FACTA UNIVERSITATIS
Series: **Mechanics, Automatic Control and Robotics** Vol. 6, N° 1, 2007, pp. 145 - 160

IONIZED GAS BOUNDARY LAYER ON A POROUS WALL OF THE BODY WHOSE ELECTROCONDUCTIVITY IS A FUNCTION OF THE VELOCITY RATIO

UDC 537.563:62-58(045)=111

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Abstract. *This paper investigates a planar ionized gas (air) flow in the boundary layer under the conditions of the so-called equilibrium ionization. The contour of the body within the fluid is porous. Ionized gas flows through the magnetic field of the strength* $B_m = B_m(x)$. *It is assumed that the ionized gas electroconductivity is a function of the ratio of the longitudinal velocity and the velocity at the outer edge of the boundary layer. The governing equations of the boundary layer are by application of the General Similarity Method brought to a generalized form. The obtained generalized equation system of the boundary layer equations, besides the transversal coordinate* η, *contains three sets of parameters. As usual in this theory, the equations are numerically solved in a four-parametric three times localized approximation. Based on the obtained numerical solutions, some conclusions about the behaviour of certain boundary layer physical values and the characteristics of a compressible fluid flow have been drawn. Some guidelines concerning further studies of this problem are also given.*

Key words: *Boundary layer, ionized gas, equilibrium ionization, ionized gas electroconductivity, porous contour, Generalized Similarity Method, porosity parameter*

1. INTRODUCTION

The paper investigates the ionized gas (air) flow in the boundary layer on the body of an arbitrary shape. The boundary layer is studied under conditions of the so-called equilibrium ionization, where the ionized gas flow is planar. The contour of the body within the fluid is *porous*.

This investigation is actually a continuation of our earlier studies of the dissociated and ionized gas flow in the boundary layer. These studies were mainly concerned with a flow when the wall of the body within the fluid was nonporous.

The primary objective of this, as with earlier investigations, is to use the Generalized Similarity Method to obtain the so-called generalized boundary layer equations of the

Received June 02, 2006

problem under study and to solve the obtained equations. The objective is to transform the governing equations into an equation system that represents a *general mathematical model* of the considered problem of the ionized gas flow in the boundary layer (the velocity at the outer edge of the boundary layer does not appear in this equation system).

When the flow velocity of the gas (air) is high, as with supersonic flight of aircrafts through the Earth atmosphere, the temperature in the viscous boundary layer increases significantly. According to the literature, these temperatures often exceed 10 000 K [2]. These high temperatures cause thermochemical reactions - first dissociation and then ionization. The increase in temperature brings about the increase in inner, i.e. kinetic energy of the gas molecules. At a certain value of the temperature, the molecule collision energy enables dissolution of the gas molecules into the constituent atoms, i.e., gas association occurs. As for the air, oxygen dissociation occurs first and then nitrogen dissociation.

With further increases in temperature, the dissociated gas atoms gain more energy. In their collisions, certain electrons in outer orbits get excited to a point that they are separated from the atoms. The thermochemical phenomenon at which the electrons are separated and positive ions are formed is called ionization. In this case, for example, air becomes a multicomponent mixture that consists of ions, electrons and atoms (oxygen and nitrogen). When the velocities of ionization and the reverse process - recombination are high enough, the thermochemical equilibrium is established in the ionized gas (air) flow.

At high temperatures, characteristic for the boundary layer at supersonic gas (air) flow, both gas dissociation and ionization occur. As a result, the ionized gas becomes electroconductive. If ionized gas flows in the magnetic field of the strength $\vec{B}_m$, then under the influence of the outer magnetic field an electric flow appears. This electric flow leads to a volumetric force, which is called "electric volumetric" or Lorentz force. Joule's heat is also generated. Therefore, at ionized gas flow in the magnetic field, due to these two physical effects, new - additional terms appear in the corresponding boundary layer equations. These terms do not exist in the homogenous (unionized) gas boundary layer equations.

As far as we know, the most important results in dissociated gas flow investigation are presented in the book by Dorrance [2]. The members of the school led by Loitsianskii [3, 4, 5] achieved significant scientific results in the field of the dissociated gas flow in the boundary layer.

2. THE GOVERNING EQUATIONS OF THE CONSIDERED FLOW PROBLEM

The above-mentioned paper [4], gives a detailed investigation of the boundary layer ionized gas flow along a planar nonporous plate when a magnetic field is present. The objective of that investigation was to obtain the so-called auto-model solution. Distribution of the strength of the outer magnetic field is defined in order to bring the governing partial boundary layer equations down to simple differential equations.

This paper, however, gives results of the investigation of the ionized gas flow in the boundary layer on the body of an arbitrary shape. The flow is planar and steady and the contour of the body within the fluid is porous. The outer magnetic field is perpendicular to the contour of the body within the fluid. Since the thickness of the boundary layer is small, it can be taken [4] that the strength of the magnetic field is $B_{my} = B_m = B_m\,(x)$ and that the magnetic Reynold's number is small enough. With the velocity $v_w(x)$, the gas is injected i.e., ejected perpendicularly to the porous wall of the body within the fluid.

For the considered flow problem, the equations of the laminar steady and planar boundary layer under conditions of equilibrium ionization [4] are:

$$\begin{aligned}
&\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0,\\
&\rho u\,\frac{\partial u}{\partial x} + \rho v\,\frac{\partial u}{\partial y} = -\frac{dp}{dx} + \frac{\partial}{\partial y}\left(\mu\,\frac{\partial u}{\partial y}\right) - \sigma B_m^2\,u,\\
&\rho u\,\frac{\partial h}{\partial x} + \rho v\,\frac{\partial h}{\partial y} = u\,\frac{dp}{dx} + \mu\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\partial}{\partial y}\left(\frac{\mu}{\Pr}\,\frac{\partial h}{\partial y}\right) + \sigma B_m^2\,u^2.
\end{aligned} \tag{1}$$

The corresponding boundary conditions, for the case of a porous wall of the body within the fluid are:

$$\begin{aligned}
&u = 0, \qquad \underline{\underline{v = v_w(x)}}, \qquad h = h_w \qquad \text{for} \quad y = 0,\\
&u \to u_e(x), \qquad\qquad\qquad h \to h_e(x) \qquad \text{for} \quad y \to \infty.
\end{aligned} \tag{2}$$

The equation system (1) represents a mathematical model of the flow problem under study. The equations of the previous governing system are: continuity equation, dynamic equation, and energy equation of the ionized gas boundary layer, respectively. In the equation system (1) and in the boundary conditions (2), the notation common in the boundary layer theory is used for the physical values and the necessary indices. Here, $u\,(x, y)$ is longitudinal projection of velocity in the boundary layer, $v\,(x, y)$ - transversal projection, ρ - ionized gas density, p - pressure, h - enthalpy, μ - coefficient of dynamic viscosity, σ - ionized gas electroconductivity and $\Pr$ - Prandtl number. The indices stand for: w - values on the wall of the body within the fluid and e - physical values at the outer edge of the boundary layer.

