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MOTION OF THE BODY IN AN INHOMOGENEOUS GRAVITATIONAL FIELD

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Milutin Marjanov

Faculty of Forestry, Belgrade, Serbia

Abstract. *Motion of an arbitrary shaped body in the gravitational field was studied in this work. Expressions for the components of the gravitational load, together with their potential were derived, and the energy extrema conditions at perihelion were established. In the cases of the closed orbits, fulfillment of these conditions at apsides produces periodic rotational motion. At the end, differential equations of motions were derived, characteristics of such motions were discussed and possibilities of obtaining solutions were investigated*.

Key words: *inhomogeneous gravitational field, gravitational potential, rotational motion: periodicity conditions*

INTRODUCTION

Rotational, together with orbital motion of the arbitrary shaped body in an inhomogeneous gravitational field was examined in this work. Gravitational load and its potential were derived. After that, the energy extrema conditions at apsides were established. In the cases of closed orbits, fulfillment of these conditions produces periodic motion. Differential equations of translational and rotational motions of the body were derived. Some conclusions about characteristics of the motion in an inhomogeneous gravitational field were made. It was followed, at the end, by the discussion about possibilities of finding solutions of the obtained nonlinear dynamical system.

MODEL OF THE FIELD

We consider an arbitrary shaped body of the mass m, moving in the field of the dominant center of gravitation of the mass m^*. Assumption was made that the body rotates around an axis arbitrary positioned with respect to the body, as well as to the orbital plane, that $m^* >> m$ and that dimensions of the body are small compared to the distance between the body and the center of gravitation.

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Study of the rotational, together with translational motion of the body in a gravitational field is only possible if the model of an inhomogeneous field was adopted. Such a model implies that the elementary gravitational forces acting on the body's particles converge toward the center of gravitation. Consequence of such a supposition is the absence of the Archimedean center of gravity, as well as the existence of the gravitational moment with respect to the center of mass C of the body [10]. If that is the case, the gravitational force does not depend on two attracting masses and distance between them only, but on the moments of masses of the higher order, as well as on the angles of relative rotation, also.

Note that a very important simplification was retained here: dominant center of gravitation was identified with its center of mass C^*. It means that inertial radial symmetry of this body was also assumed.

THREE FRAMES OF REFERENCE

As usually, orbit of the body's center of mass C is defined in the cylindrical reference frame R, Ω, z (Fig.1), with the origin at the center of gravitation C^* and anticlockwise positive direction of the angular variable Ω, beginning at C^*P.

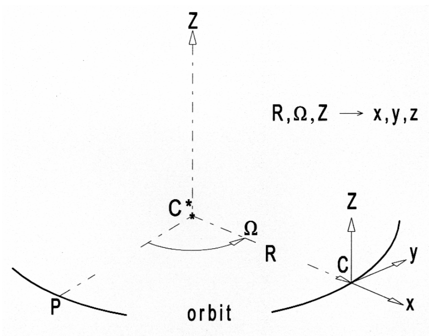
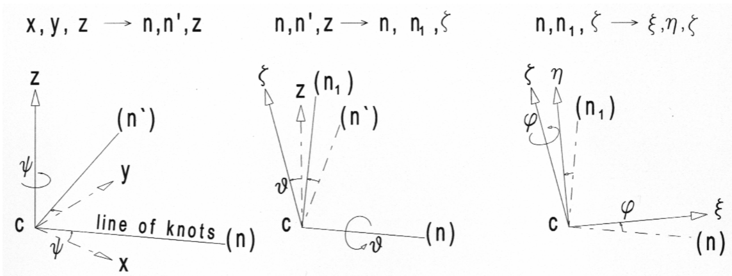


Fig. 1. Cylindrical Reference Frame.

Mass center of the body C was chosen to be the origin of two moving frames of reference x,y,z and ξ,η,ζ. The first one (translational, along the radial direction displaced cylindrical frame) is related to the geometry of the orbit. The second one is related to the geometry of mass: $C\xi$ is directed along the principal axis (1), $C\eta$ along axis (2) and $C\zeta$ along axis (3) of the ellipsoid of inertia (it was convenient to take here that $I_1 \le I_2 \le I_3$).

