FACTA UNIVERSITATIS
Series: **Mechanics, Automatic Control and Robotics** Vol. 6, Special Issue, 2007, pp. 53 - 70

QUANTITATIVE AND QUALITATIVE ESTIMATIONS OF THE MANIPULATOR END-EFFECTOR TRAJECTORY PLANNING BY SINERGETIC CRITERION

UDC 007.52+004.8(045)=111

Marina Kolesova, Aleksandr Polyakov, Mihail Kalinin

Department of the technical mechanics and machinery, Sevastopol National Technical University, SevNTU – FESTO, 99053, Sevastopol, Ukraine
e-mail: apolyakov.mail@gmail.com

Abstract. *In this work from several viewpoints the task of end-effector trajectory planning in two dimensional workspace for the model of two link planar anthropomorphous manipulator with revolute joints is considered. The set of solutions by criteria of the minimum torques-change, the minimum total work, and also by a new criterion of the minimum error of a linear synergy are received. Their relative quantitative and qualitative estimations are made. The criterion of the minimum error of a linear synergy is proved from the biomechanical point of view. It is shown that the task of a trajectory planning at its using has solutions, which correspond to a set of experimental data and in general are characterized by kinematic and dynamic invariance, which are peculiar to free movements of the human arm. The numerical solutions of the task are received by using of the Jacobi orthogonal polynomials for various directions of end-effector movements to the target and for two variants of manipulator links parameters.*

Key words: *end-effector, trajectory planning, synergetic criterion, minimum torques-change criterion, Poisson – Euler equations, orthogonal polynomials*

1. INTRODUCTION

It is known that redundancy (availability of the superfluous degrees of freedom) in the mechanisms of robots are promoted to increase dexterity and diversification of movements. However redundancy leads to ill-posed problem of inverse kinematics from the task space to the joint space. This problem formulated by N. Bernstein [4] (Bernstein's problem), till now does not remain solved both with the physiological and robotic point of view. In particular, ill-posedness appears at the task of the end-effector trajectories planning in workspace of the manipulators with redundant degrees of freedom, as first, the end-effector may approach the target along any arbitrary geometric curve (trajectory) as long as this curve connects the initial and target point; second, the progress of the end-effector along any such trajectories may be timed in an arbitrary fashion as long as over-

Received November 15, 2007

all constraints on movement duration are satisfied; and third, each spatial location of the end-effector may generally be achieved by an infinite number of joint-angle configurations [5]. The elimination specified above indeterminacy is usually made by introduction of any criteria of quality thus solving some secondary tasks. For example, these are the following criteria: minimization of joint angles sum [15], minimization of joint velocity [13], minimization of summarized joint torques [12], minimization of kinetic energy [24] and others. If the assumption that the structure and kinematic features of a human arm acquired during evolution are optimum, then at trajectories planning of manipulators end-effectors it is expedient to use various criteria including the ones justified from the biomechanical point of view.

The outcomes of numerous experiments of study of free arm movements testify that they are characterized by kinematic and dynamic invariance that in opinion of many researchers is a corollary of their optimality. So, for instance, it is established that kinematics of arm movements is highly typical under a large variety of experimental conditions. Graphic profiles of tangential velocity at arm movements from point-to-point for various amplitudes and durations are consistently bell-shaped, single-peaked, and approximately symmetrical [1, 2, 17, 18]; they generally vary little within and between subjects [16], and variability tends to decrease with practice [8]. Such characteristics are corresponding to two-phase movements consisting from phases of acceleration and braking with approximately equal time. P. Morasso [17], having investigated of arm movement restricted by a horizontal plane point-to-point, has established that trajectories of the elements (points) are close under the shape to smooth lines with small curvature.

Taking into consideration these facts, T. Flash and N. Hogan [6] for univalent trajectories planning have offered mathematical model of minimum jerk criterion (minimal jerk model). As a result of the analysis of numerous solutions it has been established, that the use of criterion leads to trajectories close to straight lines if movement begins from the center of working space and is made along an axis corresponding removal or nearing to a base of the arm but if movement begins from boundaries of working space, a trajectory is appreciably curved. In coordination with experimental data it is also determined at the movement of arm via points and at the detour of obstacles. The graphs profiles of tangential speeds, in general, have the bell-shaped form, but their maxima are displaced to the right at slow movements or to the left - at fast.

The similar model of minimum angle jerk (minimum angle jerk model) has been offered D. Rosenbaum with colleagues [20]. For using the criterion it is necessary to have the preliminary solution of an inverse kinematics that is not always possible.

The majority of deficiencies peculiar to the presented criteria are absent in offered Y. Uno, M. Kawato and R. Suzuki criteria of minimum torque-change (minimum torque-change model) [21]:

$$K_3 = \frac{1}{2}\int_0^{t_k}\sum_{i=1}^{n}\left(\frac{d\tau_i}{dt}\right)^2 dt \Rightarrow \min\,, \qquad (1)$$

here τ_i – the controlling moments in joints (torques), $i = 1...n$. According to (1) the trajectories of an arm are planned so that the change of the torques generated in each joints, was minimum. It is known, that before to justify advantages of criterion (1) in comparison with others, the authors have made a set of comparative estimations, including the criteria of minimum expenses of metabolic energy, minimum torques, minimum time of positioning, etc. It was clarified, that the criterion (1) most precisely corresponds to experimental data. Partly the authors have expressed biological usefulness which consists in minimization of wear of skeletal-muscular system. It is necessary to note, that the criteria presented above allow to receive the trajectories with peculiar kinematic invariance, but the dynamic invariance peculiar to a free movements of a human arm at it is not observed. The dynamic invariance named by a "linear synergy", consists in the approximate linear dependence between the joint torques and can be presented by expression [11]:

$$\tau_1 = K_D \cdot \tau_2 , \tag{2}$$

here $\tau_1 = \tau_s$, $\tau_2 = \tau_e$ – torques at the planar movement in a joint of an upper arm with conditionally motionless scapula (shoulder) and an upper arm with a forearm (elbow), accordingly; K_D – a constant depending from the direction of movement of a base point to the target. As a result of the statistical processing of experiments by authors of a paper [11] the formula for definition of estimations of a constant of proportionality is received:

$$K_D = 2.05 \frac{0.45\Delta\theta_2 + \Delta\theta_1 - 4.6}{\Delta\theta_2 + 0.18\Delta\theta_1 - 5.3} , \tag{3}$$

here symbol Δ means the change of a corresponding angle ($\theta_2 = \theta_e$ – an angle of a forearm rotation, $\theta_1 = \theta_s$ – an angle of an upper arm rotation) in relation to initial. The linear synergy is determined as well in experiments with various loadings and speeds of movement [9].

