovarajućih matrica. Rezultati identifikacije i upravljanja, sprovedenog na osnovu identifikovanih modela, prikazani su u obliku odgovarajućih vremenskih i frekventnih odziva.

Ključne reči: *identifikacija na bazi podprostora matrica, upravljanje levkastom strukturom*