Position of the second frame with respect to the first one is defined by the Euler's angles of relative rotation φ, ψ and ϑ, that is, by the angles of spin, precession and nutation (Fig. 2. a, b, c).



a) Precession b) Nutation c) Spin

Fig. 2. Euler's Angles of Relative Rotation.

Thus, if we introduce notations

$$x_i = x, y, z \text{ and } \xi_i = \xi, \eta, \zeta \ (i = 1, 2, 3)$$

these coordinates will be related by the transformation tensor $\alpha_{ij}(\varphi, \psi, \vartheta)$ in the following way

$$\xi_i = \alpha_{ij} x_j \quad (i, j = 1,2,3) \tag{1}$$

and, since α_{ij} is orthogonal, meaning that

$$\alpha_{ik}\alpha_{il} = \delta_{kl} \ (i, k, l = 1,2,3) ,$$

where the symbol δ_{kl} denotes the Kronecker's delta, the inverse transformation in the form

$$x_i = \alpha_{ji}\xi_j \quad (i, j = 1,2,3) \tag{2}$$

is also valid.

Elements of the transformation tensor are given by the following expressions

$$\begin{aligned}
\alpha_{11} &= \cos\varphi\cos\psi - \sin\varphi\sin\psi\cos\vartheta \\
\alpha_{21} &= -\sin\varphi\cos\psi - \cos\varphi\sin\psi\cos\vartheta \\
\alpha_{31} &= \sin\psi\sin\vartheta \\[1em]
\alpha_{12} &= \cos\varphi\sin\psi + \sin\varphi\cos\psi\cos\vartheta \\
\alpha_{22} &= -\sin\varphi\sin\psi + \cos\varphi\cos\psi\cos\vartheta \\
\alpha_{32} &= -\cos\psi\sin\vartheta \\[1em]
\alpha_{13} &= \sin\varphi\sin\vartheta \\
\alpha_{23} &= \cos\varphi\sin\vartheta \\
\alpha_{33} &= \cos\vartheta
\end{aligned} \tag{3}$$

COMPONENTS OF THE GRAVITATIONAL LOAD AND THEIR POTENTIAL

Elementary gravitational force acting on the body's particle is represented on the Figure 3.

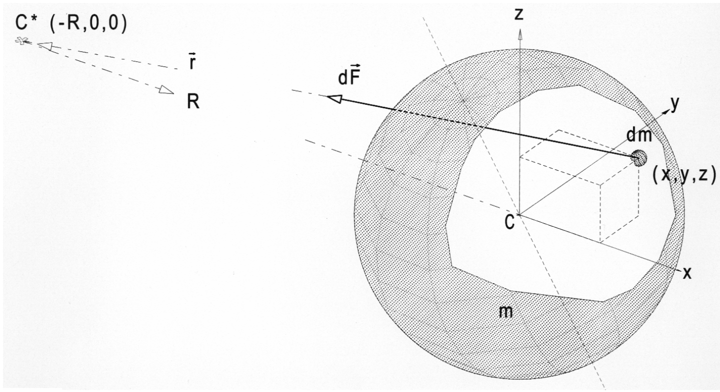


Fig. 3. Elementary Gravitational Force.

According to the Newton's law of gravitation, this force is equal

$$d\vec{F}(dX,dY,dZ)=\frac{Gm^{*}dm}{r^{2}}\frac{\vec{r}}{|\vec{r}|}\,.$$

Vector $\vec{r}$ in this expression denotes position of the center of gravitation with respect to the body's particle in the $Cxyz$ reference frame

$$\vec{r}=\vec{r}(-R-x,-y,-z)\,.$$

Having in mind that x, y, $z \ll R$ (if x, y, z belong to the body), we shall obtain, after expansion of the components of the force into the Newton's binomial series in which we retain the terms up to the exponent two, integration over the mass of the body and introduction of the transformations (2), the following components of the gravitational load in an inhomogeneous gravitational field