The observable synergy properties allow to assume, that at the execution of movements (generally spatial), at a level of the central nervous system take place separation of the degrees of freedom available for arm on a separately controlled blocks. As a result the number of independent parameters are liable to controlling less than real degrees of freedom of an arm [10]. As it is noted in article [7], such a grouping of regulated parameters in blocks gives "one of rather common and effective ways of a complex system control".

The dynamics and control of anthropomorphic mechanisms by synergy principles already find wide enough application. At the decision of a problem of N. Bernstein usually at a hypothetical level specific limitations on controlled variables in the form of linear or nonlinear ratios between them or their derivatives by means of which redundancy of mechanical system is eliminated are entered. For example, a semi-inverse synergies method [22], a method of optimizing synergies [3], a method of the arranged positioning [19], a method of inverse dynamics in control [14] in which the synergy is used implicitly are known. But, because the nature of synergy mechanisms is not studied practically, all of them, in essence, are based on heuristic approaches and, as a whole, do not allow to receive the solutions corresponding to real processes. In this connection, the further study of the nature of synergies, and the development of justified principles of a synergy control, not contradicting numerous experiments, for thecontrol of mechanical systems with redundant number of degrees of freedom are the important and actual problems.

The aim of this work is a justification of criteria for trajectories planning of an end-effector of anthropomorphic manipulator, which allows to receive the solutions with simultaneously kinematical and dynamic invariance. Such criterion of quality can be considered as synergetic since the specified properties allow to receive also unique solutions for mechanical system with a redundant number degrees of freedom, i.e. to solve a problem of its self-organizing.

2. MINIMAL TORQUE-CHANGE MODEL

The results of work presented in given paper are based on the research of two variants planned trajectories: a) from center of working space to 12 points uniformly arranged on a circle of radius *R*; and b) from a peripheral point of working space, arranged on a round of radius *R* in others 11, uniformly arranged on the same circle (fig. 1).

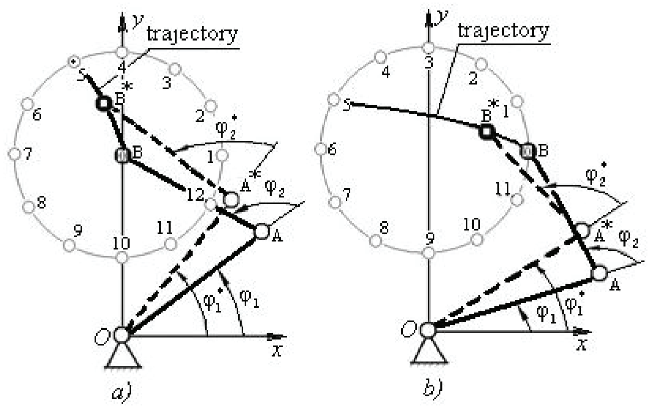

Fig. 1 The schemes of generated trajectories under manipulator end-effector movements

Except for the two variants, manipulator links parameters, presented in table 1 were examined. Relations between parameters in first variant correspond to the averaged anthropometrical data received at parameters measurements of a human arm, in the second - are selected, in common, arbitrarily, by a modification of sequence of a disposition of links in a kinematic chain.

First the regular research of a minimum torque-change model (1) has been made. Considering values of the model parameters, the optimization problem of a trajectory planning is represented in the form of:

$$\Phi = \frac{1}{2}\int_0^{t_k} F(t, \varphi_1, \varphi_2, \dot{\varphi}_1, \dot{\varphi}_2, \ddot{\varphi}_1, \ddot{\varphi}_2, \dddot{\varphi}_1, \dddot{\varphi}_2) dt \Rightarrow \min , \quad (4)$$

where function *F* is defined according to dynamics equations of the controlled movements:

$$\begin{aligned} \tau_1 &= (J_1 + J_2 + 2m_2 l_1 l_{2S} \cos\varphi_2 + m_2 l_1^2)\ddot{\varphi}_1 + \\ &+ (J_2 + m_2 l_1 l_{2S} \cos\varphi_2)\ddot{\varphi}_2 - m_2 l_1 l_{2S}(2\dot{\varphi}_1 + \dot{\varphi}_2)\dot{\varphi}_2 \sin\varphi_2 + +B_{11}\dot{\varphi}_1 + B_{12}\dot{\varphi}_2 , \\ \tau_2 &= (J_2 + m_2 l_1 l_{2S} \cos\varphi_2)\ddot{\varphi}_1 + J_2\ddot{\varphi}_2 + m_2 l_1 l_{2S}\dot{\varphi}_1^2 \sin\varphi_2 + +B_{21}\dot{\varphi}_1 + B_{22}\dot{\varphi}_2 \end{aligned} \quad (5)$$

Table 1 Values of the model parameters

Parameter	Value of parameter			
	Link 1 (upper arm)		Link 2 (forearm)	
	Var. 1	Var. 2	Var. 1	Var. 2
m, kg	1,3	1,5	1,5	1,3
J, $kg \cdot m^2$	0,07	0,1	0,1	0,07
l, m	0,25	0,35	0,35	0,25
l_S, m	--	--	0,15	0,15
B, $N \cdot m \cdot s / rad$	0,7	0,8	0,8	0,7
$B_{12} = B_{21}$, $N \cdot m \cdot s / rad$	0,18	0,18	0,18	0,18

Euler-Poisson equations give a necessary condition of an extremum of functional (4); sufficiency can be checked up by comparison of outcomes of optimization with the numerous experimental data received at the analysis of a human arm trajectories if to count them as optimal. In this task the Euler-Poisson equations are represented in the form of:

$$\begin{aligned} &\frac{\partial F}{\partial \varphi_1} - \frac{d}{dt}\frac{\partial F}{\partial \dot{\varphi}_1} + \frac{d^2}{dt^2}\frac{\partial F}{\partial \ddot{\varphi}_1} - \frac{d^3}{dt^3}\frac{\partial F}{\partial \dddot{\varphi}_1} = 0 \\ &\frac{\partial F}{\partial \varphi_2} - \frac{d}{dt}\frac{\partial F}{\partial \dot{\varphi}_2} + \frac{d^2}{dt^2}\frac{\partial F}{\partial \ddot{\varphi}_2} - \frac{d^3}{dt^3}\frac{\partial F}{\partial \dddot{\varphi}_2} = 0 \end{aligned} \qquad (6)$$

and, in view of (5), will be transformed to the system of two nonlinear differential equations of the sixth order.

