

## ROBUST ADAPTIVE CONTROL OF MECHATRONIC SYSTEMS EMPLOYING AN EMULATION OF NONLINEAR FUNCTIONS

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**Yuan-Wei Jing<sup>1</sup>, Yan-Xin Zhang<sup>1</sup>, Tatjana Kolemeisevska-Gugulovska<sup>2</sup>,  
Georgi M. Dimirovski<sup>2,3</sup>, Miomir K. Vukobratovic<sup>4</sup>**

<sup>1</sup>Northeastern University, School of Information Sciences & Engineering  
Shenyang, 110004, Liaoning, P. R. of China

<sup>2</sup>SS Cyril & Methodius University, Faculty of Electrical Eng. & Info. Technologies  
Rugjer Boskovic BB, Karpos II, 1000, Skopje, Republic of Macedonia

<sup>3</sup>Dogus University, Faculty of Engineering, Dept. of Computer Engineering  
Acibadem, Zeamet Sk. 21, Kadikoy, 34722 Istanbul, Republic of Turkey

<sup>4</sup>Mihailo Pupin Institute, Laboratory of Robotics & Automation  
Volgina 15, Karaburma, 11000, Republic of Serbia

e-mail: ywjing@mail.neu.edu.cn; yanxin@126.com; tanjakg@feit.ukim.edu.mk;  
gdimirovski@dogus.edu.tr; vuk@robot.imp.bg.ac.yu

**Abstract.** A novel robust adaptive control design synthesis, which employs both high-order neural networks and math-analytical results, for a class of mechatronic nonlinear systems possessing similarity property has been derived. This approach makes an adequate use of the structural feature of composite similarity systems and neural networks to resolve the representation issue of uncertainty interconnections and subsystem gains by on-line updating the weights. This synthesis does guarantee the real stability in closed-loop but requires skills to obtain larger attraction domains. Mechatronic example of an axis-tray drive system, possessing uncertainties, is used to illustrate the proposed technique.

**Key words:** Adaptive control, function emulation, mechatronic systems, neural networks, stability synthesis

### 1. INTRODUCTIN

Control applications in advanced mechatronic technological make are used of various types of representation models of the so-called complex systems (Astroem, et al., 2001) that are developed or chosen according to certain specific properties of the plant process to be controlled. The control of complex mechatronic systems, consisting of several subsystems, has become a rather important research focus of control community at large. In particular, the robust adaptive control of mechatronic systems that are composite

interconnected, non-linear, and uncertain is still an open problem due to various phenomena co-existing simultaneously in the plant (Narendra and Annaswamy, 1989).

Typically, with regard to non-boundary uncertainties in plant subsystems, the known adaptive control approaches are all by and large based on the model adaptive control concepts (e.g. see Narendra and Annaswamy, 1989; Ionnou and Kokotovic, 1993). Then the stability of the overall system is ensured by a certain condition imposed on the  $M$  matrix that is related to the boundary of the interconnection. Thus complete and precise knowledge of the parameters in interconnections and/or subsystems is no longer a necessity. A typical deficiency of these approaches stems from the fact that it is difficult to validate the positive definiteness of the  $M$  matrix beforehand, because elements of the  $M$  matrix depended on the subsystem uncertain parameters. Additional progress along these lines in adaptive control of strongly interconnected systems has been reported (e.g. see Han and Chewn, 1993; Osman, 1989).

The study of S.-Y. Zhang (1994) has made new discoveries about the structural impact on complex control systems of when properties of similarity or symmetry among interconnected plant subsystems exist. Further clarifying considerations about the uncertainty composite non-linear systems in motion guidance problem and control of electromechanical plants have been contributed in studies (Gao, 1988; Gao and Wang, 1994). Work (Gao, 1988) used time-varying linear composite uncertainty system to design a non-linear controller that made system real stable in closed loop by exploiting Riccati equations. Studies (Jiang, et al., 1995; Yan, et al., 1996) first provided solutions to control of non-linear composite systems that exploit essentially the plant similarity property. In (Gao, 1988) and (Qu, 1996), respectively, the guarantee for the overall system to be made real stable was obtained through employing adaptive control. However, the subsystem uncertainty input gains has not been included in these studies.

Recently (Zhou, 1999) and (Zhu, et al., 1999) gave certain solutions to this problem by making use of neural networks. These studies, however, treat only specific linear composite systems with non-varying parameters. Besides, these results are far from ideal with regard to the uncertainty input gains (boundary of which was uncertain), albeit referring to the common large-scale systems. Also recently in (Liu, 1999) a corrective solution to adaptive control for uncertain similarity composite systems has been derived. Another technique, which employs high-order neural networks (Kosmatopoulos, et al., 1995) to emulate uncertainty interconnections, was recently proposed in (Zhang, 2000). And work (Zhang, et al., 2001) proposed a similar approach for the non-linear similarity composite systems simultaneously having uncertainties in the subsystem input gains and in the interconnections. The present paper extends the latter technique for applications to electromechanical and mechatronic plants.

Further this paper is written as follows. Section 2 presents the formal problem statement and the basic assumptions adopted. Subsequent Section 3 presents the main new result along with the constructive control design. Section 4 presents the application to the case-study of a real-world complex mechatronic system of axis-tray drive to illustrate the proposed design synthesis. Conclusions and references are given thereafter.

## 2. PROBLEM STATEMENT AND ASSUMPTIONS

Consider the class of complex non-linear systems possessing similarity and uncertainties that are represented by the following set,  $i = 1, 2, \dots, N$ , of equations:

$$\dot{x}_i = A_i x_i + B_i [u_i + f_i(x_i, t)] + h_i(x). \quad (1)$$

Here, the respective symbols denote:  $x_i \in R^n$  and  $u_i \in R^m$  are the state and input vectors of the  $i$ -th subsystem, respectively; the overall system state vector is  $x = (x_1^T, x_2^T, \dots, x_N^T)^T$  by definition;  $A_i \in R^{n \times n}$  and  $B_i \in R^{n \times m}$  are subsystem state and input matrices, respectively. Further, the basic assumption about this representation model include: all pairs  $(A_i, B_i)$  are controllable; all  $f_i(x_i, t)$  are uncertain vector-valued functions; and all  $h_i(x)$  are uncertain smooth vector-valued function.

