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SANDWICH BEAM STABILITY ANALYSIS APPLYING BENDING THEORY OF THE SENDWICH CONSTRUCTIONS

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Abstract. *Sandwich constructions have two thin, elastic outer layers and a middle layer - core made of material with relatively small stiffness comparing to stiffness of the the outer layers. Calculation of these constructions is based on the supposition that all three layers deform simultaneously, and as the result a unique neutral line is formed between the outer layers. With this approach to the calculation it is possible to describe, with high accuracy, the stress and the strain state of a construction as well as the local influence in each layer. Based on the hypothesis of broken cross section line [1], this article shows the process of determining critical pressure force of a sandwich beam composed of two thin, elastic outer layers of the same thickness and a middle layer with negligible bending stiffness compared to bending stiffness of thr outer layers, Figure 1. A system of differential equilibrium equations is derived by application of the static and energetic methods as well as the contour conditions that must be fulfilled by the solution of the system.*

Key words: *Sandwich Beam, Critical Pressure Force, Differential Stability Equations; Contour Conditions*

1. INTRODUCTION

Solving the stability problem of the pressed beam is based on the solution of the equilibrium differential equations describing its bending. We obtain the adequate number of the homogenous algebraic equations by discussing boundary conditions. Using the condition that the system has an untrivial solution we obtain the equilibrium equation of the deformed form and its solutions are the values of the critical pressure forces during the buckling.

The bending theory of thin homogenous beams is established upon the Bernoulli's hypothesis of a perpendicular cross section, which during the deformation rotates as a stiff set and stays perpendicular to the beam central bending axis. This is a cinematic hypothesis; it represents the variations of the point displacements along the beam thickness and it is independent of the material properties. The normal stress is linearly

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graded along the height of cross section with the zero value on the central axis. This means that the edge fibers are totally used, and going to central axis fibers participation in bending action is becoming trivial. This resulted in appearance of sandwich constructions with two outer layers made of solid materials set at some distance and connected with ribbed middle layer made of the same material or with the middle-space filled with less solid material which provide corporate construction action. Calculation of the constructions with the solid middle layer is the same as the calculation of thin homogenous beams, and the calculation of the constructions with the middle layer made of less solid material is in accordance with the broken line hypothesis [1], [2], Fig. 1.

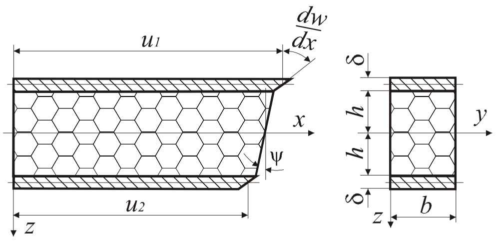

Fig. 1. Sandwich beam

Sandwich beam with two outer layers of small thickness δ and the middle layer of *2h* height made of material with trivial stiffness comparing to the stiffness of outer layers material, was analyzed in this article. Outer layers deform according to the Bernoulli's hypothesis, and the cross section of the middle layer rotates as stiff set and it doesn't need to be perpendicular to bended central beam axis.

2. Component Displacements and Deformations

If a beam cross section deforms into a broken line as shown in Fig. 1, the component displacements of the cross section random point will be:

- for the upper layer, $-(h+\delta) \le z \le -h$,

$$u_g(x,z) = u_1(x) - \left(z + h + \frac{\delta}{2}\right)\frac{dw}{dx} \ , \ w_g(x) = w(x) \ , \tag{1}$$

- for the bottom layer, $h \le z \le (h+\delta)$,

$$u_d(x,z) = u_2(x) - \left(z - h - \frac{\delta}{2}\right)\frac{dw}{dx} \ , \ w_d(x) = w(x) \ , \tag{2}$$

- for the middle layer, $-h \le z \le h$,

$$u_s(x,z) = u_\alpha(x) - \frac{z}{h}\left[u_\beta(x) - \frac{\delta}{2}\frac{dw}{dx}\right] \ , \ w_s(x) = w(x) \ , \tag{3}$$

here

$$u_\alpha = (u_1 + u_2)/2 \ , \ u_\beta = (u_1 - u_2)/2 \ . \tag{4}$$

Components of deformation, different from zero, were calculated by the known formulas of the elasticity theory

$$\varepsilon_x = \frac{\partial u}{\partial x}, \ \ \gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \ . \tag{5}$$