Electroconductivity σ is an important physical parameter of the ionized gas. In general, it is a variable that depends on the temperature [4], i.e. the gas enthalpy h. Since we do not know the exact law on variation of electroconductivity, by analogy with MHD boundary layer [1, 8] it is assumed that the electroconductivity is a function of the ratio of the longitudinal velocity and the velocity at the outer edge of the boundary layer. Therefore, it is assumed that the law on electroconductivity variation is determined by the expression

$$\sigma = \sigma_0\left(1 - \frac{u}{u_e}\right), \qquad (\sigma_0 = const.). \tag{3}$$

Based on the boundary conditions for the velocity and density at the outer edge of the boundary layer, the pressure $p(x)$ can be eliminated from the system (1), as usual. This way, the governing equation system (1) is brought down to:

$$\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)=0,$$

$$\rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial u}{\partial y}=\rho_e u_e\frac{du_e}{dx}+\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)-\sigma_0 B_m^2 u\left(1-\frac{u}{u_e}\right), \tag{4}$$

$$\rho u\frac{\partial h}{\partial x}+\rho v\frac{\partial h}{\partial y}=-u\,\rho_e u_e\frac{du_e}{dx}+\mu\left(\frac{\partial u}{\partial y}\right)^2+\frac{\partial}{\partial y}\left(\frac{\mu}{\Pr}\frac{\partial h}{\partial y}\right)+\sigma_0 B_m^2 u^2\left(1-\frac{u}{u_e}\right);$$

where the boundary conditions (2) remain unchanged.

Variation of the electroconductivity law in MHD boundary layer theory, as stated [1, 8], has enriched this theory. This is why we point out that the assumed law on variation of the ionized gas electroconductivity in the form of (3), also enriches the ionized gas boundary layer theory. Therefore, this paper is significant from the aspect of methodology.

3. The Transformation of the Governing Boundary Layer Equation System

In order to apply the General Similarity Method to the considered problem of the ionized gas flow in the boundary layer, instead of the physical coordinates x, y, new variables are introduced in the form of

$$s(x)=\frac{1}{\rho_0\mu_0}\int_0^x \rho_w\mu_w\,dx, \qquad z(x,y)=\frac{1}{\rho_0}\int_0^y \rho\,dy. \tag{5}$$

As with other already solved compressible fluid flow problems, a stream function $\psi(s, z)$ is introduced in accordance with the relations

$$u=\frac{\partial\psi}{\partial z}, \qquad \tilde{v}=\frac{\rho_0\mu_0}{\rho_w\mu_w}\left(u\frac{\partial z}{\partial x}+v\frac{\rho}{\rho_0}\right)=-\frac{\partial\psi}{\partial s}, \tag{6}$$

that are based on the continuity equation.

In transformations (5) and (6) and further on, the values ρ_0 and $\mu_0=\rho_0\nu_0$ stand for the known values of the density and dynamic viscosity of the ionized gas (air), while ν_0 represents the kinematical viscosity at a concrete point of the boundary layer. Here, ρ_w and μ_w denote the given distributions of these values on the wall of the body within the fluid.

Applying the new variables (5) and the stream function (6), the governing equation system (4), together with the boundary conditions, is transformed into the following form:

$$\frac{\partial\psi}{\partial z}\frac{\partial^2\psi}{\partial s\partial z}-\frac{\partial\psi}{\partial s}\frac{\partial^2\psi}{\partial z^2}=\frac{\rho_e}{\rho}u_e\frac{du_e}{ds}+\nu_0\frac{\partial}{\partial z}\left(Q\frac{\partial^2\psi}{\partial z^2}\right)-\frac{\sigma_0 B_m^2}{\rho u_e}\frac{\rho_0\mu_0}{\rho_w\mu_w}\left(u_e-\frac{\partial\psi}{\partial z}\right)\frac{\partial\psi}{\partial z},$$

$$\frac{\partial\psi}{\partial z}\frac{\partial h}{\partial s}-\frac{\partial\psi}{\partial s}\frac{\partial h}{\partial z}=-\frac{\rho_e}{\rho}u_e\frac{du_e}{ds}\frac{\partial\psi}{\partial z}+\nu_0\,Q\left(\frac{\partial^2\psi}{\partial z^2}\right)^2+\nu_0\frac{\partial}{\partial z}\left(\frac{Q}{\Pr}\frac{\partial h}{\partial z}\right)+$$

$$+\frac{\sigma_0 B_m^2}{\rho u_e}\frac{\rho_0\,\mu_0}{\rho_w\,\mu_w}\left(u_e-\frac{\partial\psi}{\partial z}\right)\left(\frac{\partial\psi}{\partial z}\right)^2;\tag{7}$$

$$\frac{\partial\psi}{\partial z}=0,\qquad \underline{\underline{\frac{\partial\psi}{\partial s}=-\frac{\mu_0}{\mu_w}v_w=-\tilde{v}_w}},\qquad h=h_w\qquad \text{for}\quad z=0,$$

$$\frac{\partial\psi}{\partial z}\to u_e(s),\qquad h\to h_e(s)\qquad \text{for}\quad z\to\infty.$$

The nondimensional function Q that appears in the equation system (7), is defined with the expression

$$Q=\frac{\rho\mu}{\rho_w\mu_w},\qquad (Q=1\quad \text{for}\quad z=\boldsymbol{0},\quad Q\to\frac{\rho_e\,\mu_e}{\rho_w\mu_w}=Q(s)\quad \text{for}\quad z\to\infty).\tag{8}$$

Prandtl number - Pr is determined with the known expression

$$\Pr=\frac{\mu c_p}{\lambda},\tag{9}$$

where λ - coefficient of the heat conductivity and c_p - specific heat of the ionized gas (mixture).

In order to solve the fluid flow problems in the boundary layer by means of the General Similarity Method, it is necessary to obtain the corresponding momentum equation. By the usual procedure, from the first two equations of the system (4) - by integration transversally to the boundary layer (from the inner to the outer edge of the boundary layer) and by transformation of the variables, this equation is easily obtained. With this problem of the ionized gas flow, the momentum equation can take the three following forms:

$$\frac{dZ^{**}}{ds}=\frac{F_{mp}}{u_e},\qquad \frac{df}{ds}=\frac{u_e'}{u_e}F_{mp}+\frac{u_e''}{u_e'}f,\qquad \frac{1}{\Delta^{**}}\frac{d\Delta^{**}}{ds}=\frac{u_e'}{u_e}\frac{F_{mp}}{2f};\tag{10}$$

where primes stand for the derivatives per the variable s.