*Remark 1.* In this paper, we suppose that system (1) belongs to the class of composite similarity systems.

*Definition 1* (Zhang, 1994). System (1) is said to be a similarity composite system if there is a parameter matrix  $T_i$  such that

$$T_i^{-1} A_i T_i = A, \quad T_i^{-1} B_i = B. \quad (2)$$

Also it is necessary to observe the following properties and results, respectively.

*Assumption 1.* There exist uncertain functions  $\rho_i(x_i, t) > 0$  such that

$$\|f_i(x_i, t)\| \leq \rho_i(x_i, t), \quad i = 1, 2, \dots, N, \quad (3)$$

*Lemma 1* (Zhu, et al., 1999). If pair  $(A, B)$  is controllable, then for a positive definite matrix,  $Q$ , and a positive number,  $\gamma > 0$ , there exists a unique solution  $P$  for the following equation:

$$A^T P + PA - \gamma^2 P B B^T P + Q = 0. \quad (4)$$

*Definition 2* (Qu, 1996). Suppose  $V = V(x)$  is a Lyapunov function of a given continuous system that satisfies

$$\gamma_1(\|x\|) \leq V(x, t) \leq \gamma_2(\|x\|), \quad (5)$$

$$\dot{V}(x, t) \leq -\gamma_3(\|x\|) + \varphi(t) \quad (6)$$

where  $\lim_{t \rightarrow \infty} \gamma_j(s) = \infty$ ,  $1 \leq j \leq 3$ , is a strictly continuous positive-definite function. If there exist  $\varphi(t)$  is a positive continuous function such that satisfies  $\varphi(t) \leq r^2 < \infty$ , then the system state does converge to some neighbourhood of the origin,  $\|x(t)\| \leq r$ , and the system is said to be *real stable*.

### 3. THE MAIN NEW RESULT

It should be noted first, due to the function approximation (Kolmogorov, 1957) capacity of artificial neural networks (Hecht-Nielsen, 1987; Kurkova, 1991, 1992; Katsuura and Sprecher, 1994), an arbitrary continuous function  $g(x, t)$  can be represented by an ANN, which consists of a directed graph with ideal weights,  $W_g$ , summing junction and output activation function  $\sigma(\cdot)$  (Haykin, 1999; Dimirovski and Andreeski, 2003; Jing and Dimirovski, 2003). That is  $g(x, t) = W_g^T \sigma_g(x) + \varepsilon_g(t)$ . Suppose the artificial neural network (ANN) estimation of  $g(x, t)$  is given by means of the formula  $\hat{g}(x, t) = \hat{W}_g^T \sigma_g(x)$ . Then the function estimate error  $\tilde{g}(x, t)$  can be calculated by the following equation

$$\tilde{g}(x,t) = W_g^T \sigma_g(x) - \hat{W}_g^T \sigma_g(x) + \varepsilon_g(t) \quad (7a)$$

or, putting  $\tilde{W}_g = W_g - \hat{W}_g$  yields

$$\tilde{g}(x,t) = \tilde{W}_g^T \sigma_g(x) + \varepsilon_g(t). \quad (7b)$$

Secondly, a model of the non-linear interconnections  $h_i(x)$ ,  $i = 1, 2, \dots, N$ , can be built by employing high-order neural nets (Zhang, 2000; Zhang, et al., 2001). Namely, suppose  $x$  be the input of the high-order neural network and  $y_i$  be the output; then, the respective representation model is given by

$$y_i = \hat{W}_i s(x), \quad s(x) = [s_i(x), \dots, s_L(x)]^T, \quad (8a)$$

$$s_i(x) = \prod_{k=1}^N \prod_{j \in I_i} [s(x_{kj})]^{d_j(i)}, \quad i = 1, 2, \dots, N, \quad (8b)$$

$$s(x_{kj}) = \frac{\mu_0}{1 + e^{-l_0 x_{kj}}} + \lambda_0, \quad (8c)$$

$$j = 1, 2, \dots, n, \quad k = 1, 2, \dots, N.$$

where the symbols denote:  $\hat{W}_i \in R^{n \times L}$  is the matrix of weights;  $S_i(x)$  is an element of  $s(x) \in R^{L \times 1}$ ;  $\{I_i | i = 1, 2, \dots, L\}$  is a collection of  $L$  not-ordered subsets of  $\{i = 1, 2, \dots, n\}$ ;  $d_j(i)$  is a nonnegative integer; and  $l_0$ ,  $\lambda_0$ ,  $\mu_0$  appropriate nonnegative constants. For these models of high-order neural networks, there exists an integer  $L$ , an integer  $d_j(i)$  and an optimized matrix  $W_i^*$ , such that for any  $\varepsilon > 0$ ,  $|h_i(x) - W_i^* s(x)| \leq \varepsilon$  is satisfied. In other words, if the high-order network is large enough, there exists a matrix of weights such that  $W_i^* s(x)$  can approximate  $h_i(x)$  to any degree of accuracy, moreover  $W_i^*$  is bounded, i.e.  $\|W_i^*\| \leq M_W$ ,  $M_W > 0$ .

It is therefore that the system (1),  $i = 1, 2, \dots, N$ , can be rewritten in the following form:

$$\dot{x}_i = Ax_i + B_i[u_i + f_i(x_i, t)] + W_i^* s(x) + \varepsilon_i(x) \quad (9)$$

where  $\varepsilon_i(x) = h_i(x) - W_i^* s(x)$  is the weight error estimate. Furthermore, there exists  $\varepsilon \geq 0$  such that  $|\varepsilon_i(x)| \leq \varepsilon$ ,  $i = 1, 2, \dots, N$ . Should  $W_i$  denotes the estimation of the uncertain weights matrix  $W_i^*$ ,  $i = 1, 2, \dots, N$ , then one finds:

$$\dot{x}_i = Ax_i + B_i[u_i + f_i(x_i, t)] - \tilde{W}_i s(x) + W_i s(x) + \varepsilon_i(x), \quad (10)$$

where  $\tilde{W}_i = W_i - W_i^*$ .