3. Forces and Flexural Moments

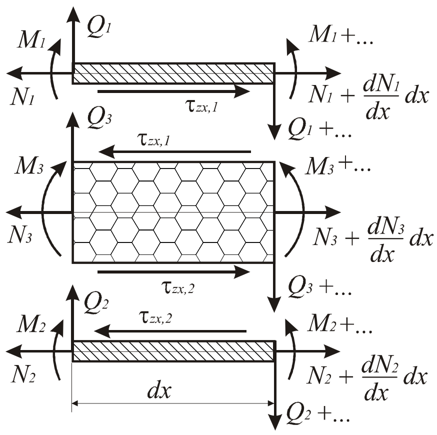

Fig. 2. Positive forces and moments in sandwich beam cross section

In outer beam layers normal stress $\sigma_x = E\varepsilon_x$ and shear stress $\tau_{xz} = G\gamma_{xz}$ will appear, and in the middle layer only the shear stress τ_{xz} . In this formulas E - is the elasticity modulus, and G - is the shearing modulus of the beam material. By integration of stresses and its moments for beam cross section central axis, along the height of the appropriate layer, we obtain forces and moments in each beam layer, Fig 2. Total forces and moments of the sandwich package we calculate by summing of the adequate forces and moments for each layer. That forces and moments can be expressed, using formulas (1) to (4), by components of displacements in the next form:

$$N_x = 2B\frac{du_\alpha}{dx}, \tag{6}$$

$$M_x = -2D\frac{d^2w}{dx^2} - 2B\left(h+\frac{\delta}{2}\right)\frac{du_\beta}{dx}, \tag{7}$$

$$Q_x = -2D\frac{d^3w}{dx^3} - 2B\left(h+\frac{\delta}{2}\right)\frac{d^2u_\beta}{dx^2}, \tag{8}$$

here $B = Eb\delta$ - is axial, and $D = Eb\delta^3/12$ - is flexural rigidity of the beam outer layers.

4. Differential Stability Equations and Contour Conditions

Differential stability equations are obtained from the static equilibrium condition for forces which appear in sections of beam elements, Fig. 2. Those equations, expressed by the components of displacements u_α, u_β and w, are reduced to the following form:

$$\frac{d^2u_\alpha}{dx^2} = 0, \tag{9}$$

$$\frac{Bh}{bG_3}\frac{d^2u_\beta}{dx^2} - u_\beta + \left(h+\frac{\delta}{2}\right)\frac{dw}{dx} = 0, \tag{10}$$

$$2D\frac{d^4w}{dx^4} + 2B\left(h+\frac{\delta}{2}\right)\frac{d^3u_\beta}{dx^3} + N_x\frac{d^2w}{dx^2} = 0. \tag{11}$$

here G_3 - is the shear modulus for the middle layer material. The equation (9) is independent of the equations (10) and (11) and it has a trivial solution with respect to u_α, so the problem of the beam stability adds up to solving the system of two differential equations (10) and (11) for unknown functions $u_\beta(x)$ and $w(x)$.

System of equations (10) and (11) can be reduced to one equation, if we introduce new displacement function $\chi(x)$ that it is:

$$w(x)=\chi(x)-\frac{Bh}{G_3 b}\frac{d^2\chi}{dx^2}\,,\quad u_\beta(x)=\left(h+\frac{\delta}{2}\right)\frac{d\chi}{dx}\,. \tag{12}$$

Then the equation (10) will be identically satisfied, and the equation (11) is reduced to

$$\left[2D+2B\left(h+\frac{\delta}{2}\right)^2\right]\frac{d^4\chi}{dx^4}-2D\frac{Bh}{G_3 b}\frac{d^6\chi}{dx^6}+N_x\left(\frac{d^2\chi}{dx^2}-\frac{Bh}{G_3 b}\frac{d^4\chi}{dx^4}\right)=0\,. \tag{13}$$

In this way resolving of the stability problems adds up to determining the solution $\chi(x)$ of the differential equation (13).