Following a technique explained in [23] solutions of system (6) are received numerically with the use of the orthogonal polynomials of Jacobi $P_n^{(\alpha,\beta)}(x)$, satisfying to a condition:

$$\int_{-1}^{1} w(x) P_m^{(\alpha,\beta)}(x) P_n^{(\alpha,\beta)}(x)dx = 0, \quad m \neq n,$$

where $w(x) = (1 - x)^{\alpha}(1 + x)^{\beta}$ is a weight function. Let's notice, that the specified polynomials are generated automatically by computer programs Maple, Matlab, etc.

Let $\alpha = \beta = 6$. Then $C_k(x) = \sqrt{w(x)} P_k^{(6,6)}(x)$ satisfies to conditions:

$$C_k(-1) = C_k(1) = \left.\frac{dC_k(x)}{dx}\right|_{x=-1} = \left.\frac{dC_k(x)}{dx}\right|_{x=+1} = \left.\frac{d^2C_k(x)}{dx^2}\right|_{x=-1} = \left.\frac{d^2C_k(x)}{dx^2}\right|_{x=+1} = 0. \qquad (7)$$

Having made substitution of a variable $t = \frac{x+1}{2}$, we shall receive:

$$C_k^*(t) = 64t^3(1-t)^3 P_k^{*(6,6)}(t). \qquad (8)$$

Similarly as (7), polynomials (8), their first and second derivatives will be converted in 0 at $t = 0$ and $t = 1$. Using this property, solutions of system (6) we shall present in the form of:

$$\varphi_i = \sum_{j=0}^{5} b_{ij} t^j + \sum_{k=0}^{K} a_{ik} C_k^*(t) \ , \qquad (9)$$

where $i = 1{,}2$ – number of links; a_{ik} – indefinite coefficients; b_{ij} – the coefficients defined from boundary conditions of a task; t – current time; K – a degree of Jacobi polynomials. The number K defines the exactitude of a task solving and, hence, and its dimension.

If necessary deriving of other intervals of time which are defining average velocity of end-effector movement point-to-point, the change of a variable is carried out under the formula: $t = (x + 1) / d_{tk}$, where d_{tk} – the number selected depending on t_k – time of movement. Clearly, that the polynomial (8) thus acquires other aspect. The interval of time $t \in [0, t_k]$, by its partition on K equal parts is discretized. For each value t_k values of the left parts of the system (6) after substitution in them of polynomials (9) are evaluated. Thus, we receive system $2(K + 1)$ of transcendental equations with $2(K + 1)$ unknown a_{ik} ($i = 1{,}2$; $k = 0..K$).

In the beginning the number K was selected by greater (order $K = 40 – 50$) that is leading to magnification of dimension and excessive difficulties at determination of solutions. However after deriving the solutions for different aspects of end-effector movements in working space has been established that with magnification of a degree of a polynomial (9) its coefficients fast decrease with magnification of a corresponding degree t and already polynomials of 9 degrees represent the solutions of a problem which is practically not differing from solutions in the form of polynomials of 50 degrees. The solutions of a problem are received in Maple system.

As a result of a regular research of criterion (1) it has been established, that at its realization the shapes of an end-effector trajectories depend on values of links parameters and relations between them at various directions of movement to the target point. In particular, trajectories noticeably are curved in cases when relations between parameters of links essentially differ from accepted in variant 1 or similar to them. As illustrations the trajectories received at two variants of links parameters, are presented in the fig. 2 and 3.

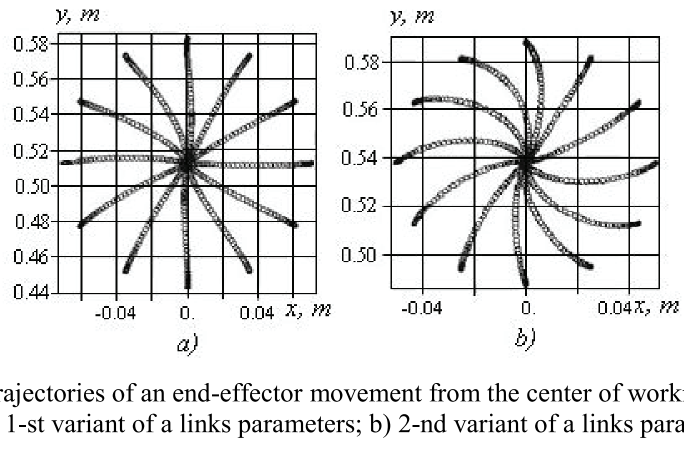

Fig. 2 Trajectories of an end-effector movement from the center of working space: a) 1-st variant of a links parameters; b) 2-nd variant of a links parameters

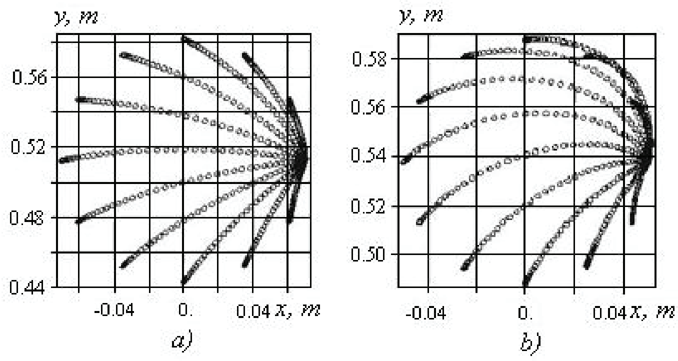

Fig. 3 Trajectories of an end-effector movement from a peripheral point of working space: a) the first variant of links parameters; b) the second variant of links parameters

Similar trajectories are peculiar to various average velocities of an end-effector movement. It is necessary to note also, that at trajectories planning by schemes (a) and (b), presented in the fig. 1, the profiles of an end-effector tangential velocity have closed to symmetric to an axis of time the bell-shaped form peculiar for free movements of a human arm in a horizontal plane. But, in all cases as it was already noted above, the relations between joints torques of a type (2) or similar to them it is not observed.

3. Synergetic Criterion (Minimum Error-Change Model Of Linear Sinergy)

Assuming, that architecture of an arm movements at a level of a central nervous system is optimum, and approximately movements possess property of a linear synergy, we shall count, that similar architecture of an end-effector movements for manipulator with anthropomorphic structure (for example, SCARA type), is also optimum with peculiar optimal trajectories from this viewpoint. In this connection, according to (2), we shall introduce the function of an error of a linear synergy

$$E_S = \tau_1 - \alpha\tau_2 - \beta , \qquad (10)$$

where α and β – some constants.

