FACTA UNIVERSITATIS
Series: **Mechanics, Automatic Control and Robotics** Vol. 4, No 17, 2005, pp. 367 - 372

Letter to Editor

MECHANICAL ROLE OF CRUCIATE LIGAMENTS IN FLEXION AND EXTENSION EXPRESSED GEOMETRICALLY

UDC 531/534:611(045)=20

Swapan Kumar Adhikari

Head of the Institution, Ghusuri Uchcha Madhyamik Vidyalaya (High)
190, 191, Naskar Para Road, Howrah, West Bengal, 711 107
E-mail: swapku_adhi@rediffmail.com
Phone : 91 33 2655-2627 (R); 91 33 2655-5370 (O); 91 33 2109-8177 (M)

Abstract. *The author tries to locate the movements of femur and tibia in flexion and extension and thereby to find curves on which centre of curvature of femur, instantaneous centre of cruciate ligaments and tibial surface move. Also find the geometrical relations among the curves to establish the mathematical stability of knee-joint during its movements.*

Key words: *Centre of curvature, envelope, evolute, four-bar linkage, hinge-motion, pedal-curve, pole, semi-cubical parabola.*

1. INTRODUCTION

Cruciate ligaments are very strong and sited a little posterior to the articular centre. These ligaments are known as cruciate because they cross anteriorly and posteriorly from their tibial attachments.

In full flexion and extension femoral condyles and tibial plateaus move in the sagittal plane during active motion. Of course in terminal stage of extension there is a rotational movement in horizontal plane to a maximum range of about 10^0.

Therefore, knee flexion-extension is not a continuous central hinge-motion like elbow-joint.

In such movements of knee-joint, cruciate ligaments play the effective role by four-joined chain or four-bar linkage system of mechanics. But such a classification is depending on following two conditions.

Received January 18, 2005

(a) The motion should be in one plane i.e., in sagittal plane but having exceptional terminal rotation.

(b) The joints of the four-joined chain must be rigidly connected i.e., cruciate ligaments connecting tibia and femur rigidly.

MATERIALS AND METHODS

In all the deductions and researches I have taken the method of using mathematics on anatomy especially on skeletal structures of bones to have mathematical precision on orthopaedic operations and to demonstrate the co-relations among the structures on mathematical basis.

RESULTS

Cruciate ligaments play a crucial role on mechanical movement of knee-joint and in this paper inter-relation among the structures of femorotibial condyles has been established to show the smooth stability of the knee-joint movements and to concretise it by geometrical illustrations and mathematical deductions.

DISCUSSION

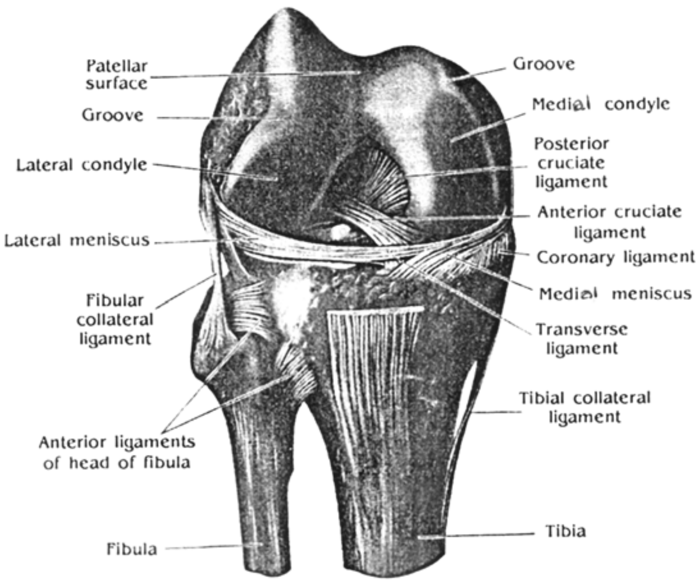


Fig. 1. Right knee-joint in full flexion (anterior aspect of dissection)

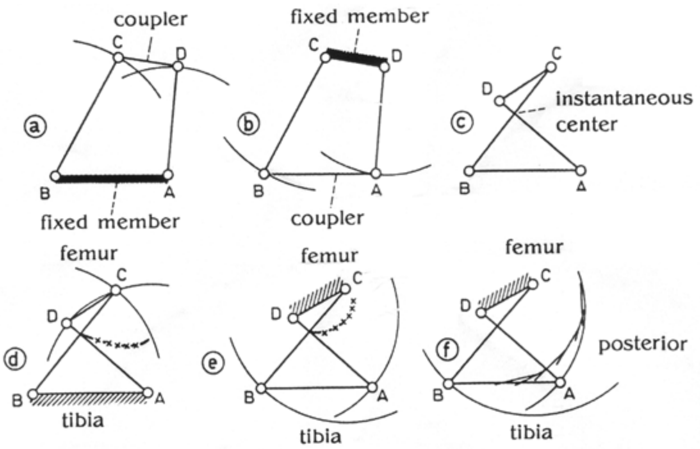


Fig. 2.