*Theorem 1.* Consider system (1) and construct the set of controllers,  $i = 1, 2, \dots, N$ , that are represented by

$$u_i = u_i^a + u_i^b + u_i^c, \quad i = 1, 2, \dots, N, \quad (11)$$

$$u_i^a = -(\gamma + 1)B^T P T_i^{-1} x_i, \quad (12)$$

$$u_i^b = -\xi_i^{-1} \hat{\rho}_i^2(x, t) B^T P T_i^{-1} x_i, \quad (13)$$

$$u_i^c = \frac{B_i^T W_i s(x)}{\lambda_i [1 + \|B_i\|^2]} + \frac{B_i^T \Theta_i}{\lambda_{1,i} [1 + \|B_i\|^2]}, \quad (14)$$

where:  $\hat{\rho}_i^2$  is the estimation of  $\rho_i^2$  by means of a neural-net model, i.e.  $\hat{\rho}_i^2(x, t) = \hat{Z}_i(t)\sigma_i(x_i)$  with  $\hat{Z}_i(t)$  as the corresponding weight vector;  $\Theta_i \in R^{n \times L}$  and  $\Theta_i = [\theta_i, 0, \dots, 0]^T$ ; and quantities defined in the course of theorem proving  $\beta_i, \gamma, \gamma_{1,j}, \lambda_i, \lambda_{1,i}$  are chosen as

$$\lambda_i \geq \frac{k_{0,i}s}{\sqrt{2\bar{k}_2^i \beta_i - sk_{0,i}}}, \quad (15a)$$

$$\lambda_{1,i} \geq \frac{k_{0,i}}{\sqrt{2k_{0,i}\bar{k}_2^i \gamma_{1,i} - k_{0,i}}}, \quad (15b)$$

$$\beta_i > \frac{s^2 k_{0,i}^2}{2\bar{k}_2^i}, \quad \gamma_{1,i} > \frac{k_{0,i}}{2\bar{k}_2^i}; \quad (15c)$$

along with the adaptation law

$$\dot{\hat{Z}}_i = -\Gamma_{i1}\hat{Z}_i + \Gamma_{i2}\sigma_i(x_i) \|B^T P T_i^{-1} x_i\|, \quad \Gamma_{i1}, \Gamma_{i2} > 0, \quad (16)$$

$$\dot{W}_i = \begin{cases} 2k_{0,i}P^T T_i^{-1}x_i S(x)^T, & \|W_i\| < M_W \\ -\beta_i W_i + 2k_{0,i}P^T T_i^{-1}x_i S(x)^T, & \|W_i\| \geq M_W \end{cases}, \quad (17)$$

$$\dot{\theta}_i = -\gamma_{1,i}\theta_i + 2k_{0,i}|P^T T_i^{-1}x_i|, \quad (18)$$

with  $\Gamma_{i1}, \Gamma_{i2}, k_{0,i}, \beta_i$ , and  $\gamma_{1,i}$  all positive constant design parameters and  $M_W$  is a large design constant that confines  $\theta_i$  within a ball of radius  $M_W$  to be chosen in the course of design. Then assuming that the neural-net approximation error  $\varepsilon_{\rho_i}(t)$  is time-varying and bounded with an uncertain boundary, and all subsystem state vectors  $x_i$ , respectively, are consistent ultimately bounded on sets

$$D_i = \left\{ x_i \in R^n \mid v_{0i}(x) \leq \frac{\mu_i}{k_{0,i}\alpha_i}, \frac{\bar{k}_2^i}{\bar{k}_1^i} \leq k_{0,i} \leq 1 \right\}. \quad (19)$$

The proof of Theorem 1 (see Appendix) evolves in two steps. In Step 1, first it is proved that there exist nominal controllers  $u_i = u_i^a + u_i^b : R^n \rightarrow R^m$  and Lyapunov functions  $V_{0i}(x_i)$  for the nominal subsystems

$$\dot{x}_i = A_i x_i + B_i [u_i + f_i(x_i, t)]$$

such that inequalities (5) and (6) are satisfied; that is, the nominal subsystems are proved to be real stable. In Step 2, the result of Step 1 is adopted as a-priori assumption, and then the proof is derived to the full via Lyapunov approach by finding the domains of attraction represented by sets (19).

*Remark 2* The assumption on uncertain boundary on time-varying approximation error  $\varepsilon_{p_i}(t)$  in this theorem is more appropriate and rational than the previous one proposed in (Qu, 1996). Hence  $\Gamma_{i1}$ ,  $\Gamma_{i2}$ ,  $k_{0,i}$ ,  $\beta_i$ ,  $\gamma_{1,j}$ , and  $M_W$  are well conceptualized.

#### 4. APPLICATION TO A MECHATRONIC SYSTEM: AXIS-TRAY DRIVE

In the sequel, the application of the above theory to a real-world case of a mechatronic system with strong interconnection is presented. The mechatronic system of the axis-tray drive (Gao and Wang, 1994), the schematic of which is depicted in Figure 1, is considered.

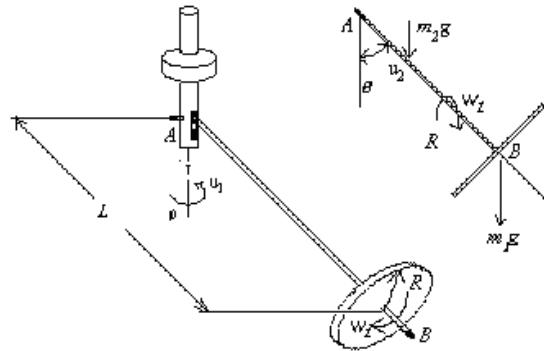


Fig. 1. Schematic diagram of the axis-tray drive mechatronic system

The equations of the system dynamics have been derived elsewhere (Gao and Wang, 1994). These are the following ones:

$$\begin{cases} \dot{x}_{11} = x_{12} \\ \dot{x}_{12} = \frac{1}{I_1 \cos^2 x_{21} + I_2 \sin^2 x_{21}} [(I_1 - I_2) \sin(2x_{21}) + I_1 \omega_1 x_{22} + u_1] \\ \dot{x}_{21} = x_{22} \\ \dot{x}_{22} = \frac{1}{I_2} \left[ -\frac{(I_1 - I_2)}{2} x_{12}^2 \sin(2x_{21}) + I_1 \omega_1 x_{12} \sin(x_{21}) + (m_1 + 0.5m_2)g/\sin(x_{21}) + u_2 \right] \end{cases}$$