The outcomes of numerous experiments testify that actually the values of a function E_S (with reference to conditions of a free arm movement) at any $t \in [t_0, t_k]$ (t_0 and t_k – times of the beginning and the ending of movement, accordingly), in common, are distinct from zero since movements of arm elements are only approximately characterized by property of a linear synergy, but are rather small. This fact testifies that the correlation coefficients of torques if to consider them as some casual variables, is accepting values close to ± 1 and, hence, experimental data can be always exactly presented to the straight line $\tau_1 = \alpha\tau_2 + \beta$ received as a result of interpolation of association $\tau_1 = f(\tau_2)$. Thus, at the trajectories planning which are possessing properties of a linear synergy, the certain interest represents not only the function (10), but also its modification

$$\frac{dE_S}{dt} = \frac{d\tau_1}{dt} - \alpha \frac{d\tau_2}{dt} . \tag{11}$$

Let's demand, that the summarized modification of the error function module for all phase of an end-effector movement from point-to-point would be minimum therefore we shall come to the criterion of quality of a trajectory planning which it is possible to consider synergetic

$$\Phi = \frac{1}{t_k - t_0} \int_{t_0}^{t_k} \left(\frac{dE_S}{dt} \right)^2 dt \Rightarrow \min . \tag{12}$$

The necessary condition of an extremum of functional (12) is the system of Euler-Poisson equations of the sixth order (6), which are depending from generally indefinite parameter α. A solution of this system at twelve boundary conditions is the set of corteges

$$B = \{ \langle \varphi_{1i}(t,\alpha), \varphi_{2i}(t,\alpha) \rangle |, i \in N, \alpha \in [\alpha_1, \alpha_2] \} , \tag{13}$$

where α_1 and α_2 – boundary values of parameter α which can be selected, for example, with the use of an estimation (2) or otherwise. All corteges of set (13) are equivalent in relation to criterion (12), but it is possible to assume, that some of them are optimum from the viewpoint of other criteria which correspond to the essence of problem. For example, at some values α a solution can be characterized by the greatest correlation between joint torques, at others - minimum average power, minimum summarized work, etc. Thus, generally, the task of a choice the best value α is multicriteria. In the further as the unique additional criterion of quality is accepted the minimum summarized work made by joints torques during at end-effector movement from point-to-point

$$A = \int_{t_0}^{t_k} \left(\tau_1 \frac{d\varphi_1}{dt} + \tau_2 \frac{d\varphi_2}{dt} \right) dt \Rightarrow \min . \tag{14}$$

The solutions of problem of a trajectory planning according to criteria (12) and (14) are received with the use of the technique stated above at twelve boundary conditions, set in initial and finite points in correspondence with the schemes which was presented in fig. 1 and the values of links parameters are presented in table 1 at different average velocities of movement.

The analysis of solutions is showing that in all cases the received trajectories are characterized by the kinematical invariance, under the shape they are close to straight lines and have rather small curvature, and the profiles of graphs of tangent velocities of an end-effector have the bell-shaped form. The correlation between joint torques considerably above, then in the solutions, is received according to criterion (1). But, thus, it is necessary to note, that optimum trajectories, from the viewpoint of criteria (12) and (14), are realized at operation on links of torques, the correlation between which varies at a modification of a direction of an end-effector movement to the target and its average velocity.

On the fig. 4, 5 and 6 as example graphs of some associations received at end-effector trajectory planning according to the scheme (a) (fig. 1) from the center of working space in position 9 at time $t = 0.5c$ are presented. Similar associations are received and for other conditions of movement.

The Graph analysis testifies the evolution of task solutions on modifications of parameter α at trajectories planning. Besides, noticeably the augmentation of correlation between torques in case of using the synergetic criterion (12). In the given example the optimum solution (according to additional criterion (14)) is received with a coefficient of correlation $k = -0.9706$. For comparison, at using of a minimum torque-change criterion (1) $k = -0.8465$. As a rule, at other directions of an end-effector movement to target the difference in correlation is more essential.

It is necessary to note, that optimum trajectories, according to criteria (12) and (14), are not always characterized by the greatest value of correlation between torques. In a surveyed example the greatest value $k = -0.9925$ is received at $\alpha = -0.110$ and, as shown in the graphs represented in the fig. 7 $\alpha < \alpha_{opt}$. The analysis of the set of the received solutions is showing, that at some value of α the correlation achieves a maximum, but in these cases, at first, the character of an end-effector movement ceases to be two-phase, secondly, the power of process is essentially above, than at $\alpha = \alpha_{opt}$. We shall note also, that from the viewpoint of the criterion (14) the optimum solutions received according to two criteria of quality surveyed above (1) and (12), in the given example differ slightly.

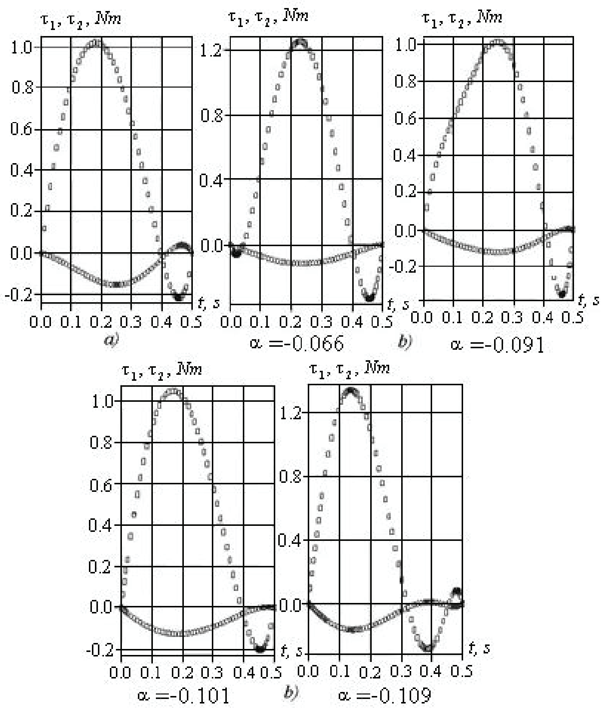

Fig. 4 The change of a joint torques at realization of the trajectories received in view of different criteria: a) minimum torque-change criterion (1); b) synergetic criterion (12)