ABCD is a closed four-bar joined kinetic chain moving on the plane of the paper.

Fig. 2 (a): AB is the fixed member. All the points on AD and BC are moving on the circular arcs about A and B as centres respectively. CD is the coupler. Middle point of CD moves on a non - circular curve.

Fig. 2 (b): If CD is the fixed member then BA is the coupler and mid-point of BA is also moving on a non-circular curve.

Fig. 2 (c): The crossed closed four-bar chain/four-joined chain represents the anatomic structure of cruciate ligaments where BA = the inter-condylar eminence (attachment surface) of the tibia; BC = the anterior cruciate ligament; AD = the posterior cruciate ligament; DC = the intercondylar fossa (attachment surface) of the femur. The centre of the axis of the knee-joint conforms the crossing point of link BC and AD and it is called 'pole' or 'instantaneous centre'. During knee flexion-extension the pole moves to form pole-curve.

Fig. 2 (d): Locus of pole during movement of femur when tibia is fixed.

Fig. 2 (e): Locus of pole during movement of tibia when femur is fixed.

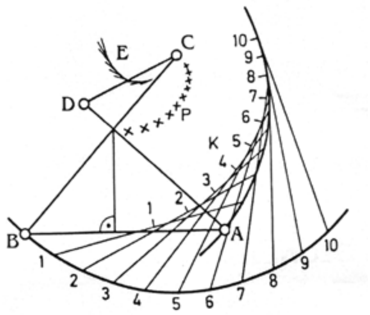
Fig. 2 (f): Coupler envelope curve yields by coupler during the movement of tibia to the fixed femur.

Due to the insertion of the anterior and posterior cruciate ligaments at the tibia, it is represented approximately by the altitude of the curve of the joint surface of the tibial condyles.

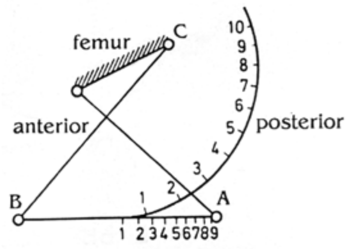
So, during flexion, the locus of the mid-point of the coupler is a curve corresponding to the contour of the femoral condyles [Fig. 1(f)].

All points of the coupler move with an instantaneous velocity directed perpendicularly to the radius from the pole. Since the velocity at the femorotibial contact point corresponds to the direction of the tibial joint surface. Then the point of contact must always lie vertically below the pole.

Therefore, it is possible to find contact points between femoral and tibial condyles during various positions of the flexion geometrically.



a)



b)

Fig. 3.

Fig. 3(a): 10 different positions of the coupler BA has been drawn between the circular arcs BC and AD. The resulting coupler envelope K corresponds to the contour of the femoral condyle whereas 10 corresponding poles produce the pole curve P. The contact points of the coupler determined by the normals from the appropriate poles. The continuation of these normals yields the evolute E (locus of the centre of curvature of the femoral condyle).

Fig. 3(b): The distance along the coupler to the contact point of each of the different positions of flexion marked on BA. The fact that as flexion increases, these points become more close to group posteriorly and it show the predominance of sliding motion in squatting.

The curve has drawn through the contact points whereas contact points are determined by the perpendiculars dropped from the poles on the line connecting the different positions in succession due to movements of tibia.

Now for mathematical deduction of the curves we take

(A) Pole-Curve P which is the envelope (a curve: tangent to each member of the family of curves) of the lines BC and AD i.e., locus of the point of intersection of the straight lines or locus of the instantaneous centre / pole.

Let us consider the general equation of the straight line $y = mx + (a/m)$, m being the variable parameter and it is different for two straight lines where $m \neq 0$.

Now, $$mx + (a/m) - y = 0$$

Differentiating with respect to m we have

$$x - (a/m^2) = 0;$$

$$m^2 = a/x;$$

$$m = \pm\sqrt{(a/x)}$$

Substituting these values of m in $mx + (a/m) - y = 0$ we have $\pm\{\sqrt{(a/x)}.x + a/\sqrt{(a/x)}\} - y = 0$;

$$y^2 = 4ax$$

which is the required envelope and it is parabola.