For reasons of comparison with the results in source reference, the case with parameter values  $m_1 = 0.5$ ,  $m_2 = 1$ ,  $I_1 = I_2 = 1$ , and parameters  $l = 1$ ,  $g = 0.098$ ,  $\omega = 0.1 \sin(10t)$  as in (Gao and Wang, 1994) is considered. Then these equations can be transformed into similarity structure system representation as follows:

$$\begin{aligned} \begin{bmatrix} \dot{x}_{11} \\ \dot{x}_{12} \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_1 + \begin{bmatrix} 0 \\ 0.1 \sin(10t) x_{22} \end{bmatrix}, \\ \begin{bmatrix} \dot{x}_{21} \\ \dot{x}_{22} \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_{21} \\ x_{22} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} (u_2 + g/l \sin x_{21}) + \begin{bmatrix} 0 \\ 0.1 \sin(10t) x_{12} \sin(x_{21}) \end{bmatrix}. \end{aligned}$$

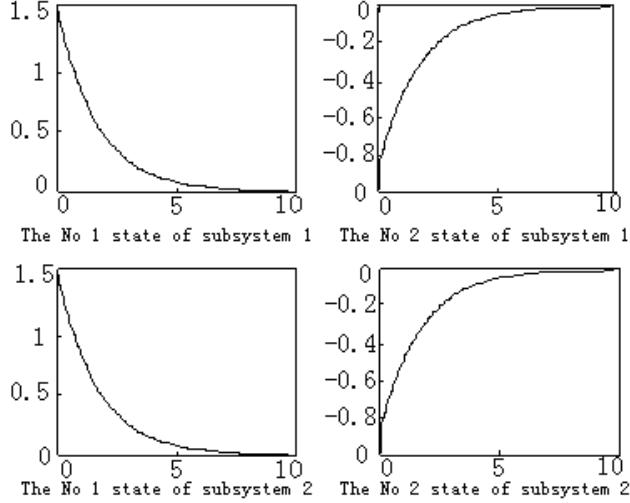


Fig. 2. Simulation results emulation the state variables of the controlled mechatronic axis-tray drive system.

Now, should one chooses  $\gamma = 1$ ,  $Q = I$ , then by solving the Riccati equation, one obtains:

$$P = \begin{bmatrix} 1.7321 & 1 \\ 1 & 1.7321 \end{bmatrix}.$$

By making use Theorem 1, the adaptive controller represented by the following set of equations:

$$u_i^a = -(\gamma + 1)B^T Px_i = -2[0 \ 1]P x_i,$$

$$u_i^b = -\xi_i^{-1}\hat{\rho}_i^2 B^T Px_i = -50\hat{\rho}_i^2[0 \ 0.2]P x_i,$$

$$u_i^c = \frac{B^T W_i s(x)}{\lambda_i [1 + \|B\|^2]} = \frac{[0 \ 1] \begin{bmatrix} W_{11} \\ W_{12} \end{bmatrix} s(x_{12}) s(x_{22})}{0.001[1 + \| [0 \ 1] \|^2]},$$

where parameters are evaluated as  $\xi_i = 0.02$ ,  $\lambda_i = 0.001$ . For the purpose of on-line emulation of  $\hat{\rho}_i^2$  one can use an artificial neural network with three layers that have 6 nodes. Similarly, for on-line emulation of  $s_i(x)$  one can make use of an artificial neural network with three layers that have 2 nodes. Thus in addition to the above equations, the learning law is defined by means of the following equations:

$$\dot{\hat{z}}_i = -\Gamma_1 \hat{z}_i + \Gamma_2 \sigma(x_i) \| B^T Px_i \| = -0.1 * \hat{z}_i + 500 * \sigma(x_i) * \| [0 \ 1] P x_i \|$$

$$W_{i,j} = 2k_{0,i} P_i^T x_i s(x) = 2 \cdot 0.4 P_i^T x_i s(x_{12}) s(x_{22})$$

It should be noted that in  $u_i^c$ , if  $\theta_i$  is counteracted, then  $\theta_i$  needs no learning emulation. For the purpose of computer simulation the following initial state values  $[1.5 - 1.0]^T$  and  $[1.5 - 1.0]^T$ , and a short time interval  $t = [0, 10]$  are chosen.

The simulation results for the controlled system states in closed loop using the controllers, which employ ANN emulation of interconnection and input gains, are given in Figure 2. It may well be seen that the overall system employing the designed controller exhibits high-quality performance in terms of both fast response and no overshooting, which is superior to the previously obtained results to the best of our knowledge. It is interesting to note, similar quality controlled performance was obtained for the plant of two elastically interconnected inverted pendulums on carts (Qu, 1996), i.e. a composite mechatronic system having elastic interconnections.

## 5. CONCLUSION

A novel robust adaptive, neural-network based, control solution is presented for complex non-linear mechatronic systems possessing uncertainties in both subsystem gains and interconnections provided plant has similarity structure. The low-order neural networks and the high-order one are banded together to resolve the uncertainty issues on both the input subsystem gains and the interconnections. Therefore the controllers designed by using our technique possess robustness and adaptability.

Two steps are adopted in solving control design problem. The first step makes use of low-order artificial neural nets to design decentralized controllers in order to make the nominal subsystems real stable. In this step, due to on-line adaptation of the ANN weights, little a-priori knowledge is needed and yet the transient performance of the system is considerably improved. The second step, in fact, makes use of the first step as an assumption, employs high-order artificial neural nets to handle the interconnections term of the system.