This equation is by its form the same for all the considered cases of the compressible fluid flow.

The usual values are introduced in order to obtain the momentum equation. They are: parameter of the form $f(s)$, magnetic parameter $g(s)$, conditional displacement thickness Δ^*, conditional momentum loss thickness Δ^{**}, conditional thickness Δ^*_1, shear stress on the wall of the body within the fluid τ_w, nondimensional friction function $\zeta(s)$ and nondimensional values H and H_1. With the ionized gas flow, these values are defined by means of the relations:

$$Z^{**} = \frac{\Delta^{**2}}{\nu_0}, \qquad f(s) = \frac{u_e' \Delta^{**^2}}{\nu_0} = u_e' Z^{**} = f_1(s),$$

$$g(s) = N_\sigma Z^{**} = g_1(s), \qquad \Delta^*(s) = \int_0^\infty \left(\frac{\rho_e}{\rho} - \frac{u}{u_e} \right) dz,$$

$$\Delta^{**}(s) = \int_0^\infty \frac{u}{u_e} \left(1 - \frac{u}{u_e} \right) dz, \qquad \Delta_1^{**}(s) = \int_0^\infty \frac{u}{u_e} \left(1 - \frac{u}{u_e} \right) \frac{\rho_e}{\rho} dz, \tag{11}$$

$$\tau_w(s) = \left(\mu \frac{\partial u}{\partial y} \right)_{y=0} = \frac{\rho_w \mu_w}{\rho_0} \frac{u_e}{\Delta^{**}} \zeta; \qquad \zeta(s) = \left[\frac{\partial (u/u_e)}{\partial (z/\Delta^{**})} \right]_{z=0},$$

$$H = \frac{\Delta^*}{\Delta^{**}}, \qquad H_1 = \frac{\Delta_1^{**}}{\Delta^{**}}, \qquad N_\sigma = \frac{\rho_0 \mu_0}{\rho_w \mu_w} \overline{N}, \qquad \overline{N} = \frac{\sigma_0 B_m^2}{\rho_e}.$$

The characteristic function of the boundary layer F_{mp}, which is very important for the numerical solution of the generalized equation system, with the flow problem under study is:

$$F_{mp} = 2[\zeta - (2 + H) f] + 2gH_1 - 2\Lambda. \tag{12}$$

Because of the porous wall of the body within the fluid, the *porosity parameter* $\Lambda(s)$ is introduced by means of the expression:

$$\Lambda(s) = -\frac{\mu_0}{\mu_w} \frac{v_w \Delta^{**}}{\nu_0} = -\frac{V_w \Delta^{**}}{\nu_0} = \Lambda_1(s); \qquad V_w(s) = \frac{\mu_0}{\mu_w} v_w = \widetilde{v}_w, \tag{13}$$

where the value $V_w(s)$ can be called *conditional transversal velocity* at the inner edge of the boundary layer.

The equation system (7) contains new variables. This system with the corresponding boundary conditions represents a mathematical model of this problem of the ionized gas flow along the porous wall (where the electroconductivity is a function of the velocity ratio). The boundary condition for the partial derivative (transversal velocity) is $\partial\psi/\partial s \neq 0$ With the application of the General Similarity Method, it is important that this boundary condition (underlined in (7)) should equal zero. Therefore, as with the incompressible fluid [5], the stream function $\psi(s, z)$ is divided into two parts. To be more correct, a new stream function $\psi^*(s,z)$ is introduced in the form of the relation

$$\psi(s,z) = \psi_w(s) + \psi^*(s,z), \qquad \psi^*(s,0) = 0 \tag{14}$$

in which $\psi(s, 0) = \psi_w(s)$ denotes the stream function along the wall of the body within the fluid ($z = 0$).

After the application of the relation (14), the system (7) is transformed into the following equation system:

$$
\begin{aligned}
&\frac{\partial\psi^*}{\partial z}\frac{\partial^2\psi^*}{\partial s\partial z}-\frac{\partial\psi^*}{\partial s}\frac{\partial^2\psi^*}{\partial z^2}-\underline{\underline{\frac{d\psi_w}{ds}\frac{\partial^2\psi^*}{\partial z^2}}}=\frac{\rho_e}{\rho}u_e u_e'+\nu_0\frac{\partial}{\partial z}\left(Q\frac{\partial^2\psi^*}{\partial z^2}\right)-\\
&-\frac{\sigma_0 B_m^2}{\rho u_e}\frac{\rho_0\,\mu_0}{\rho_w\,\mu_w}\left(u_e-\frac{\partial\psi^*}{\partial z}\right)\frac{\partial\psi^*}{\partial z},\\
&\frac{\partial\psi^*}{\partial z}\frac{\partial h}{\partial s}-\frac{\partial\psi^*}{\partial s}\frac{\partial h}{\partial z}-\underline{\underline{\frac{d\psi_w}{ds}\frac{\partial h}{\partial z}}}=-\frac{\rho_e}{\rho}u_e u_e'\frac{\partial\psi^*}{\partial z}+\nu_0\,Q\left(\frac{\partial^2\psi^*}{\partial z^2}\right)^2+\\
&+\nu_0\frac{\partial}{\partial z}\left(\frac{Q}{\mathrm{Pr}}\frac{\partial h}{\partial z}\right)+\frac{\sigma_0 B_m^2}{\rho u_e}\frac{\rho_0\,\mu_0}{\rho_w\,\mu_w}\left(u_e-\frac{\partial\psi^*}{\partial z}\right)\left(\frac{\partial\psi^*}{\partial z}\right)^2;\\
&\qquad\psi^*=0,\quad\frac{\partial\psi^*}{\partial z}=0,\qquad h=h_w\qquad\text{for}\qquad z=0,\\
&\qquad\qquad\frac{\partial\psi^*}{\partial z}\to u_e(s),\quad h\to h_e(s)\qquad\text{for}\qquad z\to\infty.
\end{aligned}
\tag{15}
$$

It can be noticed that both equations of the system (15) contain an additional (underlined) term on the left side of the equals sign. In these terms the derivative $d\psi_w / ds$, appears as a factor

$$
\frac{d\psi_w(s)}{ds}=\frac{d\psi(s,0)}{ds}=\left(\frac{\partial\psi}{\partial s}\right)_{z=0}=-\frac{\mu_0}{\mu_w}v_w=-V_w(s).
\tag{16}
$$

It can be also noticed that, for the case of a nonporous wall of the body within the fluid (for which $v_w = 0$) the underlined terms equal zero, which is quite logical, since these terms are the result of the porous contour of the body. The boundary conditions are the same as with the nonporous wall [10].