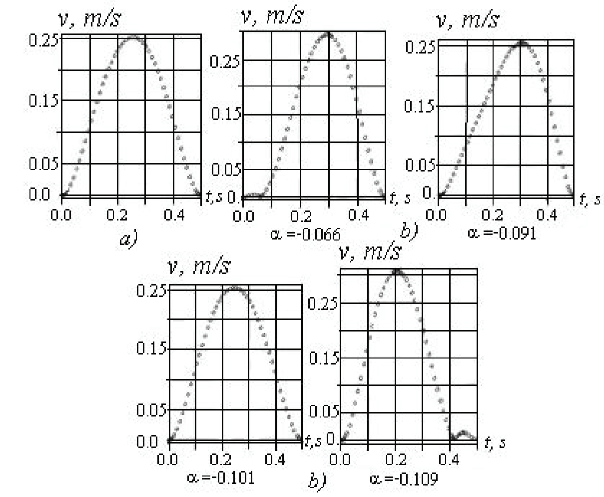

Fig. 5 The change of end-effector tangential velocities at realization of the trajectories received in view of different criteria: a) minimum torque-change criterion (1); b) synergetic criterion (12)

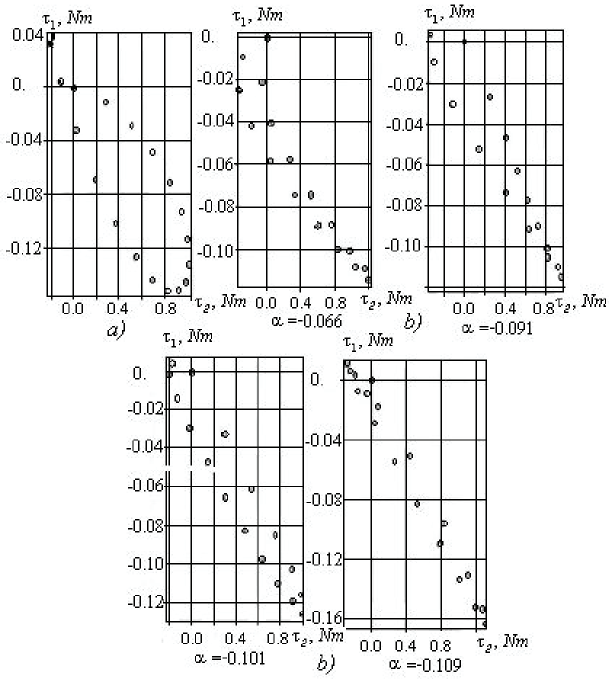

Fig. 6 The graphs of dependences $\tau_1 = f(\tau_2)$ at realization of the trajectories received in view of different criteria: a) minimum torque-change criterion (1); b) synergetic criterion (12)

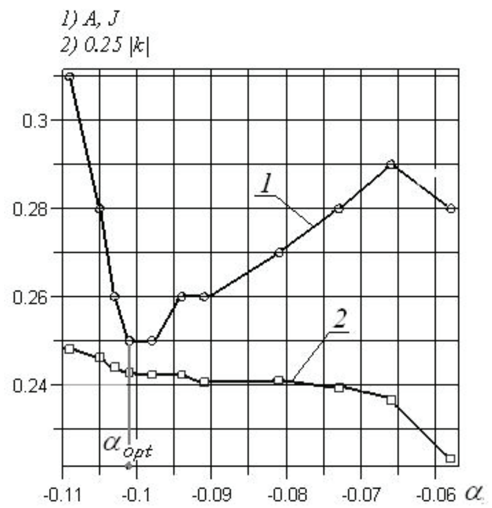

Fig. 7 The graphs of dependences: 1) $A = f(\alpha)$; 2) $k = abs(g(\alpha))$

4. DISCUSSION

The using of the synergetic criterion (12) allows to receive the additional possibilities at the optimization of task solutions of a trajectory planning. In particular, the compromise is possible at a choice of a preferable variant from the different viewpoints. As it was noted above, for some ranges of parameter α at various average velocities and directions of movement rather small curvature is inherent to trajectories, and graph profiles of a tangential velocities have the bell-shaped forms. Thus, the movement of an end-effector on such trajectories is two-phase and is characterized by the kinematical invariance inherent to free movements of a human arm. Except for it the choice of a task solutions of trajectory planning, described a high scale of a linear dependence between the torques is possible, it is also peculiar for free human arm movement in the form of observable in the experiments of a dynamic invariance. At the same time, proceeding from the analysis of set solutions, it is possible to approve, that at the use of criterion (12) to achieve an absolute linear dependence between torques is not possible, but in all cases, the correlation between them is great enough.

The evolution process of solutions at determination of a best value of parameter α represents the certain interest. Generally, at arbitrary values α the torques represent functions of time with set of extremes and as approaching $\alpha = \alpha_{opt}$ more and more aspire to two-extreme. Thus the other performances of solutions vary also: the shapes of trajectories, the graph profiles of an end-effector tangential velocity and others. The evolution process of solutions depending from a parameter α is showed in the fig. 8-11 on the example of an end-effector trajectory planning at movement from the center of working space in position 3 (scheme (a), fig.1); a variant of parameters 1. The numerical values of the parameters describing solutions are presented in table 2.

Table 2 Numerical values of some parameters characterizing trajectories of an end-effector movement according to the scheme (a) from the center of working space in position 3 at different values of parameter α

α_0	Regression τ_2 on τ_1	*A*, *J*	*k*
-0.25	$\tau_1 = -0.28\tau_2 - 0.06$	2.13	-0.99
-0.35	$\tau_1 = -0.35\tau_2 - 0.10$	0.92	-0.975
-0.55	$\tau_1 = -0.43\tau_2 - 0.16$	0.615	-0.94
-0.65	$\tau_1 = -0.44\tau_2 - 0.17$	0.585	-0.92
-0.80	$\tau_1 = -0.46\tau_2 - 0.18$	**0.578**	**-0.92**
-0.90	$\tau_1 = -0.49\tau_2 - 0.20$	0.580	-0.92
-1.50	$\tau_1 = -0.68\tau_2 - 0.34$	0.609	-0.94
-2.20	$\tau_1 = -0.90\tau_2 - 0.50$	0.633	-0.94
-3.00	$\tau_1 = -1.10\tau_2 - 0.66$	0.650	-0.93
Solutions according to criterion 1			
--	$\tau_1 = -0.41\tau_2 - 0.14$	**0.57**	**0.87**