## APPENDIX – PROOF OF THEOREM 1

*Step 1.* Due to the known condition that  $(A_i, B_i)$  is controllable and the formula (2), it can be deduced that  $(A, B)$  is controllable. Moreover by virtue of Lemma 1, there exists a  $P$  such that

$$A^T P + PA - \gamma^2 PBB^T P + Q = 0$$

is satisfied. Next, consider Lyapunov functions

$$v_{0i}(x_i) = x_i^T (T_i^{-1})^T P T_i^{-1} x_i + \hat{Z}_i^T \Gamma_{i2}^{-1} \hat{Z}_i \quad (20)$$

Then by using (1), the time derivative of Lyapunov functions (20) can be expressed as follows:

$$\begin{aligned} \dot{v}_{0i}(x_i) &= \{x_i^T A_i^T + [u_i^T + f_i^T(x_i, t)] B_i^T\} [(T_i^{-1})^T P T_i^{-1}] x_i + \\ &\quad x_i^T [(T_i^{-1})^T P T_i^{-1}] \{A_i x_i + B_i [u_i + f_i(x_i, t)]\} + 2 \dot{\hat{Z}}_i^T \Gamma_{i2}^{-1} \hat{Z}_i = \end{aligned}$$

$$\begin{aligned}
& x_i^T A_i^T (T_i^{-1})^T P T_i^{-1} x_i + x_i^T (T_i^{-1})^T P T_i^{-1} A_i x_i + 2x_i^T (T_i^{-1})^T P T_i^{-1} B_i [u_i + f_i(x_i, t)] + 2\dot{\hat{Z}}_i^T \Gamma_{i2}^{-1} \hat{Z}_i = \\
& x_i^T (T_i^{-1})^T [T_i^T A_i^T (T_i^{-1})^T] P T_i^{-1} x_i + x_i^T (T_i^{-1})^T P [T_i^{-1} A_i T_i] T_i^{-1} x_i + \\
& 2x_i^T (T_i^{-1})^T P T_i^{-1} B_i [u_i + f_i(x_i, t)] + 2\dot{\hat{Z}}_i^T \Gamma_{i2}^{-1} \hat{Z}_i = \\
& x_i^T (T_i^{-1})^T [A^T P + PA] T_i^{-1} x_i + 2x_i^T (T_i^{-1})^T P B [u_i + f_i(x_i, t)] + 2\dot{\hat{Z}}_i^T \Gamma_{i2}^{-1} \hat{Z}_i
\end{aligned}$$

Furthermore, by using (10) and (11), one can obtain

$$\begin{aligned}
& \dot{v}_{0i}(x_i) = -x_i^T (T_i^{-1})^T Q T_i^{-1} x_i - \\
& x_i^T (T_i^{-1})^T \gamma^2 P B B^T P T_i^{-1} x_i + 2x_i^T (T_i^{-1})^T P B \\
& [-(\gamma+1)B^T P T_i^{-1} x_i - \xi_i^{-1} \hat{\rho}_i^2 (x, t) B^T P T_i^{-1} x_i + f_i(x_i, t)] + 2\dot{\hat{Z}}_i^T \Gamma_{i2}^{-1} \hat{Z}_i = \\
& -x_i^T (T_i^{-1})^T Q T_i^{-1} x_i - x_i^T (T_i^{-1})^T \gamma^2 P B B^T P T_i^{-1} x_i + 2x_i^T (T_i^{-1})^T P B \gamma B^T P T_i^{-1} x_i \\
& - 2x_i^T (T_i^{-1})^T P B B^T P T_i^{-1} x_i + 2x_i^T (T_i^{-1})^T P B [-\xi_i^{-1} \hat{\rho}_i^2 (x, t) B^T P T_i^{-1} x_i + f_i(x_i, t)] + 2\dot{\hat{Z}}_i^T \Gamma_{i2}^{-1} \hat{Z}_i = \\
& -x_i^T (T_i^{-1})^T Q T_i^{-1} x_i - x_i^T (T_i^{-1})^T \gamma^2 P B B^T P T_i^{-1} x_i - 2x_i^T (T_i^{-1})^T P B \gamma B^T P T_i^{-1} x_i - \\
& 2x_i^T (T_i^{-1})^T P B B^T P T_i^{-1} x_i + 2[f_i(x_i, t) B^T P T_i^{-1} x_i - \xi_i^{-1} (\rho_i^2 - \varepsilon_{\rho_i})] \\
& \|B^T P T_i^{-1} x_i\|^2 + 2\dot{\hat{Z}}_i^T \Gamma_{i2}^{-1} \hat{Z}_i.
\end{aligned}$$

By making use of (16), the above formulas can be rewritten as

$$\begin{aligned}
& \dot{v}_{0i}(x_i) \leq -x_i^T (T_i^{-1})^T Q T_i^{-1} x_i - x_i^T (T_i^{-1})^T P B B^T P T_i^{-1} x_i + \\
& \varepsilon_{\rho_i} x_i^T (T_i^{-1})^T P B B^T P T_i^{-1} x_i - \hat{Z}_i^T \Gamma_{i2}^{-1} \Gamma_{i1} \hat{Z}_i + \xi_i \\
& \leq -x_i^T (T_i^{-1})^T Q T_i^{-1} x_i + \varepsilon_{\rho_i}^2 - \hat{Z}_i^T \Gamma_{i2}^{-1} \Gamma_{i1} \hat{Z}_i + \xi_i \leq -\lambda_i^* v_{0i} + \varepsilon_{\rho_i}^2 + \xi_i
\end{aligned} \tag{21}$$

with  $\lambda_i^* = \min\left\{\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}, \lambda_{\min}(\Gamma_{i1})\right\}$ . Moreover, there can be found another form for the derivatives of Lyapunov functions:

$$\begin{aligned}
& \dot{v}_{0i}(x_i) \leq -x_i^T Q x_i + \varepsilon_{\rho_i}^2 - \hat{Z}_i^T \Gamma_{i2}^{-1} \Gamma_{i1} \hat{Z}_i + \xi_i \leq -x_i^T Q x_i + \varepsilon_{\rho_i}^2 + \xi_i \\
& \leq -\frac{\lambda_{\min}(Q)}{4\lambda_{\max}(PP^T)} x_i^T (T_i^{-1})^T P P^T T_i^{-1} x_i + \varepsilon_{\rho_i}^2 + \xi_i \leq -\bar{k}_i \left| \frac{\partial v_{0i}(x_i)}{\partial x_i} \right|^2 + \varepsilon_{\rho_i}^2 + \xi_i
\end{aligned} \tag{22}$$

with  $\bar{k}_i = \frac{\lambda_{\min}(Q)}{4\lambda_{\max}(PP^T)}$ . From (21) and (22) along with Definition 1, it is readily inferred all the nominal subsystems are real stable in closed loop.