4. The Generalized ionized gas boundary layer equations

The obtained equation system (15) is analyzed in details in our investigation. The ideas applied both with incompressible and compressible fluid are not useful if we want to obtain the so-called generalized boundary layer equations [10] by means of the new stream function of the form $\Phi(s, \eta)$, where η is a new variable. Therefore, to obtain the generalized boundary layer equations of the considered ionized gas flow, it is necessary to introduce new transformations in the form of the following expressions:

$$
\begin{aligned}
&s=s,\qquad \eta(s,z)=\frac{u_e^{b/2}}{K(s)}z,\qquad \psi^*(s,z)=u_e^{1-b/2}K(s)\Phi[\eta,\,\kappa,\,(f_k),\,(g_k),\,(\Lambda_k)],\\
&h(s,z)=h_1\cdot\bar{h}[\eta,\,\kappa,\,(f_k),\,(g_k),\,(\Lambda_k)];\qquad h_e+\frac{u_e^2}{2}=h_1=const.,\\
&\qquad K(s)=(a\nu_0\int_0^s u_e^{b-1}ds)^{1/2};\qquad a,\,b=const.
\end{aligned}
\tag{17}
$$

In the defined similarity transformations, the following notation is used: $\eta(s, z)$ - newly introduced transversal variable, Φ - newly introduced stream function, $\bar{h}$ - non-dimensional enthalpy; while a and b are constants that will be discussed later.

With this flow problem, based on the expressions for the newly introduced variable $\eta(s, z)$, certain important values and characteristics of the boundary layer (11) can be written in the form of the suitable relations. These relations are:

$$
\begin{gathered}
u = u_e \frac{\partial \Phi}{\partial \eta}, \qquad \Delta^{**}(s) = \frac{K(s)}{u_e^{b/2}} B(s), \qquad B(s) = \int\limits_0^\infty \frac{\partial \Phi}{\partial \eta}\left(1 - \frac{\partial \Phi}{\partial \eta}\right) d\eta, \\
\frac{\Delta^*(s)}{\Delta^{**}(s)} = H = \frac{A(s)}{B(s)}, \qquad A(s) = \int\limits_0^\infty \left(\frac{\rho_e}{\rho} - \frac{\partial \Phi}{\partial \eta}\right) d\eta, \qquad \zeta = B\left(\frac{\partial^2 \Phi}{\partial \eta^2}\right)_{\eta=0}, \\
\frac{\Delta_1^{**}(s)}{\Delta^{**}(s)} = H_1 = \frac{A_1(s)}{B(s)}; \qquad A_1(s) = \int\limits_0^\infty \frac{\partial \Phi}{\partial \eta}\left(1 - \frac{\partial \Phi}{\partial \eta}\right) \frac{\rho_e}{\rho}\, d\eta, \\
\frac{f}{B^2} = \frac{a u_e'}{u_e^b} \int\limits_0^s u_e^{b-1}\, ds.
\end{gathered}
\tag{18}
$$

Here, it is assumed that the already defined values A, B and A_1 are continual functions of the variable s.

With the nondimensional functions Φ and $\bar{h}$, in the general similarity transformations (17), we introduced a local parameter of the ionized gas compressibility $\kappa = f_0$, a set of parameters of the form f_k of Loitsianskii type [5], a set of magnetic parameters g_k and a set of *porosity parameters* Λ_k [10]. These sets are determined by the following expressions:

$$
\begin{gathered}
\kappa = f_0(s) = \frac{u_e^2}{2h_1}, \qquad f_k(s) = u_e^{k-1} u_e^{(k)} Z^{**k}, \qquad g_k(s) = u_e^{k-1} N_\sigma^{(k-1)} Z^{**k}, \\
\Lambda_k(s) = -u_e^{k-1}\left(\frac{V_w}{\sqrt{\nu_0}}\right)^{(k-1)} Z^{**k-1/2} \qquad (k = 1, 2, 3, ...);
\end{gathered}
\tag{19}
$$

and, as it is known, they play a role of independent variables instead of the longitudinal variable s.

For $k = 1$ we will get $f_1(s) = u'_e Z^{**}$ - the already introduced parameter of the form (11). The first in the set of the porosity parameters $\Lambda_1(s) = -(V_w/\sqrt{\nu_0})\, Z^{**1/2} = -(V_w \Delta^{**}/\nu_0)$ is the same as the already defined parameter (13). Just like with incompressible fluid, the parameters of the sets (19), satisfy the corresponding simple recurrent differential equations of the form:

$$
\begin{aligned}
&\frac{u_e}{u_e'}\,f_1\,\frac{d\kappa}{ds} = 2\,\kappa\,f_1 = \theta_0\,,\\
&\frac{u_e}{u_e'}\,f_1\,\frac{df_k}{ds} = [(k-1)f_1 + kF_{mp}]\,f_k + f_{k+1} = \theta_k\,,\\
&\frac{u_e}{u_e'}\,f_1\,\frac{dg_k}{ds} = [(k-1)\,f_1 + k\,F_{mp}]\;g_k + g_{k+1} = \gamma_k\,,\\
&\frac{u_e}{u_e'}\,f_1\,\frac{d\Lambda_k}{ds} = \{(k-1)f_1 + [(2k-1)/2]F_{mp}\}\;\Lambda_k + \Lambda_{k+1} = \chi_k\,.\\
&(k = 1, 2, 3, \ldots)
\end{aligned}
\tag{20}
$$

Applying the similarity transformations (17) and (19) to the equation system (15) after some calculation, the so-called *generalized boundary layer equation system* of the considered problem has been obtained. The obtained equation system, together with the transformed boundary condition, is:

$$
\begin{aligned}
&\frac{\partial}{\partial\eta}\left(Q\,\frac{\partial^2\Phi}{\partial\eta^2}\right) + \frac{aB^2+(2-b)f_1}{2B^2}\,\Phi\,\frac{\partial^2\Phi}{\partial\eta^2} + \frac{f_1}{B^2}\left[\frac{\rho_e}{\rho} - \left(\frac{\partial\Phi}{\partial\eta}\right)^2\right] - \frac{g_1}{B^2}\frac{\rho_e}{\rho}\left(1-\frac{\partial\Phi}{\partial\eta}\right)\frac{\partial\Phi}{\partial\eta} +\\
&+\underline{\underline{\frac{\Lambda_1}{B}\,\frac{\partial^2\Phi}{\partial\eta^2}}} = \frac{1}{B^2}\left[\sum_{k=0}^{\infty}\theta_k\left(\frac{\partial\Phi}{\partial\eta}\,\frac{\partial^2\Phi}{\partial\eta\partial f_k} - \frac{\partial\Phi}{\partial f_k}\,\frac{\partial^2\Phi}{\partial\eta^2}\right) + \sum_{k=1}^{\infty}\gamma_k\left(\frac{\partial\Phi}{\partial\eta}\,\frac{\partial^2\Phi}{\partial\eta\partial g_k} - \frac{\partial\Phi}{\partial g_k}\,\frac{\partial^2\Phi}{\partial\eta^2}\right) +\right.\\
&\left.+\sum_{k=1}^{\infty}\chi_k\left(\frac{\partial\Phi}{\partial\eta}\,\frac{\partial^2\Phi}{\partial\eta\partial\Lambda_k} - \frac{\partial\Phi}{\partial\Lambda_k}\,\frac{\partial^2\Phi}{\partial\eta^2}\right)\right],\\
&\frac{\partial}{\partial\eta}\left(\frac{Q}{\Pr}\,\frac{\partial\bar{h}}{\partial\eta}\right) + \frac{aB^2+(2-b)f_1}{2B^2}\,\Phi\,\frac{\partial\bar{h}}{\partial\eta} - \frac{2\kappa f_1}{B^2}\,\frac{\rho_e}{\rho}\,\frac{\partial\Phi}{\partial\eta} + 2\kappa Q\left(\frac{\partial^2\Phi}{\partial\eta^2}\right)^2 +\\
&+\frac{2\kappa g_1}{B^2}\,\frac{\rho_e}{\rho}\left(1-\frac{\partial\Phi}{\partial\eta}\right)\left(\frac{\partial\Phi}{\partial\eta}\right)^2 + \underline{\underline{\frac{\Lambda_1}{B}\,\frac{\partial\bar{h}}{\partial\eta}}} = \frac{1}{B^2}\left[\sum_{k=0}^{\infty}\theta_k\left(\frac{\partial\Phi}{\partial\eta}\,\frac{\partial\bar{h}}{\partial f_k} - \frac{\partial\Phi}{\partial f_k}\,\frac{\partial\bar{h}}{\partial\eta}\right) +\right.\\
&\left.+\sum_{k=1}^{\infty}\gamma_k\left(\frac{\partial\Phi}{\partial\eta}\,\frac{\partial\bar{h}}{\partial g_k} - \frac{\partial\Phi}{\partial g_k}\,\frac{\partial\bar{h}}{\partial\eta}\right) + \sum_{k=1}^{\infty}\chi_k\left(\frac{\partial\Phi}{\partial\eta}\,\frac{\partial\bar{h}}{\partial\Lambda_k} - \frac{\partial\Phi}{\partial\Lambda_k}\,\frac{\partial\bar{h}}{\partial\eta}\right)\right];\\
&\qquad\Phi = \frac{\partial\Phi}{\partial\eta} = 0\,,\qquad \bar{h} = \bar{h}_w = const.\qquad \text{for}\qquad \eta = 0\,,\\
&\qquad\qquad\frac{\partial\Phi}{\partial\eta}\to 1,\qquad \bar{h}\to\bar{h}_e = 1-\kappa\qquad \text{for}\qquad \eta\to\infty\,.
\end{aligned}
\tag{21}
$$

Distribution of the outer velocity $u_e(s)$ appears neither in the equations of the obtained system nor in the corresponding boundary conditions of the flow problem under study. Therefore, the equation system (21) represents a general mathematical model for the case of the ionized gas flow in the boundary layer along a porous wall of the body within the fluid when the law on electroconductivity variation is given with the expression (3).

It can be noticed that both of the equations of the system (21), on the left side of the equals sign, contain one term that depends on the porosity parameter Λ_1 (13). Each of them contains a sum of terms on the right side of the equals sign that are multiplied with the function χ_k (20). If the wall of the body within the fluid is nonporous ($v_w = 0$), all the porosity parameters equal zero. Then, all the mentioned terms in the system (21) equal zero. In that case, the equation system, obtained in this paper, comes down to the corresponding equation system for the case of the ionized gas flow along a nonporous wall [10].

5. THE NUMERICAL SOLUTION OF THE OBTAINED EQUATION SYSTEM

As with other already solved flow problems, numerical solution of the obtained equation system (21) is possible only in the so-called n - parametric approximation. Assuming that all the parameters equal zero, starting from the second one, i.e., if $f_k = 0$, $g_k = 0$ and $\Lambda_k = 0$ when $k \geq 2$, the obtained equation system is considerably simplified. The system (21) comes down to the system of partial differential equations with five independent variables: η, κ, f_1, g_1, Λ_1; and it represents a four-parametric approxima-tion. In the boundary layer theory, it is common [5, 9] to neglect the first derivatives per some of the mentioned parameters, i.e., to perform the so-called localization. It is clear that in a detailed analysis, with any flow problem, it is necessary to justify the localization per each of the parameters. Here, the localization is justified due to difficulties of mathematical nature. Therefore, the equation system for numerical solution in the four-parametric three times localized approximation ($\partial/\partial\kappa = 0$, $\partial/\partial g_1 = 0$, $\partial/\partial\Lambda_1 = 0$) together with the corresponding boundary conditions has the following form:

$$\begin{gathered}
\frac{\partial}{\partial\eta}\left(Q\frac{\partial^2\Phi}{\partial\eta^2}\right)+\frac{aB^2+(2-b)f}{2B^2}\Phi\frac{\partial^2\Phi}{\partial\eta^2}+\frac{f}{B^2}\left[\frac{\rho_e}{\rho}-\left(\frac{\partial\Phi}{\partial\eta}\right)^2\right]-\frac{g}{B^2}\frac{\rho_e}{\rho}\left(1-\frac{\partial\Phi}{\partial\eta}\right)\frac{\partial\Phi}{\partial\eta}+ \\
+\underline{\underline{\frac{\Lambda}{B}\frac{\partial^2\Phi}{\partial\eta^2}}}=\frac{F_{mp}f}{B^2}\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial^2\Phi}{\partial\eta\partial f}-\frac{\partial\Phi}{\partial f}\frac{\partial^2\Phi}{\partial\eta^2}\right), \\
\frac{\partial}{\partial\eta}\left(\frac{Q}{\Pr}\frac{\partial\bar{h}}{\partial\eta}\right)+\frac{aB^2+(2-b)f}{2B^2}\Phi\frac{\partial\bar{h}}{\partial\eta}-\frac{2\kappa f}{B^2}\frac{\rho_e}{\rho}\frac{\partial\Phi}{\partial\eta}+2\kappa Q\left(\frac{\partial^2\Phi}{\partial\eta^2}\right)^2+ \\
+\frac{2\kappa g}{B^2}\frac{\rho_e}{\rho}\left(1-\frac{\partial\Phi}{\partial\eta}\right)\left(\frac{\partial\Phi}{\partial\eta}\right)^2+\underline{\underline{\frac{\Lambda}{B}\frac{\partial\bar{h}}{\partial\eta}}}=\frac{F_{mp}f}{B^2}\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial\bar{h}}{\partial f}-\frac{\partial\Phi}{\partial f}\frac{\partial\bar{h}}{\partial\eta}\right); \\
\Phi=\frac{\partial\Phi}{\partial\eta}=0,\qquad \bar{h}=\bar{h}_w=const.\qquad \text{for}\qquad \eta=0, \\
\frac{\partial\Phi}{\partial\eta}\to 1,\qquad \bar{h}\to\bar{h}_e=1-\kappa\qquad \text{for}\qquad \eta\to\infty.
\end{gathered} \tag{22}$$