$$X=-\frac{Gmm^{*}}{R^{2}}-\frac{9Gm^{*}}{4R^{4}}(I_2-I_1)(\cos 2\varphi\cos 2\psi-\sin 2\varphi\sin 2\psi\cos\vartheta+ \\ +\cos 2\varphi\sin^{2}\psi\sin^{2}\vartheta)-\frac{3Gm^{*}}{4R^{4}}(2I_3-I_2-I_1)(1-3\sin^{2}\psi\sin^{2}\vartheta) \tag{4}$$

$$Y=\frac{3Gm^{*}}{2R^{4}}\Big[\,(I_2-I_1)(\cos 2\varphi\sin 2\psi+\sin 2\varphi\cos 2\psi\cos\vartheta- \\ -\frac{1}{2}\cos 2\varphi\sin 2\psi\sin^{2}\vartheta)+\frac{2I_3-I_2-I_1}{2}\sin 2\psi\sin^{2}\vartheta\,\Big] \tag{5}$$

$$Z = \frac{3Gm^*}{2R^4}\Big[\,(I_2 - I_1)(\sin 2\varphi \cos\psi \sin\vartheta + \frac{1}{2}\cos 2\varphi \sin\psi \sin 2\vartheta) - \\ - \frac{2I_3 - I_2 - I_1}{2}\sin\psi \sin 2\vartheta\,\Big] \tag{6}$$

$I_i\ (i = 1,2,3)$ are the principal moments of inertia. As said, it was assumed here that $I_1 \le I_2 \le I_3$.

As seen, components of the gravitational load depend on the masses m and m^*, on the principal moments of inertia of the body and on four variables: radial coordinate of the orbit and three Euler's angles of the relative rotation. Implicitly, of course, they depend on the time, as well.

Concerning the moment of this force with respect to the center of mass, it has two components in the same frame of reference

$$\vec{M}^C = \vec{M}^C(0, M_y, M_z)\,, \tag{7}$$

where $M_y = RZ$ and $M_z = -RY$.

Gravitational potential is a four-dimensional function, also:

$$U = -\frac{Gmm^*}{R} - \frac{3Gm^*}{4R^3}(I_2 - I_1)(\cos 2\varphi \cos 2\psi - \sin 2\varphi \sin 2\psi \cos\vartheta + \\ + \cos 2\varphi \sin^2\psi \sin^2\vartheta) - \frac{Gm^*}{4R^3}(2I_3 - I_2 - I_1)(1 - 3\sin^2\psi \sin^2\vartheta). \tag{8}$$

Negative partial derivative of the potential with respect to the radial coordinate is the attracting component of the gravitational force, while the negative partial derivatives, with respect to the angular variables represent components of the gravitational moment in the corresponding directions ζ, z and n (line of knots), respectively (Fig. 2):

$$-\frac{\partial U}{\partial R} = X\,, \qquad -\frac{\partial U}{\partial \varphi} = M_\zeta = \alpha_{32}M_y + \alpha_{33}M_z\,,$$

$$-\frac{\partial U}{\partial \psi} = M_z \text{ and } -\frac{\partial U}{\partial \vartheta} = M_n = M_y \sin\psi \quad \text{(see Fig. 2a).}$$

ENERGY EXTREMA CONDITIONS

Let us say something about motion of the body in a closed and stable orbit.

It was shown in the work [11] that in the case of such a motion, energy extrema conditions have to be satisfied at perihelion and aphelion. Potential energy must be minimal and kinetic energy maximal at perihelion, while the reversed situation has to appear at aphelion.

