FACTA UNIVERSITATIS
Series: **Mechanics, Automatic Control and Robotics** Vol. 6, Special Issue, 2007, pp. 145 - 158

MATHEMATICAL MODEL OF FLEXIBLE SPACECRAFT AND PHYSICALLY REALIZABLE ADAPTIVE-RELAY CONTROL FOR ITS ORIENTATION

UDC 629.787:681.584.7:510.24(045)=111

Vladislav Yu. Rutkovsky, Victor M. Sukhanov,
Stanislav D. Zemlyakov, Victor M. Glumov

Institute of Control Sciences, Russian Academy of Sciences,
Profsoyuznaya 65, 117997, Moscow, Russia
e-mail: rutkov@ipu.ru

Abstract. *Spacecrafts with flexible construction are considered. The angular motion equations decomposition and procedures of the control systems design are suggested. Some problems of stabilization processes dynamics of such kind control objects are investigated. Essential flexibility of the construction, variations of the mathematical model parameters and disconnected character of the handwheels control actions of the orientation system are taken into account. Physically realizable adaptive attitude control system providing robustness with respect to the elastic oscillations is suggested. Kalman estimations of the elastic modes are used. The instants of the control action switching are chosen with regard to the condition of the dominant mode optimal phase.*

Key words: *flexible spacecrafts, oscillation, modeling, adaptive algorithm, relay control*

1. INTRODUCTION

Very often an attitude control of flexible spacecraft (FS) implies a conflict between the main goal of the control of the flexible object as a rigid body and the necessity to restrict the magnitude of structural vibrations that are caused by control actions of the main regulator [1,2]. Particularly highly the tendency to excitation and accumulation of the vibrations occurs at the using of the relay type actuator devices (gas-jet nozzles, handwheels). A level of the vibrations that exceeds a critical value leads to a system instability, where the flexible oscillations capture the regulator [3]. The crux of the problem is the lack of information with regard to the state of the flexible body as structural vibration detectors are not available and the mathematical model of the object is often defined poorly.

In this paper some tasks of the FS control are considered. Among them are: algorithm's synthesis of computer derivation of the FS spatial angular motion, transformation of obtained Lagrange model to the modal-physical form and its decomposition into three

Received December 20, 2007

"almost" independent models for the case of using handwheels for FS control and design adaptive system of orientation. This system must be robust with regard to the inaccurate given of the FS vector of parameters. It must not excite vibrations of high level and must guarantee minimum time of their damping.

2. COMPUTER AIDED DERIVATION OF MOTION EQUATIONS FOR FLEXIBLE SPACECRAFT

Let us consider the problem of computer aided derivation and transformations of motion equations for spacecrafts with flexible structure [1, 2].

We take the kinematical structure of a mechanical system as a main carrying body with s rigid bodies to be resiliently connected to the main one. In such a statement the problem is solved in [4]. Namely, in [4] the mathematical model (MM) of the mechanical system is derived as the Lagrange's equation

$$A(q)\ddot{q} + \sum_{s=1}^{n}[\dot{q}^T D_s(q)\dot{q}]e_s + Cq = S(q)M \tag{1}$$

where $q^T = (q_1, q_2,...,q_n)$ is the vector of generalized coordinate; $M \in R^6$ is the vector of control forces and moments acting with respect to the main body axes. In [4] concrete mathematical formulas for matrices $A(q), D_s(q)$ $(s = \overline{1,n})$,C, $S(q)$ are presented for computer calculating.

In this work we will turn our attention on the problem how from MM (1) to go to more simple MM that would be more convenient for a system control synthesis in the case of observation spacecrafts. For this goals we need to find mathematical formulas for:
- linearization of the MM (1) taking into account small deviations from a desired motion;
- transfer of linearized MM (1) to the normal coordinates; - decomposition of the linearized MM to a series partial MM; - transformation of a partial MM to a modally-physical form [6] that is the most convenient for the synthesis and analysis of a control system.

2.1. Linearization of the mathematical model

A control system has to guarantee the prescribed dynamic of an orientation. It could be supposed before those deviations of controlled coordinates are not so large relatively desired ones. Then it is possible to linearize the MM (1) relatively desired motions, for example $q = q^0$, $\dot{q} = 0$, where all components of the vector q^0 are zeros except $q_1 = q_1^0 =$ const $\neq 0$. In this case, the MM (1) could be rewritten in the form

$$A\ddot{q} + Cq = SM \tag{2}$$

where $A = A(q)$, $S = S(q)$ for $q = q^0$.