Note that in the equations of the system (22), the index 1 is, for practical reasons, left out from some (first) parameters. Both of the equations of the system (22) contain one (underlined) term that characterizes the porous wall of the body within the fluid.

Therefore, the equation system (22), obtained in this investigation, represents a general mathematical model of the considered problem of the ionized gas flow, which is to be solved numerically. For numerical integration of the obtained system of differential partial equations of the third order, it is necessary first to decrease the order of the dynamic equation. By the usual transformation [9, 10]

$$\frac{u}{u_e} = \frac{\partial \Phi}{\partial \eta} = \varphi = \varphi(\eta,\, \kappa,\, f,\, g,\, \Lambda), \tag{23}$$

the order of the dynamic equation has been decreased, hence the corresponding equation system of this flow problem now is:

$$\begin{gathered}
\frac{\partial}{\partial \eta}\left(Q \frac{\partial \varphi}{\partial \eta}\right) + \frac{aB^2 + (2-b)f}{2B^2}\, \Phi \frac{\partial \varphi}{\partial \eta} + \frac{f}{B^2}\left[\frac{\rho_e}{\rho} - \varphi^2\right] - \frac{g}{B^2}\frac{\rho_e}{\rho}(1-\varphi)\,\varphi + \\
+ \underline{\underline{\frac{\Lambda}{B}\frac{\partial \varphi}{\partial \eta}}} = \frac{F_{mp} f}{B^2}\left(\varphi \frac{\partial \varphi}{\partial f} - \frac{\partial \Phi}{\partial f}\frac{\partial \varphi}{\partial \eta}\right), \\
\frac{\partial}{\partial \eta}\left(\frac{Q}{\Pr}\frac{\partial \overline{h}}{\partial \eta}\right) + \frac{aB^2 + (2-b)f}{2B^2}\, \Phi \frac{\partial \overline{h}}{\partial \eta} - \frac{2\kappa f}{B^2}\frac{\rho_e}{\rho}\varphi + 2\kappa Q \left(\frac{\partial \varphi}{\partial \eta}\right)^2 + \\
+ \frac{2\kappa g}{B^2}\frac{\rho_e}{\rho}(1-\varphi)\,\varphi^2 + \underline{\underline{\frac{\Lambda}{B}\frac{\partial \overline{h}}{\partial \eta}}} = \frac{F_{mp} f}{B^2}\left(\varphi \frac{\partial \overline{h}}{\partial f} - \frac{\partial \Phi}{\partial f}\frac{\partial \overline{h}}{\partial \eta}\right); \\
\Phi = \varphi = 0, \qquad \overline{h} = \overline{h}_w = const. \qquad \text{for} \qquad \eta = 0, \\
\varphi \to 1, \qquad \overline{h} \to \overline{h}_e = 1 - \kappa \qquad \text{for} \qquad \eta \to \infty.
\end{gathered} \tag{24}$$

A concrete numerical solution of the obtained system of nonlinear and conjugated differential partial equations (24) is done by finite differences method, i.e., "passage method" or TDA method. According to the well-known scheme [9] of the planar integration grid, derivatives of the functions φ, Φ and $\overline{h}$ are substituted with finite differences ratios. This way the equation system (24) is transformed into the equivalent system of algebraic equations that are:

$$\begin{gathered}
(I) \quad a^i_{M,K+1}\varphi^i_{M-1,K+1} - 2b^i_{M,K+1}\varphi^i_{M,K+1} + c^i_{M,K+1}\,\varphi^i_{M+1,K+1} = g^i_{M,K+1}\,, \\
(II) \quad a^j_{M,K+1}\overline{h}^j_{M-1,K+1} - 2b^j_{M,K+1}\overline{h}^j_{M,K+1} + c^j_{M,K+1}\overline{h}^j_{M+1,K+1} = g^j_{M,K+1}; \\
M = 2, 3, \ldots, N-1; \qquad K = 0, 1, 2, \ldots; \qquad i, j = 0, 1, 2, \ldots \\
\Phi^i_{1,K+1} = \varphi^i_{1,K+1} = 0, \qquad \overline{h}^j_{1,K+1} = \overline{h}_w = const. \qquad \text{for} \qquad M = 1, \\
\varphi^i_{N,K+1} = 1, \qquad \overline{h}^j_{N,K+1} = 1 - \kappa \qquad \text{for} \qquad M = N.
\end{gathered} \tag{25}$$

In the previous system the coefficient $a^i_{M,K+1}$ of the dynamic equation (I) is determined with the expression:

$$a^i_{M,K+1} = Q^{j-1}_{M,K+1} - \frac{1}{4}(Q^{j-1}_{M+1,K+1} - Q^{j-1}_{M-1,K+1}) -$$
$$-\frac{\Delta\eta}{2(B^{i-1}_{K+1})^2}\left\{\left[a(B^{i-1}_{K+1})^2 + (2-b)f_{K+1}\right]\frac{\Phi^{i-1}_{M,K+1}}{2} + F^{i-1}_{mp,K+1}\cdot f_{K+1}\frac{\Phi^{i-1}_{M,K+1} - \Phi_{M,K}}{\Delta f}\right\} - \Lambda\frac{\Delta\eta}{2B^{i-1}_{K+1}}.$$

During transformation into an algebraic equation, for each layer $(K+1)$ and at each discrete point M of the planar integration grid, special attention was paid to the order of calculation of certain functions and to linearization [9]. As with other flow problems, the system (25) is solved by an iterative procedure, where i, j- stand for the number of iterations. For the nondimensional function Q [10] and for the density ratio ρ_e / ρ that appear in the equations (24), approximate formulas are used in the forms of the following expressions:

$$Q = Q(\bar{h}) = \left(\frac{\bar{h}_w}{\bar{h}}\right)^{1/3}, \qquad \frac{\rho_e}{\rho} \approx \frac{\bar{h}}{1-\kappa}. \tag{26}$$