*Step 2:* Construct Lyapunov function for the system (1) in the following form

$$V = V(x, \tilde{W}_1, \tilde{W}_2, \dots, \tilde{W}_N, \tilde{\theta}_1, \tilde{\theta}_2, \dots, \tilde{\theta}_N) = \sum_{i=1}^N V_i(x, \tilde{W}_i, \tilde{\theta}_i), \tag{23}$$

$$V_i(x, \tilde{W}_i, \tilde{\theta}_i) = k_{0,i} v_{0i}(x_i) + \frac{1}{2} \text{tr}\{\tilde{W}_i^T \tilde{W}_i\} + \frac{1}{2} \tilde{\theta}_i^2, \tag{24}$$

with  $\tilde{\theta}_i = \theta_i - \varepsilon$ . By virtue of (10), the derivatives of  $V_i(x, \tilde{W}_i, \tilde{\theta}_i)$  are found to be:

$$\begin{aligned}\dot{V}_i &= k_{0,i} \frac{\partial v_{0i}}{\partial x_i} \{ A_i x_i + B_i [u_i^a + u_i^b + f_i(x_i, t)] \} + k_{0,i} \frac{\partial v_{0i}}{\partial x_i} B_i u_i^c - k_{0,i} \frac{\partial v_{0i}}{\partial x_i} \tilde{W}_i S(x) + \\ &\quad k_{0,i} \frac{\partial v_{0i}}{\partial x_i} W_i S(x) + k_{0,i} \frac{\partial v_{0i}}{\partial x_i} \varepsilon(x) + \text{tr}\{\dot{W}_i^T \tilde{W}_i\} + \tilde{\theta}_i \dot{\theta}_i.\end{aligned}\quad (25)$$

By making use of (17), one obtains

$$\dot{V}_i = k_{0,i} \dot{v}_{0i} + k_{0,i} \frac{\partial v_{0i}}{\partial x_i} B_i u_i^c + k_{0,i} \frac{\partial v_{0i}}{\partial x_i} W_i S(x) + k_{0,i} \frac{\partial v_{0i}}{\partial x_i} \varepsilon_i(x) - \beta_i I_W \text{tr}\{W_i^T \tilde{W}_i\} + \tilde{\theta}_i \dot{\theta}_i, \quad (26)$$

where  $I_W$  is the indicator function of  $W$  that satisfies

$$I_W = \begin{cases} 1 & \text{if } \|W_i\| \geq M_W \\ 0 & \text{if } \|W_i\| < M_W \end{cases}. \quad (27)$$

Since  $\text{tr}\{W_i^T \tilde{W}_i\} = \frac{1}{2} \|W_i\|^2 + \frac{1}{2} \|\tilde{W}_i\|^2 - \frac{1}{2} \|W_i^*\|^2$ , one can derive:

$$\begin{aligned}\dot{V}_i &= k_{0,i} \dot{V}_i + k_{0,i} \frac{\partial v_{0i}}{\partial x_i} B_i u_i^c + k_{0,i} \frac{\partial v_{0i}}{\partial x_i} W_i S(x) + k_{0,i} \frac{\partial v_{0i}}{\partial x_i} \varepsilon_i(x) - \\ &\quad \frac{\beta_i}{2} \text{tr}\{\tilde{W}_i^T \tilde{W}_i\} + \frac{\beta_i}{2} (1 - I_W) \text{tr}\{\tilde{W}_i^T \tilde{W}_i\} - \frac{\beta_i}{2} I_W \|W_i\|^2 + \frac{\beta_i}{2} I_W \|W_i^*\|^2 + \tilde{\theta}_i \dot{\theta}_i.\end{aligned}\quad (28)$$

The  $u_i^c(\lambda_i, \lambda_{1,i})$  (notice that these two parameters can be adjusted) are put into (24) and, due to Assumption 1, it is found in turn that the time derivatives of  $V_i$  satisfy the following inequality:

$$\begin{aligned}\dot{V}_i &\leq k_{0,i} \bar{k}_i \left| \frac{\partial v_{0i}}{\partial x_i} \right|^2 + k_{0,i} \left| \frac{\partial v_{0i}}{\partial x_i} \right| \frac{\|B_i\|^2 \|W_i\| |S(x)|}{\lambda_i [1 + \|B_i\|^2]} + k_{0,i} \left| \frac{\partial v_{0i}}{\partial x_i} \right| \|W_i\| |S(x)| k_{0,i} \left| \frac{\partial v_{0i}}{\partial x_i} \right| \frac{\|B_i\|^2 \|W_i\| |\theta_i|}{\lambda_i [1 + \|B_i\|^2]} + \\ &\quad \frac{\beta_i}{2} (1 - I_W) \text{tr}\{\tilde{W}_i^T \tilde{W}_i\} - \frac{\beta_i}{2} I_W \|W_i\|^2 + \frac{\beta_i}{2} I_W \|W_i^*\|^2 + k_{0,i} (\xi_i + \varepsilon_{p_i}^2(t)).\end{aligned}\quad (29)$$