In terms of solving a sandwich beam stability problem completely it is necessary to determine the contour conditions which displacement function $\chi(x)$, forces and moments should satisfy. Those conditions, as well as the differential stability equations, can be derived according to the energetic method, using the possible displacement principal. As it is known, according to this principal, the virtual work of all external and internal forces of an elastic system in equilibrium equals zero. So for the beam of the length l we can write:

$$\delta A_e+\delta A_i=0\,, \tag{14}$$

here $\delta A_e=\delta\left(\frac{1}{2}\int_0^l N_x\left(\frac{dw}{dx}\right)^2 dx\right)$ - is the work of active external forces, and

δA_i - is the work of internal elastic forces during the transfer from one considered equilibrium of bended form to the other, very similar to the previous form. For a sandwich beam loaded with the force N_x , equation (14), in virtual displacements δw, δu_α, δu_β and $\delta(dw/dx)$, is:

$$\begin{aligned}
&\int_0^l\left\{-2B\frac{d^2u_\alpha}{dx^2}\delta u_\alpha-\left[2B\frac{d^2u_\beta}{dx^2}+\frac{2G_3 b}{h}\left(-u_\beta+\left(h+\frac{\delta}{2}\right)\frac{dw}{dx}\right)\right]\delta u_\beta\right\}dx+\\
&+\int_0^l\left\{\left[2D\frac{d^4w}{dx^4}+2B\left(h+\frac{\delta}{2}\right)\frac{d^3u_\beta}{dx^3}-N_x\frac{d^2w}{dx^2}\right]\delta w\right\}dx+\\
&+\left\{\left[-2D\frac{d^3w}{dx^3}-2B\left(h+\frac{\delta}{2}\right)\frac{d^2u_\beta}{dx^2}+N_x\frac{dw}{dx}\right]\delta w\right\}_{x=0}^{x=l}+\\
&+\left\{2D\frac{d^2w}{dx^2}\delta\left(\frac{dw}{dx}\right)+2B\frac{du_\alpha}{dx}\delta u_\alpha+2B\frac{du_\beta}{dx}\delta u_\beta\right\}_{x=0}^{x=l}=0\,.
\end{aligned} \tag{15}$$

The equation (15) will be satisfied if the expressions being integrated which are multiplied by the virtual displacements δu_α, δu_β, δw are equal to zero, which corresponds to the system of differential equations (9), (10) and (11). Last two components define the needed conditions on the beam ends $x = 0$ and $x = l$, which must be satisfied by demanded function $\chi(x)$, or forces and moments in those sections. As we can see, we need to know three contour conditions in case of the sandwich beam, different from the homogeneous beam in which case we can give two contour conditions. Contour conditions can be expressed with displacement components or appropriate transversal forces and moments.

For freely leaned end of the beam variations of the vertical displacement equals zero ($\delta w=0$), and the terms δu_β and $\delta(dw/dx)$ have random values, and according to the equation (15), the contour conditions for this relation will be:

$$w = 0,\ \frac{d^2 w}{dx^2} = 0\,,\ \frac{du_\beta}{dx} = 0\,. \tag{16}$$

For the constraint end of the beam, the following geometric conditions state:

$$w = 0,\ \frac{dw}{dx} = 0\,,\ u_\beta = 0, \tag{17}$$

so the variations of this term at that end equals zero, and the equation (15) is satisfied.

On the free end of the beam the variations δu_β and $\delta(dw/dx)$ have random values, so according to the equation (15) on this end, it must be:

$$-2D\frac{d^3 w}{dx^3} - 2B\left(h + \frac{\delta}{2}\right)\frac{d^2 u_\beta}{dx^2} + N_x\frac{dw}{dx} = 0\,,\ \frac{du_\beta}{dx} = 0\,,\ \frac{d^2 w}{dx^2} = 0\,. \tag{18}$$

5. Numerical results and conclusions

Calculation of the differential stability equations of the beam is shown on the basic - Euler's cases of buckling of the beam having the length l, loaded with the pressure force $N_x = -N$, Fig. 3. If we introduce dimensionless coordinate

$$\xi = \frac{\pi}{l}x \tag{19}$$

the differential stability equation (13) reduces to

$$\frac{d^4\chi}{d\xi^4} - r\,k\frac{d^6\chi}{d\xi^6} + \varphi\left(\frac{d^2\chi}{d\xi^2} - k\frac{d^4\chi}{d\xi^4}\right) = 0\,, \tag{20}$$

here φ - is the axial pressure force coefficient, k- is the shear coefficient

$$\varphi = \frac{N\,l^2}{\left[2D + 2B\left(h + \frac{\delta}{2}\right)^2\right]\pi^2}\,,\quad k = \frac{Bh}{G_3 b}\frac{\pi^2}{l^2}\,, \tag{21}$$