These expressions are more appropriate for the dissociated gas (air) flow, and in the paper [3] they are determined based on the tables of the thermodynamic functions for the air. Determination of more correct laws on distribution of these ionized gas values may be the subject of some other investigation. For the numerical solution of the equation system (24), i.e., of the corresponding algebraic system, a programme is written in FORTRAN programming language. The basis of this program is the one used in the investigation [9]. A segment of this programme is presented in the Fig. 1. Since the objective of this investigation is not to write a program, the notation used in the program is not given here. All the calculations in this paper are done for the concrete values of the constants a and b when $a = 0.4408$; $b = 5.7140$ that according to [9] represent the optimal values. Prandtl number is taken to be constant – $\Pr = 0.712$. For the calculations of the characteristic functions B and F_{mp} at a zero iteration the values $B^0_{K+1} = 0.469$ and $F^0_{mp,K+1} = 0.4411$ are accepted (as used in [9]).

Since the equation system (22), i.e., (24) is localized per compressibility, porosity and magnetic parameter; all these parameters have the role of simple parameters. That is why the equation system (24) is solved by the usual procedure [9], starting from the value $f = 0.00$ (flat plate), for the given values of the parameters κ, g and Λ.

6. THE OBTAINED RESULTS AND CONCLUSIONS

In this investigation, the numerical solutions of the system (24) are obtained in the form of tables defined by the written programme. Each of the tables represents the solution of the equation system in the corresponding cross-section of the boundary layer for the given values of the input parameters (κ, g, Λ). This paper presents only some of the most important results in the form of diagrams. Fig. 2 shows a diagram of nondimensional velocity u / u_e, Fig. 3 shows a diagram of nondimensional enthalpy $\bar{h}$, while Figs. 4 and 5 give diagrams of the characteristic B of the boundary layer and distribution of the nondimensional friction function ζ. A diagram of distribution of the nondimensional velocity (Fig. 6) and a diagram of nondimensional friction function ζ (Fig. 7) for different values of the porosity parameter Λ are also given. Based on the given and other diagrams [10] the following important conclusions have been drawn:

- The general conclusion is that profiles of the obtained solutions of the boundary layer equations of the considered ionized gas problem, according to their behaviour, are the same as with other problems of the compressible fluid flow [3].
- The nondimensional velocity u / u_e in each of the cross-sections of the boundary layer (different f) very quickly converges towards unity (Figs. 2 and 6).
- A significant influence of the magnetic parameter g on the characteristics of the boundary layer A, B, F_{mp} and ζ is noticed.
- The local compressibility parameter of the ionized gas $\kappa = f_0$ does not have a significant influence on the nondimensional friction function ζ.
- At low values, the porosity parameter Λ has a minor influence on profiles of the nondimensional velocities u / u_e, while at higher values this influence is significant (Fig. 6).
- A change in compressibility parameter has a great influence on distribution of the nondimensional enthalpy $\bar{h}$ in the boundary layer of the ionized gas.
- The porosity parameter Λ, however, has a greater influence on nondimensional friction function ζ (Fig. 7), as well as on the characteristic function F_{mp}. Therefore, the porosity parameter also has a significant influence on the boundary layer separation point.

Once again, it is pointed out that, for calculation of the ionized gas boundary layer, we have used the method based on the application of the Saljnikov's version [9] of the General Similarity Method. This method is actually based on solution of the corresponding generalized equation system of the considered flow problem. The solutions of the obtained equations explain several general rules, some of which are stated above.

However, there are some difficulties in the application of the General Similarity Method to the compressible fluid flow problems. These difficulties are usually of mathematical nature.

The investigation of the dissociated gas flow in the boundary layer [3, 7], has showed that the compressibility parameter has a great influence on distribution of the nondimensional enthalpy at the cross-section of the boundary layer. It has been determined that this parameter changes even the general characteristics of the behaviour of this distribution. Therefore, it can be concluded that with the considered ionized gas flow problem the corresponding equation system (21) should be solved without localization per this parameter. Thus, we would obtain more correct results in terms of quantity. This, however, means more mathematical, i.e., numerical difficulties.

Finally, let us point out, that determination of more correct laws on distribution of the physical values Q and ρ_e / ρ (26) in the ionized gas boundary layer would be of great interest in further studies. It is obvious that a solution of the ionized gas boundary layer equations without localization per parameter g and especially per porosity parameter Λ would mean a great contribution to the boundary layer theory.

```
C      SOLUTION OF THE DYNAMIC EQUATION
C
C      COEFFICIENT OF THE DYNAMIC DIFFERENTIAL EQUATION
C
   11  X=DELET/(2.*BKI**2)
       X1=X*DELET
       X2=2.*X1
       DO 20 M=2,K3
       R1=FRSPI*F1R*(BRPI(M)-BPS(M))/DELFR+BRPI(M)*(VA*BKI**2+
      1   F1R*(2.-VB))/2.
       R1=R1*X+(VFQR(M+1)-VFQR(M-1))/4.
       AKJIM=VFQR(M)-R1
       AKJIM=AKJIM-VL1*DELET/(2.*BKI)
       CKJIM=VFQR(M)+R1
       CKJIM=CKJIM+VL1*DELET/(2.*BKI)
       R2=F1R*ARPI(M)*(1.+FRSPI/DELFR)+(G1*CRPI(M)/(1.-F0))*(1.-ARPI(M))
       BKJIM=VFQR(M)+X1*R2
       R3=F1R*CRPI(M)/(1.-F0)+FRSPI*F1R*ARPI(M)*APS(M)/DELFR
       GKJIM=-X2*R3
C
C      PASSAGE COEFFICIENTS
C
       APVIM=2.*BKJIM-AKJIM*EKP(M-1)
       F33=DABS(APVIM)
       IF(F33-EPS3)8,7,7
    8  APVIM=EPS3
    7  CONTINUE
       DKP(M)=(AKJIM*DKP(M-1)-GKJIM)/APVIM
   20  EKP(M)=CKJIM/APVIM
C
C      SOLUTIONS OF THE DYNAMIC EQUATION BY THE PASSAGE METHOD
C
       M=N-1
   30  ARRI(M)=DKP(M)+EKP(M)*ARRI(M+1)
       M=M-1
       IF(M-2) 31,30,30
   31  DELAM=0.
       N1=N-1
       DO 45 M=2,N1
       DELA=DABS(ARRI(M)-ARPI(M))
       IF(DELAM-DELA) 44,45,45
   44  DELAM=DELA
   45  CONTINUE
C
C      NUMERICAL DETERMINATION OF THE FUNCTION FI
C
       BRRI(2)=3.*DELET/8.*(ARRI(1)+3.*ARRI(2)+3.*ARRI(3)+ARRI(4))
       BRRI(2)=BRRI(2)-DELET/3.*(ARRI(2)+4.*ARRI(3)+ARRI(4))
       BRRI(2)=BRRI(2)+BRRI(1)
       N2=N-2
       DO 50 M=1,N2
       BRRI(M+2)=DELET/3.*(ARRI(M)+4.*ARRI(M+1)+ARRI(M+2))+BRRI(M)
   50  CONTINUE
       DELBM=0.
       DO 55 M=2,N
       DELB=DABS(BRRI(M)-BRPI(M))
       IF(DELBM-DELB) 59,55,55
   59  DELBM=DELB
   55  CONTINUE
```