As seen, gravitational potential along the orbit depends on the radial coordinate, and on three Euler's angles of relative rotation. Regarding the change of the radial variable, potential is minimal at perihelion, where $R = R_P$ and maximal at aphelion, where $R = R_A$ $R = R_A$. For the rest necessary extrema conditions, they have the form

$$\frac{\partial U(R_P, \varphi, \psi_P, \vartheta_P)}{\partial \varphi} = \frac{\partial U(R_P, \varphi_P, \psi, \vartheta_P)}{\partial \psi} = \frac{\partial U(R_P, \varphi_P, \psi_P, \vartheta)}{\partial \vartheta} = 0 \ , \tag{9}$$

at perihelion and the similar form at aphelion.

Conditions (9) imply that the components M_ζ, M_z and M_n, as well as the vector $\vec{M}^C$ composed of them have to become zeroes at characteristic points of the orbit. Thus, ***the gravitational force must pass through the center of mass of the body,*** that is***, it must lie on the line of apsides at perihelion (Fig. 4) and aphelion***. The same extrema condition was obtained in [11], for the case of planar motion. In accordance with (7), expressions (9) may be written in the form

$$Y_{aps} = Z_{aps} = 0 \text{, as well.} \tag{10}$$

Close inspection of the expressions (5) and (6) exposes the fact that, if the angle of precession Ψ_{aps} is equal zero, for $\varphi_{aps} = \pm k\pi/2$ ($k = 0, 1, 2, \ldots$), conditions (9) are satisfied identically. Hence, we conclude that with regard to the attitude of the body the extrema necessary condition at perihelion and aphelion requires that ***the line of apsides (Cx axis), takes its place in one of the planes of inertial symmetry of the body, that is, in the plane*** ξ, ζ ***or*** η, ζ (Fig. 4).

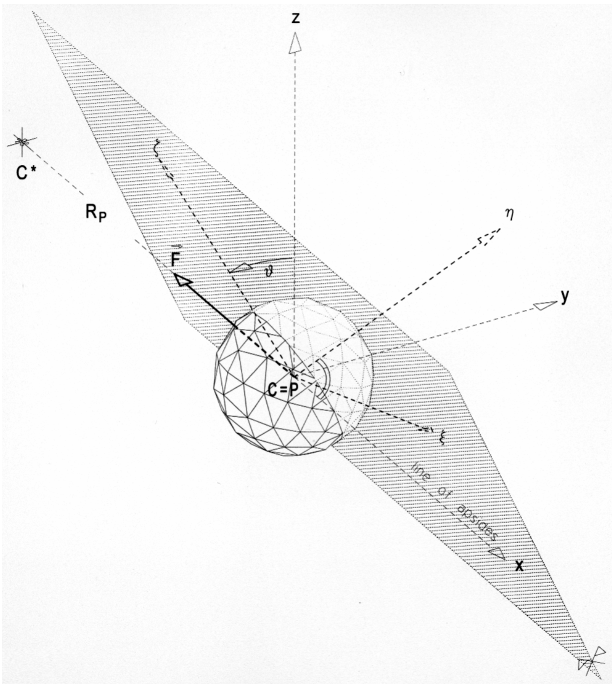


Fig. 4. Energy Extrema Condition at Perihelion

The analogous energy extrema condition obtained for the planar motion in the work [11] requires that the principal axis, corresponding to the minimal moment of inertia (denoted ξ, that is (1), here) becomes coincident with the line of apsides. It was shown in that work, that fulfillment of the extrema conditions at apsides produces periodic and moreover resonant motions: rotational and orbital periods of these bodies become related as the rational fractions.
If $\psi_{aps} \neq 0$, such positioning of the body requires, obviously, $\eta \; or \; \xi \perp x$ at apsides and hence

$$\vec{\mu} \cdot \vec{i} = \alpha_{21} = -\sin\varphi\cos\psi - \cos\varphi\sin\psi\cos\vartheta = 0 \,, \quad \text{or,} \tag{11 a}$$

$$\vec{\lambda} \cdot \vec{i} = \alpha_{11} = \cos\varphi\cos\psi - \sin\varphi\sin\psi\cos\vartheta = 0 \,. \tag{11 b}$$

Fulfillment of the conditions (10), (11 a), or (10), (11 b) at apsides provides periodic motion of the body.