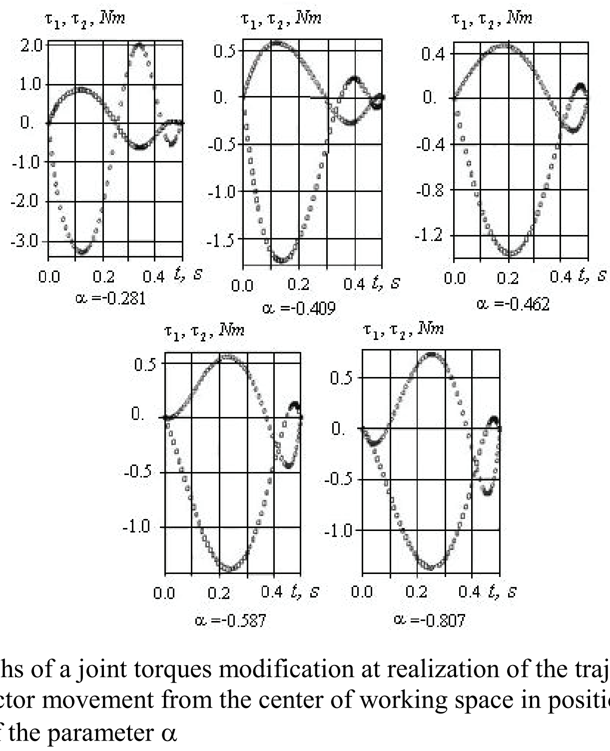

Fig. 8 The graphs of a joint torques modification at realization of the trajectories of an end-effector movement from the center of working space in position 3 at different values of the parameter α

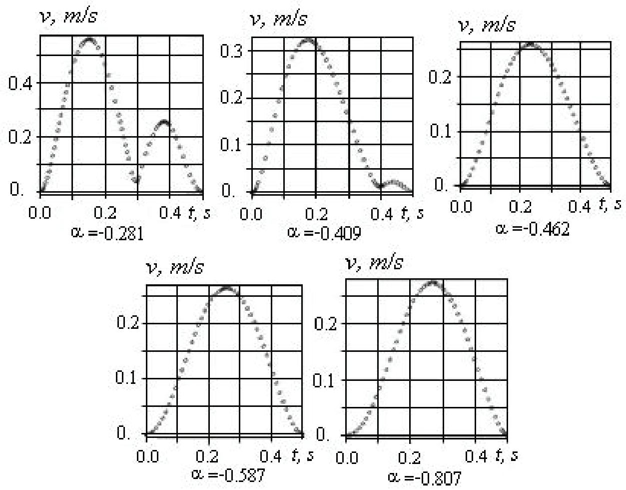

Fig. 9 The modification of a tangential velocity at realization of the trajectories of an end-effector movement from the center of working space in position 3 at different values of the parameter α

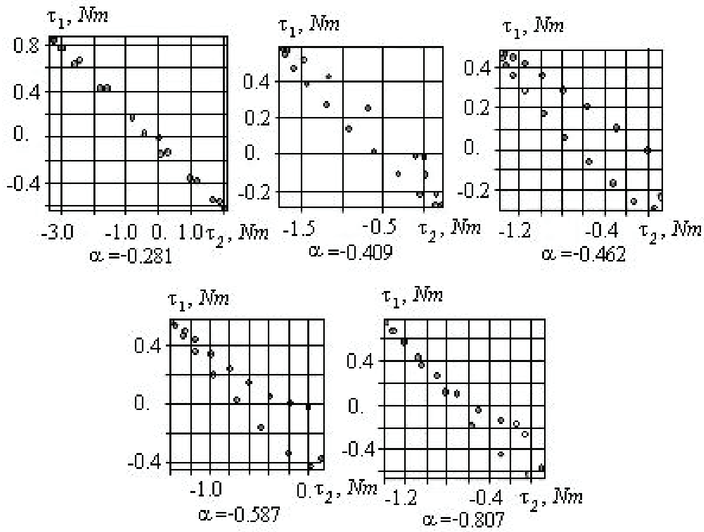

Fig. 10 The graphs of dependences $\tau_1 = f(\tau_2)$ at realization of the trajectories of an end-effector movement from the center of working space in position 3 at different values of a parameter α

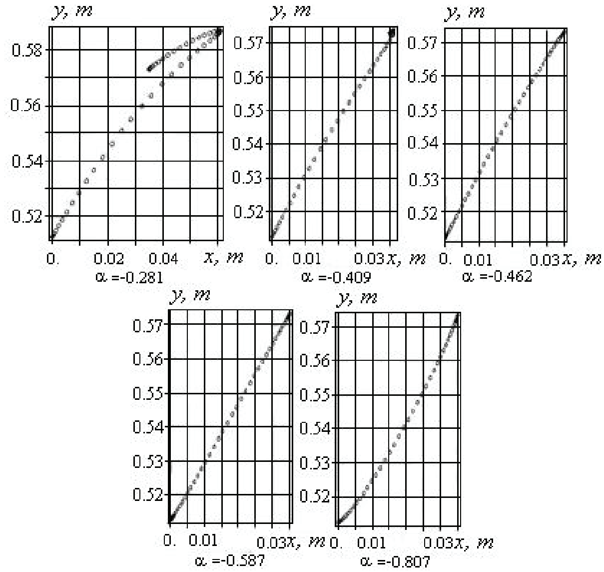

Fig. 11 Trajectories of an end-effector movement from the center of working space in position 3 at different values of the parameter α realized with the use of the criterion (12)

Such an evolution of solutions is similar to the process of tutoring to rational movements. So, for example, F. Zaal with colleagues during long observation of children-babies measured gyrating moments in concatenations of their hands at the execution of the elementary movements linked with the grasping effect of a toy and its migration in a horizontal plane [25]. The analysis of observations has displayed, that in the given process, beginning from the inept movements up to quite successful the gyrating moments from multiextreme under the shape more and more came nearer to two-phase. Thus correlation between them was observed during all phases of observations. Moreover, the angular coefficient of a regression line τ_1 on τ_2 remained during all phases practically fixed and was within the limits of 2,5 … 3.0 (angular coefficient of a regression line τ_2 on τ_1 - 0,4 … 0.33, accordingly).

The data presented in table 2, testify that from the viewpoint of the summarized work, the criterion (1) is more preferable than criterion (12). But, such situation does not always take place. For example, at a modification of relations between links parameters, relations between characteristics of trajectories vary also. In the example below the results of an end-effector trajectory planning at movement with a weight in mass m = 1kg (parameters of the links corresponding to variant 1 (table 1)) are presented. Movement, as well as in the previous example, is carried out from the center of working area in position 3. As the first approximation the weight is accepted in the form of a particle arranged on the extremity of an end-effector.

On the fig. 12 are presented graphs of associations $\tau_1 = f(\tau_2)$ received at the conditions specified above.