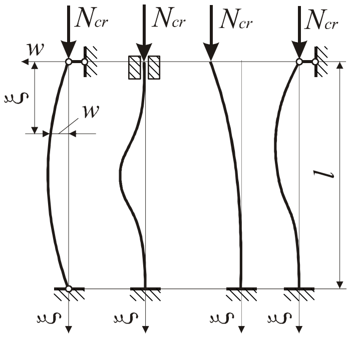

Fig. 3. Critical pressure force for fore basic cases of a beam ends leaning

and r - is the dimensionless geometric coefficient

$$r = \frac{\delta^2}{\delta^2 + 12\left(h + \frac{\delta}{2}\right)^2}. \tag{22}$$

Characteristic equation for the differential equation (16), for $\chi(\xi) = e^{\sqrt{s}\xi}$, is

$$s[r\,k\,s^2 + \varphi(k\,s - 1) - s] = 0. \tag{23}$$

Equation (23) has one solution equal to zero and two real solutions with different sign. If we assume the solutions of the characteristic equation in the following form

$$s_1 = -\lambda^2,\ s_2 = \nu^2, \tag{24}$$

then

$$\varphi = \frac{\lambda^2(1 + rk\lambda^2)}{1 + k\lambda^2},\quad \nu^2 = \frac{1 + rk\lambda^2}{rk\,(1 + k\lambda^2)}. \tag{25}$$

The value of the characteristic equation solution λ depends on the conditions of the beam leaning. For the beam freely leaned on its ends, according to (16), (12) and (19), the boundary conditions that the displacement function $\chi(\xi)$ has to satisfy are:

$$\chi = 0,\quad \frac{d^2\chi}{d\xi^2} = 0,\quad \frac{d^4\chi}{d\xi^4} = 0,\ \text{for } \xi = 0 \text{ and } \xi = \pi. \tag{26}$$

If we try to find a common solution of differential equation (20) in the form

$$\chi(\xi) = C_1\sin(\lambda\xi) + C_2\cos(\lambda\xi) + C_3\,\mathrm{sh}(\nu\xi) + C_4\,\mathrm{ch}(\nu\xi) + C_5\xi + C_6 \tag{27}$$

and put it in the boundary conditions (26) we obtain the following system of algebraic equations in unknown constants C_i, i=1..6:

$$\begin{gathered}
C_2 + C_4 + C_6 = 0,\\
-C_2\lambda^2 + C_4\nu^2 = 0,\\
C_2\lambda^4 + C_4\nu^4 = 0,\\
C_1\sin(\lambda\pi) + C_2\cos(\lambda\pi) + C_3\,\mathrm{sh}(\nu\pi) + C_4\,\mathrm{ch}(\nu\pi) + C_5\pi + C_6 = 0,\\
-\lambda^2(C_1\sin(\lambda\pi) + C_2\cos(\lambda\pi)) + \nu^2(C_3\,\mathrm{sh}(\nu\pi) + C_4\,\mathrm{ch}(\nu\pi)) = 0,\\
\lambda^4(C_1\sin(\lambda\pi) + C_2\cos(\lambda\pi)) + \nu^4(C_3\,\mathrm{sh}(\nu\pi) + C_4\,\mathrm{ch}(\nu\pi)) = 0
\end{gathered} \tag{28}$$

Since the untrivial solution of the system of homogeneous algebraic equations is possible only when the system determinant, formed of the coefficients multiplying the unknowns, is equals to zero, that the condition results in the stability equation, the solutions of which correspond to the critical loads. The condition under which the system (28) has untrivial solutions reduces to the equation:

$$\lambda^4\nu^4\pi\sin(\lambda\pi)\ \mathrm{sh}(\nu\pi)(\lambda^4+2\lambda^2\nu^2+\nu^4)=0\,,$$