Fig. 1. A segment of the written programme in FORTRAN

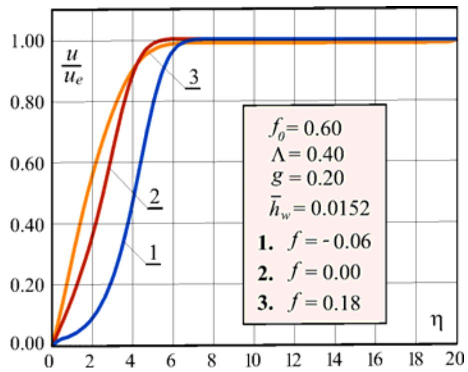

Fig. 2. Diagram of the nondimensional velocity u / u_e

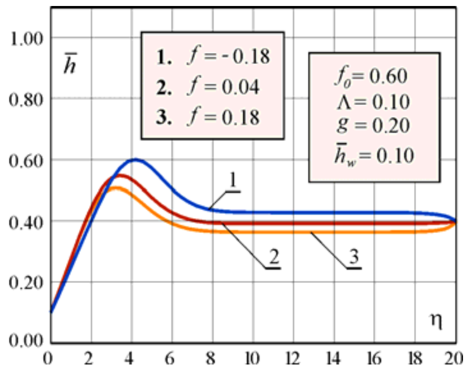

Fig. 3. Diagram of the nondimensional enthalpy $\bar{h}$

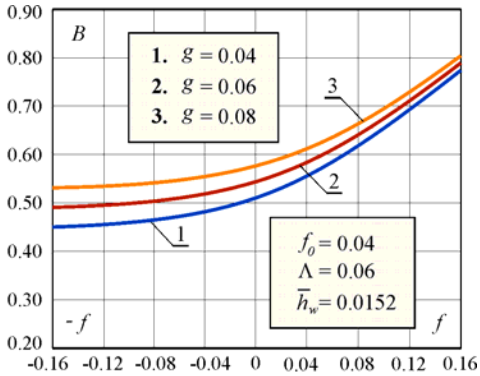

Fig. 4. Graphic of the characteristic B of the boundary layer

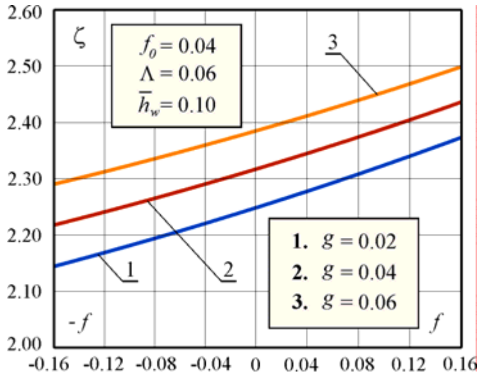

Fig. 5. Distribution of the nondimensional friction function ζ

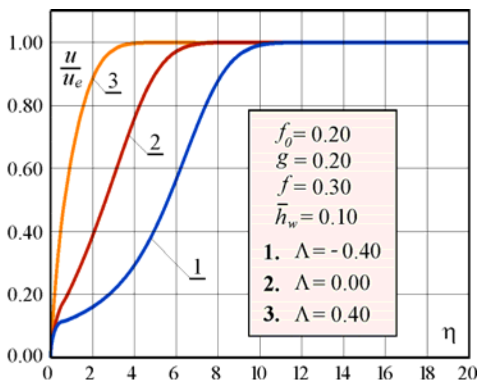

Fig. 6. Diagrams of the nondimensional velocity u / u_e for different values of Λ ($f = 0.30$)

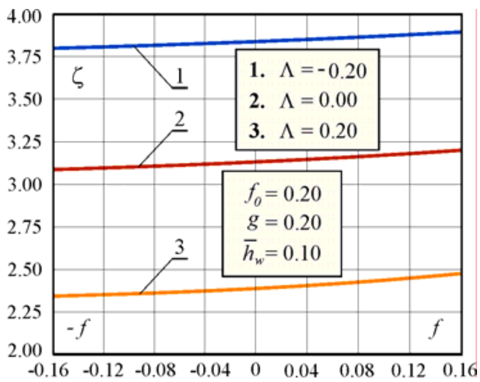

Fig. 7. Distribution of the nondimensional friction function ζ for different values of the parameter Λ

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GRANIČNI SLOJ JONIZOVANOG GASA NA POROZNOM ZIDU TELA ČIJA JE ELEKTROPROVODNOST FUNKCIJA ODNOSA BRZINA

Branko R. Obrović, Slobodan R. Savić

U radu se istražuje ravansko strujanje jonizovanog gasa (vazduha) u graničnom sloju u uslovima tzv. ravnotežne jonizacije. Kontura opstrujavanog tela je porozna. Jonizovani gas se kreće u magnetnom polju jačine $B_m = B_m(x)$. Pri tome se pretpostavlja da je elektroprovodnost jonizovanog gasa funkcija odnosa podužne i brzine na spoljašnjoj granici graničnog sloja. Polazne jednačine graničnog sloja su primenom Metode uopštene sličnosti dovedene na uopšteni oblik. Dobijeni uopšteni sistem jednačina graničnog sloja, pored poprečne koordinate η, sadrži i tri skupa parametara. Kako je to uobičajeno u ovoj teoriji ove jednačine numerički su rešene u četvoroparametarskom tri puta lokalizovanom približenju. Na bazi numeričkih rešenja izvedeni su zaključci o ponašanju pojedinih fizičkih veličina i karakteristika graničnog sloja razmatranog problema strujanja stišljivog fluida. Date su i smernice za dalja moguća istraživanja ovog problema strujanja fluida.

Ključne reči: *Granični sloj, jonizovani gas, ravnotežna jonizacija, elektroprovodnost jonizovanog gasa, porozna kontura, metoda uopštene sličnosti, parametar poroznosti*