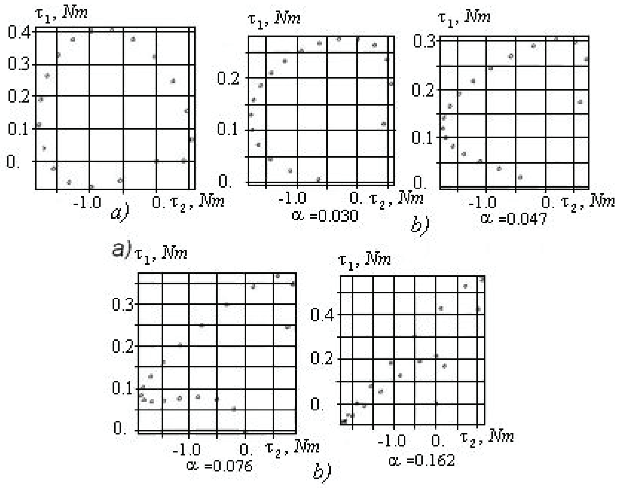

Fig. 12 The graphs of dependences $\tau_1 = f(\tau_2)$ at realization of trajectories of an end-effector movement from the center of working space in position 3 according to different criteria and at various values of the parameter α: a) minimum torque-change criterion (1); b) synergetic criterion (12)

Their analysis is showing that at realization of the criterion (1), the correlation between the torques is absent generally. The coefficient of correlation corresponding to data, presented graphically in fig. 12 (a) is equal $k = -0.035$. At the use of criterion (12) already at $\alpha = 0.076$ (data of fig. 12 (b)) the coefficient of correlation achieves value $k = 0.611$ and is augmented at corresponding magnification α. Somewhat the optimum solution for the given example is received at $\alpha = 0.047$ ($k = 0.397$). Thus summarized work $A = 0.583$ *J*. For comparison, at the use of criterion (1) $A = 0.584$ *J*. We shall note also, that in all cases of the shape of trajectories have small curvature, and graphs profiles of end-effector tangential velocities have the strongly pronounced bell-shaped form. The data of the presented example is showing that at the use of criterion (12), unlike criterion (1) there is a possibility of a choice of the conciliatory proposals, which correspond to the essence of the solved task at different values of parameters and the conditions of movement.

In spite of the fact that the outcomes of a task solutions of trajectories planning, precisely enough correspond to the set of the experimental data received at research of the free movements of a human arm, there are no foundations to consider, that realization of the trajectories corresponding to them is optimum from the energetic viewpoint. The set of solutions (13) is restricted by a necessary condition of an extremum of functional (12) and, hence, generally, does not contain energetically optimum solution. On the other hand, the set (13) is restricted by a necessary condition of an extremum of functional (14) and, generally, does not contain a solution with the maximum expressed correlation between the joint torques, affirming that the data are represented in table 2. For comparison of energetic costs of different processes the analysis of the results of a trajectory planning in view of only criterion (14) has been carried out. The outcomes of a solution of the given task in view of the conditions accepted in the previous example, in the graphic form are presented in fig. 13. The introduced outcomes testify that at realization of criterion (14) a three-phase movement of an end-effector is tracked: acceleration - the steady movement - braking. The trajectory is curved slightly, but the profile of the graph of a tangential velocity differs from the bell-shaped form. The correlation between the joint torques is high enough also its value more, than at sharing criteria (12) and (14): $k = -0.954$. The summarized work made by torques $A = 0.452$ J, that more than on 20 % is less, than at sharing criteria (12) and (14).

It is necessary to note, that similar outcomes (except for a coefficient of correlation between the torques) are peculiar also to other conditions of movement.

Thus, the using of the criterion (14) without side conditions allows to plan trajectories, whose realization is energetically more favorable in comparison with criteria (1) and (12) (together with (14)). But, at first, the practical realization of the torques is more complicated and, secondly, the beginning and the ending of movement will be accompanied in this case by soft shock processes as the torques in the beginning and the end of movement vary spasmodically on the finite magnitude. In practice it can lead, at least, to the essential loss of exactitude of positioning of an end-effector. The observable distinctions of the received solutions with experimental data allows to assume, that at the biological objects, which carry out the movement by means of extremities, at a level of central nervous system, the multicriteria problem of a movement optimization is solved and, as a result, the trajectory and a mode of its realization satisfying to the set of criteria of quality is selected.

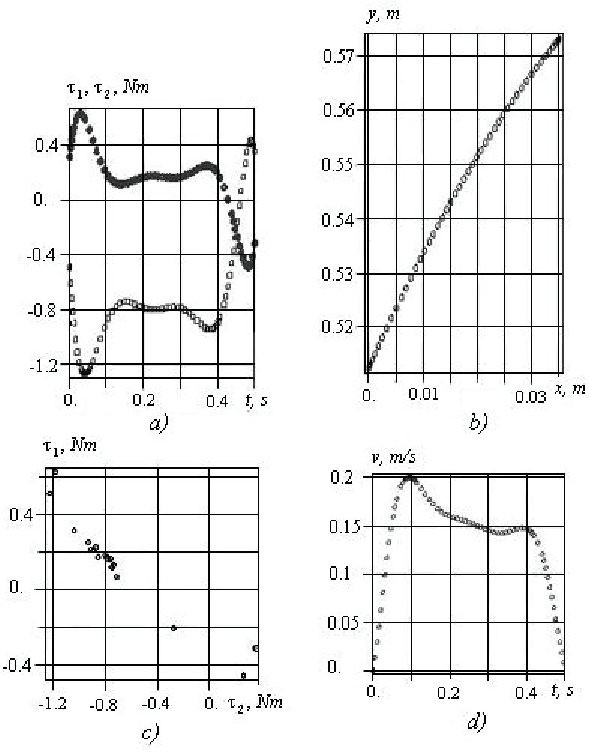

Fig. 13 The results of an end-effector trajectory planning with using of criterion (14): a) the torques; b) a trajectory of a base point of end-effector; c) the graph of dependence $\tau_1 = f(\tau_2)$; d) the graph of tangential velocity of a base point of an end-effector

5. CONCLUSION

The regular analysis of possible criteria of a trajectories planning of manipulator end-effector allows to make the following conclusions:

1. The greatest conformity with experimental data is peculiar to the trajectories which are planned as a result of the combined use of criteria (12) and (14).
2. From the energetic viewpoint the most preferable are the trajectories which are planned according to criterion (14). Criteria (1) and (12) in this sense are practically equal in rights.
3. The combination of criteria (12) and (14) allows to make a choice of conciliatory proposals, optimal from the various viewpoints. The search for decisions can be made as well in view of other criteria corresponding to the nature of the solved problem.
4. In the further research it is expedient to effect analysis of the trajectories which are planned according to various criteria of quality on the basis of the models of manipulators with flexible links and elastic joints.