It is satisfied for $\lambda = 0,1,2,3,...$, and the minimal value of the critical load is obtained for $\lambda = 1$. The axial force coefficient (25), calculated for the obtained value of the solution λ, is then:

$$\varphi=\frac{1+rk}{1+k}\,. \tag{29}$$

For the beam whose ends are constrained by the contour conditions, according to (17), (12) and (19) it will be

$$\chi-k\frac{d^2\chi}{d\xi^2}=0\,,\quad \frac{d\chi}{d\xi}=0\,,\quad \frac{d^3\chi}{d\xi^3}=0\,,\ \text{for for } \xi=0 \text{ and } \xi=\pi. \tag{30}$$

These contour conditions are reduced to the system of algebraic equations in unknown constants C_i, $i = 1..6$

$$\begin{gathered}
C_2(1+k\lambda^2)+C_4(1-k\nu^2)+C_6=0\,,\\
C_1\lambda+C_3\nu+C_5=0\,,\\
-C_1\lambda^3+C_3\nu^3=0\,,\\
C_1(1+k\lambda^2)\sin(\lambda\pi)+C_2(1+k\lambda^2)\cos(\lambda\pi)+\\
+C_3(1-k\nu^2)sh(\nu\pi)+C_4(1-k\nu^2)ch(\nu\pi)+C_5\pi+C_6=0,\\
C_1\lambda\cos(\lambda\pi)-C_2\lambda\sin(\lambda\pi)+C_3\nu\,\mathrm{ch}(\nu\pi)+C_4\nu\,\mathrm{sh}(\nu\pi)+C_5=0\,,\\
-C_1\lambda^3\cos(\lambda\pi)+C_2\lambda^3\sin(\lambda\pi)+C_3\nu^3\,\mathrm{ch}(\nu\pi)+C_4\nu^3\,\mathrm{sh}(\nu\pi)=0\,.
\end{gathered} \tag{31}$$

The condition under which the system (31) has untrivial solutions results from the equation

$$\sin\left(\frac{\lambda\pi}{2}\right)=0\,, \tag{32}$$

and the smallest value different from zero, for which the equation (32) is satisfied, is $\lambda = 2$. So the axial force coefficient is:

$$\varphi=\frac{4(1+4rk)}{1+4k}\,. \tag{33}$$

In the third stability case, when one end of the beam is constrained and the other is free, contour conditions (17) and (18) are reduced to the following form:

$$\begin{gathered}
\chi-k\frac{d^2\chi}{d\xi^2}=0\,,\quad \frac{d\chi}{d\xi}=0\,,\quad \frac{d^3\chi}{d\xi^3}=0\,,\ \text{for } \xi=0 \text{ and}\\
\frac{d^2\chi}{d\xi^2}=0\,,\quad \frac{d^4\chi}{d\xi^4}=0\,,\\
rk\frac{d^5\chi}{d\xi^5}-\frac{d^3\chi}{d\xi^3}-\varphi\left(\frac{d\chi}{d\xi}-k\frac{d^3\chi}{d\xi^3}\right)=0\,,\ \text{for } \xi=\pi.
\end{gathered} \tag{34}$$

Algebraic equations obtained from the condition (34) are:

$$
\begin{gathered}
C_2(1+k\lambda^2)+C_4(1-k\nu^2)+C_6=0\,,\\
C_1\lambda+C_3\nu+C_5=0\,,\\
-C_1\lambda^3+C_3\nu^3=0\,,\\
-C_1\lambda^2\sin(\lambda\pi)-C_2\lambda^2\cos(\lambda\pi)+C_3\nu^2\,\mathrm{sh}(\nu\pi)+C_4\nu^2\,\mathrm{ch}(\nu\pi)=0\,,\\
C_1\lambda^4\sin(\lambda\pi)+C_2\lambda^4\cos(\lambda\pi)+C_3\nu^4\,\mathrm{sh}(\nu\pi)+C_4\nu^4\,\mathrm{ch}(\nu\pi)=0\,,\\
C_1(\mathrm{rk}\,\lambda^5+\lambda^3-\varphi\lambda-\varphi k\lambda^3)\cos(\lambda\pi)-\\
-C_3(\mathrm{rk}\,\lambda^5+\lambda^3-\varphi\lambda-\varphi k\lambda^3)\sin(\lambda\pi)+\\
+C_3(\mathrm{rk}\,\nu^5-\nu^3-\varphi\nu+\varphi k\nu^3)\,\mathrm{ch}(\nu\pi)+\\
+C_4(\mathrm{rk}\,\nu^5-\nu^3-\varphi\nu+\varphi k\nu^3)\,\mathrm{sh}(\nu\pi)-C_5\pi=0\,.
\end{gathered}
\tag{35}
$$