Because of  $\frac{\|B_i\|^2}{1 + \|B_i\|^2} \leq 1$ , relationship (25) can be rewritten as follows:

$$\begin{aligned}\dot{V}_i &\leq k_{0,i} \bar{k}_i \left| \frac{\partial v_{0i}}{\partial x_i} \right|^2 + k_{0,i} \left| \frac{\partial v_{0i}}{\partial x_i} \right| \|W_i\| |S(x)| \left( 1 + \frac{1}{\lambda_i} \right) + \\ &\quad k_{0,i} \left| \frac{\partial v_{0i}}{\partial x_i} \right| \frac{|\theta_i|}{\lambda_{1,i}} + k_{0,i} \left| \frac{\partial v_{0i}}{\partial x_i} \right| \theta_i - k_{0,i} \left| \frac{\partial v_{0i}}{\partial x_i} \right| \theta_i + k_{0,i} \left| \frac{\partial v_{0i}}{\partial x_i} \right| \varepsilon_i + \tilde{\theta}_i \dot{\theta}_i - \frac{\beta_i}{2} \text{tr}\{\tilde{W}_i^T \tilde{W}_i\} + \\ &\quad \frac{\beta_i}{2} (1 - I_W) \text{tr}\{\tilde{W}_i^T \tilde{W}_i\} - \frac{\beta_i}{2} I_W \|W_i\|^2 + \frac{\beta_i}{2} I_W \|W_i^*\|^2 + k_{0,i} (\xi_i + \varepsilon_{p_i}^2(t)).\end{aligned}\quad (30)$$

Now, should the following definition  $\bar{k}_i = \bar{k}_1^i + \bar{k}_2^i + \bar{k}_3^i$  is introduced, then relationship (26) can be transformed into the following one:

$$\begin{aligned} \dot{V}_i &\leq -k_{0,i}\bar{k}_1^i\left|\frac{\partial v_{0i}}{\partial x_i}\right|^2 - k_{0,i}\bar{k}_2^i\left|\frac{\partial v_{0i}}{\partial x_i}\right|^2 - k_{0,i}\bar{k}_3^i\left|\frac{\partial v_{0i}}{\partial x_i}\right|^2 + k_{0,i}s\left|\frac{\partial v_{0i}}{\partial x_i}\right|\|W_i\|\left(1 + \frac{1}{\lambda_i}\right) + \\ &k_{0,i}\left|\frac{\partial v_{0i}}{\partial x_i}\right|\theta_i\left(1 + \frac{1}{\lambda_{1,i}}\right) - \frac{\gamma_{1,i}}{2}\tilde{\theta}_i^2 - \frac{\gamma_{1,i}}{2}\theta_i^2 + \frac{\gamma_{1,i}}{2}\varepsilon^2 - \frac{\beta_i}{2}\text{tr}\{\tilde{W}_i^T\tilde{W}_i\} + \\ &\frac{\beta_i}{2}(1-I_W)\text{tr}\{\tilde{W}_i^T\tilde{W}_i\} - \frac{\beta_i}{2}I_W\|W_i\|^2 + \frac{\beta_i}{2}I_W\|W_i^*\|^2 + k_{0,i}(\xi_i + \varepsilon_{\rho_i}^2(t)). \end{aligned} \quad (31)$$

Via appropriate choice of  $\lambda_i \geq \frac{k_{0,i}s}{\sqrt{2\bar{k}_2^i\beta_i - sk_{0,i}}}$ ,  $\lambda_{1,i} \geq \frac{k_{0,i}}{\sqrt{2k_{0,i}\bar{k}_2^i\gamma_{1,i}} - k_{0,i}}$ ,  $\beta_i > \frac{s^2k_{0,i}^2}{2\bar{k}_2^i}$ ,

(see inequalities (15 a-c)) and  $\gamma_{1,i} > \frac{k_{0,i}}{2\bar{k}_2^i}$ , from (31) it can be found:

$$\begin{aligned} \dot{V}_i &\leq -k_{0,i}\bar{k}_1^i\left|\frac{\partial v_{0i}}{\partial x_i}\right|^2 - k_{0,i}\bar{k}_3^i\left|\frac{\partial v_{0i}}{\partial x_i}\right|^2 - \left[\bar{k}_2^i\left|\frac{\partial v_{0i}}{\partial x_i}\right|^2 - 2\sqrt{\frac{\bar{k}_2^i\beta_i}{2}}\left|\frac{\partial v_{0i}}{\partial x_i}\right|\|W_i\| + \frac{\beta_i}{2}\|W_i\|^2\right] + \\ &\left[\left(\sqrt{k_{0,i}\bar{k}_2^i}\left|\frac{\partial v_{0i}}{\partial x_i}\right|\right)^2 - 2\sqrt{\frac{k_{0,i}\bar{k}_2^i\gamma_{1,i}}{2}}\theta_i\left|\frac{\partial v_{0i}}{\partial x_i}\right| + \frac{\gamma_{1,i}}{2}\theta_i^2\right] + \frac{\beta_i}{2}\|W_i\|^2 + \bar{k}_2^i\left|\frac{\partial v_{0i}}{\partial x_i}\right|^2 - \frac{\gamma_{1,i}}{2}\tilde{\theta}_i^2 - \frac{\gamma_{1,i}}{2}\theta_i^2 + \\ &\frac{\gamma_{1,i}}{2}\varepsilon^2 - \frac{\beta_i}{2}\text{tr}\{\tilde{W}_i^T\tilde{W}_i\} + \frac{\beta_i}{2}(1-I_W)\text{tr}\{\tilde{W}_i^T\tilde{W}_i\} - \frac{\beta_i}{2}I_W\|W_i\|^2 + \frac{\beta_i}{2}I_W\|W_i^*\|^2 + k_{0,i}(\xi_i + \varepsilon_{\rho_i}^2(t)). \end{aligned} \quad (32)$$

Notice that: if  $k_{0,i} \geq \frac{\bar{k}_2^i}{\bar{k}_1^i}$  is satisfied, then  $-k_{0,i}\bar{k}_1^i + \bar{k}_2^i \leq 0$  ( $\bar{k}_1^i \geq 0$ ) is true; otherwise,

if  $k_{0,i} \leq 1$  is true, then  $k_{0,i} \geq \frac{\bar{k}_2^i}{\bar{k}_1^i}$ . Thus, if  $\frac{\bar{k}_2^i}{\bar{k}_1^i} \leq k_{0,i} \leq 1$  is satisfied, then (32) can be modified to give:

$$\begin{aligned} \dot{V}_i &\leq -k_{0,i}\bar{k}_3^i\left|\frac{\partial v_{0i}}{\partial x_i}\right|^2 - \frac{\beta_i}{2}\|\tilde{W}_i\|^2 + \frac{\beta_i}{2}\|W_i^*\|^2 + \\ &\beta_i(1-I_W)\text{tr}\{\tilde{W}_i^T\tilde{W}_i\} - \frac{\gamma_{1,i}}{2}\tilde{\theta}_i^2 + \frac{\gamma_{1,i}}{2}\varepsilon^2 + (1-I_W)\frac{\beta_i}{2}M_W^2 + k_{0,i}(\xi_i + \varepsilon_{\rho_i}^2(t)). \end{aligned} \quad (33)$$