So, pole-curve P is a parabola.

(B) Curve E is the envelope of the normals to the parabola $y^2 = 4ax$ or E is evolute of the parabola.

The equation of the normal to the parabola at any point 'm' is $y = mx - 2am - am^3$ where for envelope we consider 'm' to be the parameter.

Differentiating with respect to m, we have

$$0 = x - 2a - 3am^2;$$

or, $$m^2 = (x - 2a) / (3a).$$

i.e., $$27ay^2 = 4(x - 2a)^3.$$

which is the envelope of the normals to the parabola i.e., evolute of the curve P (i.e., parabola $y^2 = 4ax$) and it is a semi-cubical parabola.

(C) For the curve K straight line BA is moving on circles or family of circles whose centre is on the parabola $y^2 = 4ax$ i.e., Pole-curve P.

Let us consider the equation of family of circles, having centre (α, β), which is also variable. Centre is lying on the parabola $y^2 = 4ax$ where the circle passes through the origin and is given by

$$x^2 + y^2 - 2\alpha x - 2\beta y = 0$$

where $$\beta^2 = 4a\alpha.$$

It is the case of two-parameter family (α, β) where parameters are connected by the relation $\beta^2 = 4a\alpha$.

Now eliminating α in the equation of the circle it becomes

$$x^2 + y^2 - 2 \cdot (\beta^2/4a) \cdot x - 2\beta y = 0 \quad [\text{since } \alpha = \beta^2/4a]$$

$$\text{or, } 2a(x^2 + y^2) - \beta^2 x - 4a\beta y = 0$$

Differentiating with respect to β we have

$$0 - 2\beta x - 4ay = 0$$

or, $$\beta = -2ay/x$$

Now, putting the value of β in the equation $2a(x^2 + y^2) - \beta^2 x - 4a\beta y = 0$; we get

or, $$2a(x^2 + y^2) - (4a^2y^2/x^2) \cdot x + 8a^2 \cdot (y^2/x) = 0$$

$$\text{or, } x(x^2 + y^2) + 2ay^2 = 0$$

This curve is known to be the pedal-curve of the parabola $y^2 = 4ax$ i.e., the curve is traced by the foot of the perpendicular from the centres to the line BA.

Thus knee flexion occurs in accordance with the law of kinematics of the crossed four-joined chain under the geometrically anatomical arrangement of cruciate ligaments whereas flexion corresponds to the pole and the contour of joints, the lateral ligaments, the contact points and the centre of curvature are secondary to the kinematics of the cruciate ligaments.

Therefore, from the above deductions we can conclude that movements of knee in flexion and extension operated by cruciate ligaments are based on mathematical relation and it is operated on the basis of movement of instantaneous centre / pole i.e., cross point of anterior and posterior cruciate ligaments.

CONCLUSION

So, knee-joint is stable geometrically as well as mathematically and this ability will give us direction of movement of knee in flexion-extension after operation in relation to surface of the condyles and it will help us to locate the mode of exercise after operation in the knee-joint.

REFERENCES

1. Adhikari., Swapan Kumar: The Role of Mathematics on Human Structure, Dipali Publication, Howrah, West Bengal, India, 711 107 ; 2003.
2. Das., B.C. & Mukherjee., B.N.: Differential Calculus, 47th Edition, U.N.Dhur & Sons Private Ltd., Kolkata, West Bengal, India, 700 073; 2001.
3. Dennis Lawrence., J.: A catalog of Special Plane Curves, Dover Publications, Inc., New York, USA; 1992.
4. Hastings., D.E.: The Knee: Ligament and Articular Cartilage Injuries (Progress in Orthopaedic Surgery 3), Springer-Verlag, New York, USA; 1978.

GEOMETRIJSKI OPIS MEHANIČKE ULOGE UKRŠTENIH LIGAMENATA U SAVIJANJU I OPRUŽANJU

Swapan Kumar Adhikari

Autor je pokušao da lokalizuje pokrete golenjače i butne kosti u savijanju i opružanju, kao i da nadje centre krivih butne kosti, kao i trenutni centar ukrštenog ligamenta i kretanje gornje površine golenjače. Takodje da nadje geometrijska odnose izmedju krivih radi uspostavljanja matematičkih kriterijuma stabilnosti kolenog zgloba u njegovom kretanju.

Ključne reči: *Centar krive, anvelopa, evoluta, zglobni četvorougao, zglobno kretanje, kriva zglobne površi, pol, semi kubna parabola.*