The condition under which the equations (35) have untrivial solutions in constants C_i, $i = 1..6$, is reduced to the equation $\cos(\lambda\pi) = 0$, and the smallest value of the solution for which it is satisfied is $\lambda = 1/2$. According to (25), the smallest value of the axial force coefficient in this case of the beam stability is:

$$\varphi=\frac{4+rk}{16+4k}\,. \tag{36}$$

For the beam which is constrained on one end and freely leaned on the other contour, the conditions (16) and (17), expressed in displacement component $\chi(\xi)$, are:

$$\chi-k\frac{d^2\chi}{d\xi^2}=0\,,\quad \frac{d\chi}{d\xi}=0\,,\quad \frac{d^3\chi}{d\xi^3}=0\,,\ \text{ for } \xi=0 \text{ and}$$

$$\chi=0,\qquad \frac{d^2\chi}{d\xi^2}=0,\qquad \frac{d^4\chi}{d\xi^4}=0\,,\ \text{ for } \xi=\pi. \tag{37}$$

According to (27), these conditions are reduced to the system of algebraic equations in unknown constants C_i, $i = 1..6$:

$$
\begin{gathered}
C_2(1+k\lambda^2)+C_4(1-k\nu^2)+C_6=0\,,\\
C_1\lambda+C_3\nu+C_5=0\,,\\
-C_1\lambda^3+C_3\nu^3=0\,,\\
C_1\sin(\lambda\pi)+C_2\cos(\lambda\pi)+C_3\,\mathrm{sh}(\nu\pi)+C_4\,\mathrm{ch}(\nu\pi)+C_5\pi+C_6=0\,,\\
-\lambda^2(C_1\sin(\lambda\pi)+C_2\cos(\lambda\pi))+\nu^2(C_3\,\mathrm{sh}(\nu\pi)+C_4\,\mathrm{ch}(\nu\pi))=0\,,\\
\lambda^4(C_1\sin(\lambda\pi)+C_2\cos(\lambda\pi))+\nu^4(C_3\,\mathrm{sh}(\nu\pi)+C_4\,\mathrm{ch}(\nu\pi))=0\,.
\end{gathered}
\tag{38}
$$

The system of algebraic equations (38) will have untrivial solutions if the determinant of that system is equal to zero. That condition is reduced to the equation:

$$tg(\lambda\pi)=\frac{\lambda\pi(\lambda^2+\nu^2)}{\nu^2(1+k\lambda^2)}-\frac{\lambda^3(1-k\nu^2)\,th(\nu\pi)}{\nu^3(1+k\lambda^2)}\,. \tag{39}$$

The value of parameter λ can be determined graphically, or through an iterative process, by changing the value of the parameter until the value of the functions on the left and right hand side of the equation (39) become equal to the preset error. Unlike in the previous cases, in the case of the beam constrained on one end and freely leaned on the other, the value of the parameter λ depends on the shear coefficient k and trivially depends on the geometric coefficient r, Fig. 4. The minimal value of the parameter is defined with the formula $\lambda_{min} = 1.43 - 0.105k$.

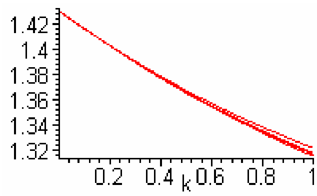

Fig. 4. Parameter $\lambda = \lambda(k)$ change

Introducing the concept of the reduced beam length $l_r = l / \lambda$, like in the case of homogenous beams, the parameter λ represents the slenderness ratio of the beam, so the critical pressure force can be calculated using the following formula

$$N_{cr} = \frac{\pi^2 \left[2D + 2B\left(h + \frac{\delta}{2} \right)^2 \right]}{l_r^2} \left(\frac{1 + r\,k_r}{1 + k_r} \right), \tag{40}$$

here

$$k_r = k \frac{l^2}{l_r^2} \,. \tag{41}$$

Change of axial pressure force coefficient $\varphi = \varphi(k)$.