Since

$$\beta_i(1-I_W)\text{tr}\{\tilde{W}_i^T\tilde{W}_i\} = \begin{cases} \beta_i\text{tr}\{\tilde{W}_i^T\tilde{W}_i\}, & \text{if } |W_i| < M_W \\ 0, & \text{if } |W_i| \geq M_W \end{cases}, \quad (34)$$

it follows

$$\beta_i(1 - I_W) \operatorname{tr}\{\tilde{W}_i^T \tilde{W}_i\} \leq \beta_i M_W^2. \quad (35)$$

Moreover, because of

$$(1 - I_W) \frac{\beta_i}{2} M_W^2 \leq \frac{\beta_i}{2} M_W^2, \quad (36)$$

inequality (33) can be transformed into the following form:

$$\begin{aligned} \dot{V}_i &\leq -k_{0,i} \bar{k}_3 \left| \frac{\partial v_{0i}}{\partial x_i} \right|^2 - \frac{\beta_i}{2} \|\tilde{W}_i\|^2 + \\ &\quad \frac{\beta_i}{2} M_W^2 + \beta_i M_W^2 + \frac{\beta_i}{2} M_W^2 - \frac{\gamma_{1,i}}{2} \tilde{\theta}_i^2 + \frac{\gamma_{1,i}}{2} \varepsilon^2 + k_{0,i} (\xi_i + \varepsilon_{\rho_i}^2(t)). \end{aligned} \quad (37)$$

Hence, making use of (21) and (22), one obtains

$$\dot{V}_i \leq -\frac{k_{0,i} \bar{k}_3 \lambda_i^*}{\bar{k}_i} v_{0i}(x_i) - \frac{\beta_i}{2} \|\tilde{W}_i\|^2 - \frac{\gamma_{1,i}}{2} \tilde{\theta}_i^2 + 2\beta_i M_W^2 + \frac{\gamma_{1,i}}{2} \varepsilon^2 + k_{0,i} (\xi_i + \varepsilon_{\rho_i}^2(t)), \quad (38)$$

and therefore

$$\dot{V}_i \leq -\alpha_i V_i + \mu_i, \quad \dot{V} = \sum_{i=1}^N \dot{V}_i \quad (39)$$

with

$$\alpha_i = \min \left\{ \frac{\bar{k}_3 \lambda_i^*}{\bar{k}_i}, \beta_i, \gamma_{1,i} \right\}, \quad (40a)$$

$$\mu_i = 2\beta_i M_W^2 + \frac{\gamma_{1,i}}{2} \varepsilon^2 + k_{0,i} (\xi_i + \varepsilon_{\rho_i}^2(t)). \quad (40b)$$

Integration of both sides of inequality (39) yields

$$V_i(t) \leq \frac{\mu_i}{\alpha_i} + \left[ V_i(0) - \frac{\mu_i}{\alpha_i} \right] e^{-\alpha_i t}, \quad \forall t \geq 0. \quad (41)$$

Thus, it is readily deduced from (41) that  $x$ ,  $\theta_i(x)$ ,  $W_i(x)$  are bounded consistently. On the other hand, from (24) it is seen that

$$k_{0,i} v_{0i}(x_i) \leq V_i \quad (42)$$

hence for all  $i = 1, 2, \dots, N$  and  $\forall t \geq 0$  it is valid

$$v_{0i}(x_i) \leq \frac{\mu_i}{k_{0,i} \alpha_i} + \frac{1}{k_{0,i}} \left[ V_i(0) - \frac{\mu_i}{\alpha_i} \right] e^{-\alpha_i t}. \quad (43)$$

From inequality (43) it follows at once that all vector-valued state variables  $x_i$ ,  $i = 1, 2, \dots, N$ , are consistently ultimately bounded on the sets

$$D_i = \left\{ x_i \in R^n : v_{0i}(x) \leq \frac{\mu_i}{k_{0,i} \alpha_i}, \frac{\bar{k}_2^i}{\bar{k}_1^i} \leq k_{0,i} \leq 1 \right\} \quad (44)$$

defining the attraction domains for all the subsystems, respectively. Thus the original class of composite similarity systems (1) under synthesized controls (11)-(18) is real stable in closed loop on sets  $D_i$ , which completes the proof.

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**ROBUSTNO ADAPTIVNO UPRAVLJANJE  
MEHATRONIČNIH SISTEMA KOJI KORISTE  
ANN EMULACIJU NONLINEARNIH FUNKCIJA**

**Yuan-Wei Jing, Yan-Xin Zhang, Tatjana Kolemeisevska-Gugulovska,  
Georgi M. Dimirovski, Miomir K. Vukobratovic**

*Izvodi se nova sinteza robustnog adaptivnog upravljanja koje koristi kako neuronske mreže visokog reda tako i matematičko-analitičke rezultate za klasu mehatroničkih nonlinearnih sistema sa sličnostima u ovom radu. Ovaj pristup predstavlja adekvatnu upotrebu strukturalnih osobina složenih sistema sa sličnostima i neuronskih mreža da bi razrešile neizvesno predstavljanje interkonekcija i pojačanja podsistema onlajn obnavljanjem težina. Ova sinteza garantuje realnu stabilnost u zatvorenim petljama i zahteva projektantsku veština da bi se postigli veći domeni privlačenja. Koristi se mehatronički primer prenosno-spregajuceg sistema ose, koji ima neizvesnosti, da bi se ilustrovala predložena tehnika.*

Ključne reči: *Adaptivna upravljanje, emulacija funkcija, mehatronički sistemi, neuronske mreže, sinteza stabilnosti*