REFERENCES

1. Abend, W., E. Bizzi, and P. Morasso, 1982, *Human arm trajectory formation,* Brain, 105, pp. 331–348.
2. Atkeson, C. G., and J.M. Hollerbach, 1985*, Kinematic features of unrestrained arm movements,* Journal of Neuroscience, 5, pp. 2318–2330.
3. Berbyuk, V.E., 1989, *Dynamics and optimization of robotics systems,* Naukova Dumka, Kiev (in Russia),
4. Bernstein, N.A., 1967, *The coordination and regulation of movements,* Pergamon, London
5. Engelbrecht, S.E., 2001, *Minimum Principles in Motor Control,* Journal of Mathematical Psychology, 45, pp. 497– 542.
6. Flash, T., and N. Hogan, 1985, *The coordination of arm movements; an experimentally confirmed mathematical model,* Journal of Neuroscience, 5, pp. 1688–1703.
7. Fomin, S.V., 1970, *Some mathematical problems of biology,* Biophysics. XV(2), pp. 344–351 (in Russia).
8. Georgopoulos, A. P., J.F. Kalaska, and J.T. Massey, 1981, *Spatial trajectories and reaction times of aimed movements: Effects of practice, uncertainty, and change in target location,* Journal of Neurophysiology, 46(4), pp. 725–743.
9. Gottlieb, G.L., Q. Song, D.A. Hong, and D.M. Corcos, 1996a, *Coordinating two degrees of freedom during human arm movement: load and speed invariance of relative joint torques,* Journal of Neurophysiology, 76(5), pp. 3196 –3206.
10. Gottlieb, G.L., Q. Song, D. Hong, G.L. Almeida, and D.M. Corcos, 1996b, *Coordinating movement at two joints: A principle of linear covariance,* Journal of Neurophysiology, 75, pp. 1760–1764.
11. Gottlieb, G.L., Q. Song, G.L. Almeida, D. Hong, and D. Corcos, 1997, *Directional Control of Planar Human Arm Movement,* Journal of Neurophysiology, 78, pp. 2985–2998.
12. Hollerbach, J.M., and K.C. Suh, 1985, *Redundancy resolution of manipulators through torque optimization,* Proc. IEEE Int. Conf. on Robotics and Automation, pp. 1016-1021.
13. Kazerounian, K., and Z. Wang, 1988, *Global versus local optimization in redundancy resolution of robotic manipulators,* Int. J. of Robotics Research, 7(5), pp. 3–12.
14. Krutko, P.D. and N.A. Lakota, 1982, *Synthesis of control algorithms by movement of robots on a method of inverse problems of dynamics. The coordinate shape of the definition of a trajectory,* Technical Cybernetics, 1, pp. 154–159 (in Russia).
15. Liegeois, A., 1977, *Automated supervisory control of the configuration and behaviour of the multibody mechanism,* Proc. IEEE Trans. on Systems, Man and Cybernetics, SMC-7(12), pp. 868-871.
16. Miall, R.C., and P.N. Haggard, 1995, *The curvature of human arm movements in the absence of visual experience,* Experimental Brain Research, 103, pp. 421–428.
17. Morasso, P., 1981, *Spatial control of arm movements,* Experimental Brain Research, 42, pp. 223–227.
18. Morasso, P., 1983, *Three dimensional arm trajectories,* Biological Cybernetics, 48, pp. 187–194.
19. Potkonjak, V., M. Popović, M. Lazarević, and J. Sinanović, 1988, *Redundancy problem in writing: from human to anthropomorphic robot arm,* IEEE Trans. on Systems, Man and Cybernetics: part B: Cybernetics, 28(6), pp. 790-805.

20. Rosenbaum, D.A., L.D. Loukopoulos, R.G. Meulenbrok, J. Vaughan, and S.E. Engelbrecht, 1995, *Planning reaches by evaluating stored posture,* Psychological Review, 102, pp. 28–67.
21. Uno, Y., M. Kawato, and R. Suzuki, 1989, *Formation and control of optimal trajectory in human arm movement – minimum torque-change model,* Biological Cybernetics, 61, pp. 89–101.
22. Vukobratovich, M, 1976, *Walking robots and anthropomorphic mechanisms,* Mir, Moscow, (in Russia).
23. Wada, Y., Y. Kaneko, E. Nakano, R. Osu, and M. Kawato, 2001, *Quantitative examination for multi joint arm trajectory planning – using a robust calculation algorithm of the minimum commanded torque,* Neural networks, 14, pp. 381–393.
24. Whitney D.E., 1969, *Resolved motion rate control of manipulators and human prostheses,* Trans. Man-Machine Systems, MMS(10), pp. 47-53.
25. Zaal, F., K. Daigle, G.L. Gottlieb, and E.Thelen, 1999, *An unlearned principle for controlling natural movements,* Journal of Neurophysiology. 82, pp. 255–259.

KVANTITATIVNE I KVALITATIVNE PROCENE PUTANJE KRAJNJEG EFEKTORA MANIPULATORA PLANIRANE SINERGETSKIM KRITERIJUMOM

Marina Kolesova, Aleksandr Polyakov, Mihail Kalinin

U ovom radu razmatra se putanja krajnjeg efektora planirana u dvodimenzionalnom radnom prostoru za primer dvosegmentong antropomorfnog manipulatora sa okretnim spojevima. Set rešenja dobija se kriterijumima minimalne obrtne promene, minimalnog celokupnog rada, i takodje novim kriterijumom minimalne greške linearne sinergije. Dobijene su njihove relativne kvantitativne i kvalitativne procene. Kriterijum minimalne greške linearne sinergije dokazan je biomehaničkim stanovištem. Pokazano je da zadatak putanje planirane za upotrebu ima rešenja koja odgovaraju setu eksperimentalnih podataka i generativno su obeležena kinematičkom i dinamičnom invarijantom koja je karakteristična za slobodne pokrete ljudske ruke. Brojna rešenja zadataka dobijena su Jacobi ortogonalnim polinomom za različite pravce pokreta krajnjeg efektora i za dve varijante manipulatora link parametara.

Ključne reči: *krajnji efektor, planirana putanja, sinergetski kriterijum, kriterijum minimalne obrtne promene, Poisson-Euler jednačine, ortogonalni polinomi*