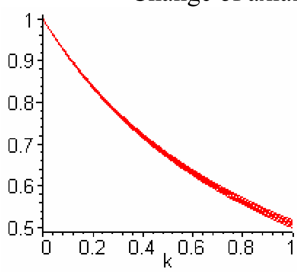

Fig. 5. Ends of the beam freely leaned

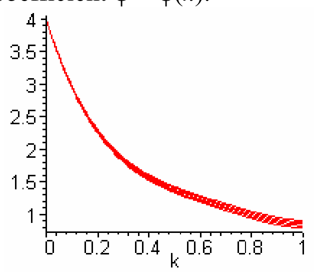

Fig. 6. Ends of the beam constrained

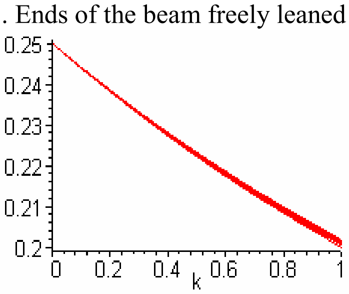

Fig. 7. One end of beam is constrained,
constrained, other is free

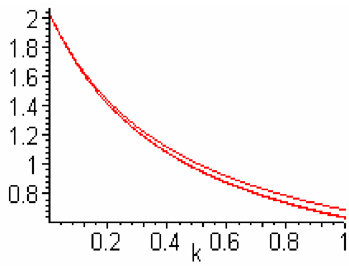

Fig. 8. One end of beam is
and other is freely leaned

Figures 5, 6, 7 and 8 show the change of the axial pressure force coefficient φ as a function of the shear coefficient k and the geometric coefficient r. The value of the shear coefficient is from the interval from zero to one, and the value of the geometrical coefficient r, according to the formula (22), is defined for the ratio between the thickness of the outer layers δ and the thickness of the middle layer h in the interval ($1/20 \le \delta/h \le 1$).

We can see that the influence of the geometrical coefficient r on the critical pressure force value is trivial, so it can be neglected in the process of solving the sandwich beam stability problems. In this way the solving of sandwich beam stability problem is reduced to solving the following differential equation

$$\frac{d^4\chi}{d\xi^4} + \varphi\left(\frac{d^2\chi}{d\xi^2} - k\frac{d^4\chi}{d\xi^4}\right) = 0\,, \tag{42}$$

which rank is for two lower than the rank of the differential equation (20), and lower is the number of the contour conditions that must be satisfied. According to the formula (40), the critical pressure forces are defined by the formula

$$N_{cr} = \frac{\pi^2\left[2D + 2B\left(h + \frac{\delta}{2}\right)^2\right]}{l_r^2(1+k_r)}\,. \tag{43}$$

The shear coefficient k has a practical application for $k < 1$. The value of the critical pressure force coefficient φ, when the shear coefficient is equal to zero corresponds to the values of the same coefficient for the homogenous beam with the flexural rigidity $D + B\,(h + \delta/2)^2$.

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ANALIZA STABILNOSTI TROSLOJNE GREDE PRIMENOM TEORIJE SAVIJANJA TROSLOJNIH KONSTRUKCIJA

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Troslojne konstrukcije imaju dva tanka spoljašna sloja od elastičnog materijala i srednji sloj - jezgro relativno male krutosti u odnosu na krutost spoljašnih slojeva. Proračun ovakvih konstrukcija zasniva se na pretpostavci da se sva tri sloja istovremeno deformišu i imaju zajedničku neutralnu liniju koja se nalazi između spoljašnih slojeva. Ovakvim pristupom proračuna moguće je opisati, sa velikim stepenom tačnosti, kako naponsko i deformabilno stanje konstrukcije tako i lokalne uticaje kod svakog njenog sloja. Polazeći od hipoteze o izlomljenoj liniji poprečnog preseka [1], u radu je pokazan postupak odredjivanja kritične sile pritiska pri izvijanju troslojne grede sastavljene od dva spoljašna tanka elastična sloja iste debljine i srednjeg sloja zanemarljive savojne krutosti u odnosu na savojne krutostii spoljašnih slojeva, slika 1. Statičkom i energijskom metodom izveden je sistem diferencijalnih jednačina ravnoteže, kao i konturni uslovi koje rešenje tog sistema mora da zadovolji.

Ključne reči: *Troslojna greda, kritična sila pritiska, diferencijalne jednačine stabilnosti, konturni uslovi*