Besides periodicity of three variables defining a stable orbit, such a motion of the body in the gravitational field requires periodicity of three Euler's angles of relative rotation, also. Those six variables are coupled (see the next chapter) and their periods have to be related in the mutual resonance: if T_i ($i = 1,2,...$) are periods of these variables, than all the relations $T_i / T_j \;\; (i \neq j = 1,2,...)$ have to represent rational fractions.

DIFFERENTIAL EQUATIONS OF MOTION

Translational motion of the body is to be described by the differential equations formulated in the cylindrical frame of reference. It may be done easily, because the components of the gravitational force (4), (5) and (6) already have directions of the corresponding coordinates. But formulation of the differential equations of rotation (of the, so cold Euler's equations) requires that all related quantities were represented in the ξ,η,ζ reference frame. First of all, here follow components of the gravitational moment in this frame, obtained by application of the transformation (1) on the expressions (7).

$$\begin{aligned} \vec{M}^C &= \vec{M}^C(M_\xi, M_\eta, M_\zeta) \\ M_\xi &= \alpha_{12}M_y + \alpha_{13}M_z \\ M_\eta &= \alpha_{22}M_y + \alpha_{23}M_z \\ M_\zeta &= \alpha_{32}M_y + \alpha_{33}M_z \end{aligned}$$

$$M_1 = M_\xi = -\frac{3Gm^*}{2R^3}(I_3 - I_2)(\sin\varphi\sin 2\psi\sin\vartheta + \cos\varphi\sin^2\psi\sin 2\vartheta) \tag{12}$$

$$M_2 = M_\eta = -\frac{3Gm^*}{2R^3}(I_3 - I_1)(\cos\varphi\sin 2\psi\sin\vartheta - \sin\varphi\sin^2\psi\sin 2\vartheta) \tag{13}$$

$$M_3 = M_\zeta = -\frac{3Gm^*}{2R^3}(I_2 - I_1)(\sin 2\varphi\cos 2\psi + \cos 2\varphi\sin 2\psi\cos\vartheta + \sin 2\varphi\sin^2\psi\sin^2\vartheta) \tag{14}$$

Concerning angular velocity of the body, it is composed of two components. The first one is orbital and the second one is relative angular velocity

$$\vec{\omega} = \dot{\Omega}\vec{k} + \dot{\varphi}\vec{\nu} + \dot{\psi}\vec{k} + \dot{\vartheta}\vec{n} \,. \tag{15}$$

Components of the absolute angular velocity in the body's reference frame are products of the expression (15) and the unit vectors of the corresponding coordinate axes.

$$\omega_1 = \omega_\xi = \vec{\omega}\cdot\vec{\lambda} = (\dot{\Omega}+\dot{\psi})\vec{k}\cdot\vec{\lambda} + \dot{\vartheta}\vec{n}\cdot\vec{\lambda} = (\dot{\Omega}+\dot{\psi})\sin\varphi\sin\vartheta + \dot{\vartheta}\cos\varphi \tag{16}$$

$$\omega_2 = \omega_\eta = \vec{\omega}\cdot\vec{\mu} = (\dot{\Omega}+\dot{\psi})\vec{k}\cdot\vec{\mu} + \dot{\vartheta}\vec{n}\cdot\vec{\mu} = (\dot{\Omega}+\dot{\psi})\cos\varphi\sin\vartheta - \dot{\vartheta}\sin\varphi \tag{17}$$

$$\omega_3 = \omega_\zeta = \vec{\omega}\cdot\vec{\nu} = (\dot{\Omega}+\dot{\psi})\vec{k}\cdot\vec{\nu} + \dot{\varphi}\vec{\nu}\cdot\vec{\nu} = (\dot{\Omega}+\dot{\psi})\cos\vartheta + \dot{\varphi} \tag{18}$$

It's possible, now, to write down the differential equations of motions of the body in an inhomogeneous gravitational field. Here they follow:

$$\begin{aligned} m(\ddot{R} - R\dot{\Omega}^2) = -\frac{Gmm^*}{R^2} - \frac{9Gm^*}{4R^4}(I_2 - I_1)(\cos 2\varphi\cos 2\psi - \sin 2\varphi\sin 2\psi\cos\vartheta \\ + \cos 2\varphi\sin^2\psi\sin^2\vartheta) - \frac{3Gm^*}{4R^4}(2I_3 - I_2 - I_1)(1 - 3\sin^2\psi\sin^2\vartheta) \end{aligned} \tag{19}$$

$$\begin{aligned} m(R\ddot{\Omega} + 2\dot{R}\dot{\Omega}) = \frac{3Gm^*}{2R^4}[(I_2 - I_1)(\cos 2\varphi\sin 2\psi + \sin 2\varphi\cos 2\psi\cos\vartheta - \\ -\frac{1}{2}\cos 2\varphi\sin 2\psi\sin^2\vartheta) + \frac{2I_3 - I_2 - I_1}{2}\sin 2\psi\sin^2\vartheta] \end{aligned} \tag{20}$$

$$\begin{aligned} m\ddot{z} = \frac{3Gm^*}{2R^4}[(I_2 - I_1)(\sin 2\varphi\cos\psi\sin\vartheta + \frac{1}{2}\cos 2\varphi\sin\psi\sin 2\vartheta) - \\ -\frac{2I_3 - I_2 - I_1}{2}\sin\psi\sin 2\vartheta] \end{aligned} \tag{21}$$

$$I_1\dot{\omega}_\xi + (I_3 - I_2)\omega_\eta\omega_\zeta = -\frac{3Gm^*}{2R^3}(I_3 - I_2)(\sin\varphi\sin 2\psi\sin\vartheta + \cos\varphi\sin^2\psi\sin 2\vartheta) \tag{22}$$

$$I_2\dot{\omega}_\eta - (I_3 - I_1)\omega_\xi\omega_\zeta = -\frac{3Gm^*}{2R^3}(I_3 - I_1)(\cos\varphi\sin 2\psi\sin\vartheta - \sin\varphi\sin^2\psi\sin 2\vartheta) \tag{23}$$

$$\begin{aligned} I_3\dot{\omega}_\zeta + (I_2 - I_1)\omega_\xi\omega_\eta = -\frac{3Gm^*}{2R^3}(I_2 - I_1)(\sin 2\varphi\cos 2\psi + \cos 2\varphi\sin 2\psi\cos\vartheta + \\ + \sin 2\varphi\sin^2\psi\sin^2\vartheta) \end{aligned} \tag{24}$$

This system of the six coupled and nonlinear second-order differential equations describes motion of an arbitrary shaped body in the gravitational field GF(2) [11], [12]. The first three are equations of translation and the second three, equations of rotation.

Close examination of the equations (19) – (24) leeds to some important conclusions concerning motion of the body.

Influence of the (inhomogeneous) field

- Forcing terms appearing on the right hand sides of the equations (19), (20) and (21) produce the ***(three-dimensional) meander course of the mass center path around the corresponding Kepler's orbit.*** Thus, ***the orbit of an arbitrary shaped body's mass center does not lie in one plane.***
- Taking into account that the right hand side of the equation (20) is a periodic function we can conclude that the sectorial velocity $\sigma = \frac{1}{2} R^2 \dot{\Omega}$ ***is not constant in an inhomogeneous gravitational field***, but "almost constant" wavy line, periodic in time (Fig. 5).

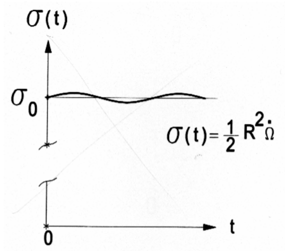


Fig. 5. Sectorial Velocity in an Inhomogeneous Gravitational Field.

- Motion of the body is planar if the spin axis ζ is perpendicular at the orbital plane. Namely, if the angle and the angular velocity of nutation are zeroes $\vartheta = \dot{\vartheta} = 0$, from eq. (16) and (17) follows that ω_ξ and ω_η are zeroes too. That model of motion was studied in the work [11].
- As said, if the orbit is closed and the energy extrema conditions fulfilled at apsides***, motion of the body is periodic***, otherwise it's ***chaotic***.

Let's analyze the influence of the mass distribution of the body on its motion now.

Influence of the mass distribution

- When the body's moments of inertia are radial symmetrical, then $I_1 = I_2 = I_3$ and the motion of its center of mass is described by the classical (Keplerian) equations of motion. If that is the case, ***the absolute angular velocity is a constant vector, indifferent to the influence of gravitation and equal to the "initial" angular velocity.***
- In the case $I_1 = I_2 \neq I_3$ the component ω_ζ ***(spin) of the absolute angular velocity is constant and the equation (24) is homogeneous.***
- In the case $I_1 \neq I_2 = I_3$ ***the corresponding component*** ω_ξ ***is constant and the equation (22) is homogeneous.***
- It's worth noting the impossibility of $\omega_\eta (= \omega_2)$ becoming constant, excepting the case of inertial radial symmetry.

On possibilities of obtaining solution of the system (19) – (24)

If somebody is seeking periodic, or the short term solutions, he may try to apply the perturbation technique. The small parameter $\delta = \max \delta_2$ ($= \frac{I_3}{mR_P^2}$ in this case), already existing on the right hand sides of the equations (and used for ranking of the gravitational fields in the work [11]) may be utilized to expand all the variables in the power series. As known, the time variable has to be expanded also; otherwise the influence of the nonlinearity on the angular velocity would be lost. In that case, the leading terms of the obtained solutions would, obviously, represent the Keplerian solutions. Smallness of the parameter δ is a guaranty of the fast convergence of such series in a relatively short period of time.

Another way to attack the problem lays in the combination of phenomenological and causal approach: it's possible to introduce some of the observed facts into equations and, after that, to resolve the rest. It would be, doubtless, of the great help in a better understanding of the motion.

However, if somebody intends to resolve the complete system of equations trying to find the long term solutions, consisting chaotic motions as well, he has to bear in mind that numerical integration of the nonlinear dynamical systems requires not only the perfectly precise mathematical model (including, probably, influence of the here disregarded dissipative forces), but practically unreachable accuracy of the input data, also.

CONCLUSION

Rotational, together with orbital motion of the arbitrary shaped body in the gravitational field was studied in this work. Gravitational load and its potential in an inhomogeneous gravitatitational was derived. After that, the energy extrema conditions at apsides were established. In the cases of closed orbits, fulfillment of these conditions produces periodic motion. Differential equations of translational and rotational motions of the body were derived. Some conclusions about characteristics of the motion in an inhomogeneous gravitational field were made. It was followed, at the end, by the discussion about possibilities of finding solutions of the obtained nonlinear dynamical system.

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KRETANJE TELA U NEHOMOGENOM GRAVITACIONOM POLJU

Milutin Marjanov

Proučavanje opšteg slučaja kretanja tela u gravitacionom polju je moguće samo ako se za matematički model usvoji nehomogeno polje. U radu su izvedeni izrazi za gravitaciono opterećenje tela, kao i njegov potencijal u takvom polju. Ti izrazi su funkcije orbitalnih koordinata centra mase, tri Ojlerova ugla relativne rotacije i, posredno, vremena. Ispisane su diferencijalne jednačine kretanja: one su spregnute, nelinearne i numerički teško rešive, jer za vremenske intervale od ineresa zahtevaju praktično nedostižno precizne ulazne podatke. Razmotreni su slučajevi kada se te jednačine uprošćavaju i postavljen je uslov ekstremuma kinetičke i potencijalne energije u perihelu (i afelu, za zatvorene orbite). Ispunjavanje tog uslova daje periodična i rezonantna kretanja u zatvorenim orbitama.

Ključne reči: *Nehomogeno gravitaciono polje, gravitacioni potencijal, rotaciono kretanje, uslovi periodičnosti*