2.2. Transformation of the linearized mathematical model to the normal coordinates

As a rule for the desired observation spacecraft orientation only coordinates of the vector $q_0^T = (q_1, q_2,..., q_6)$ and their velocities are measured. Then the MM (2) more comfortable to present in the form

$$A\ddot{q} + Cq = SM, \quad q_0 = (E_6 \quad O_{6\times(n-6)})q, \tag{3}$$

where $O_{6\times(n-6)}$ is $6 \times (n-6)$ zero matrix.

In (2) and (3) matrices A and C are positive and negative definite, respectively. Then there exists [6] nonsingular transformation $q = \Phi s$, that reduces MM (3) to the normal coordinates $s^T = (s_1, s_2,...,s_n)$. To find the matrix Φ, we represent MM (3) to the form

$$\begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}\begin{pmatrix} \ddot{q}_0 \\ \ddot{q}_c \end{pmatrix} + \begin{pmatrix} O_6 & O_{6\times(n-6)} \\ O_{(n-6)\times 6} & C_{22} \end{pmatrix}\begin{pmatrix} q_0 \\ q_c \end{pmatrix} = \begin{pmatrix} S_{11}(q) \\ O_{(n-6)\times 6} \end{pmatrix} M_a, \quad q_0 = (E_6 \ \ O_{6\times(n-6)})\begin{pmatrix} q_0 \\ q_c \end{pmatrix}. \tag{4}$$

Proposition 1 [3]. *To reduce MM* (4) *to normal coordinates s, one should apply nonsingular transformation* $q = \Phi s$, *where the matrix* Φ *is given by the equality*

$$\Phi = \begin{pmatrix} A_{11}^{-1/2} & -A_{11}^{-1}A_{12}Q \\ O_{(n-6)\times 6} & Q \end{pmatrix}, \; Q = a^{-1/2}T, \; a = A_{22} - A_{12}^{\mathrm{T}} A_{11}^{-1} A_{12}, \tag{5}$$

where the orthogonal matrix T is obtained from the relation $T^{\mathrm{T}} a^{-1/2} C_{22} a^{-1/2} T = \Omega$.

In the vector s we note subvectors $s_0^{\mathrm{T}} = (s_1, s_2,..., s_6)$ and $s_c^{\mathrm{T}} = (s_7, s_8,..., s_n)$. Then the MM (3) relatively normal coordinates takes the form

$$\begin{pmatrix} \ddot{s}_0 \\ \ddot{s}_c \end{pmatrix} + \begin{pmatrix} O_6 & O_{6\times(n-6)} \\ O_{(n-6)\times 6} & \Omega \end{pmatrix}\begin{pmatrix} s_0 \\ s_c \end{pmatrix} = \begin{pmatrix} RS_{11}(q) \\ HS_{11}(q) \end{pmatrix} M_a, \quad q_0 = (R \ \ H^{\mathrm{T}})\begin{pmatrix} s_0 \\ s_c \end{pmatrix}, \tag{6}$$

where $R = A_{11}^{-1/2}$, $H = -T^{\mathrm{T}} a^{-1/2} A_{12}^{\mathrm{T}} A_{11}^{-1}$.

Now we introduce for the MM (6) the transformation $\begin{pmatrix} s_0 \\ s_c \end{pmatrix} = \begin{pmatrix} R^{-1} & O_{6\times(n-6)} \\ O_{(n-6)\times 6} & E_{(n-6)} \end{pmatrix}\begin{pmatrix} \overline{x} \\ s_c \end{pmatrix}$

and as a result, we receive a MM that contains a coordinate of the "rigid" motion $\overline{x}$

$$\begin{gathered} \begin{pmatrix} \ddot{\overline{x}} \\ \ddot{s}_c \end{pmatrix} + \begin{pmatrix} O_6 & O_{6\times(n-6)} \\ O_{(n-6)\times 6} & \Omega \end{pmatrix}\begin{pmatrix} \overline{x} \\ s_c \end{pmatrix} = \begin{pmatrix} A_{11}^{-1}S_{11}(q) \\ HS_{11}(q) \end{pmatrix} M_a, \\ q_0 = (E_6 \ \ H^{\mathrm{T}})\begin{pmatrix} \overline{x} \\ s_c \end{pmatrix}. \end{gathered} \tag{7}$$

2.3. Construction of mathematical models of partial motion with respect to each of the measurable coordinates

With the MM (7) it appears the possibility to separate the partial MM that represents the spatial motion of our mechanical system with respect to only one of the coordinates of the vector q_0, for example , $q_i \ (i = \overline{1, 6})$

$$\begin{pmatrix} \ddot{\overline{x}}_i \\ \ddot{s}_c \end{pmatrix} + \begin{pmatrix} 0 & O_{1\times(n-6)} \\ O_{(n-6)\times 1} & \Omega \end{pmatrix}\begin{pmatrix} \overline{x}_i \\ s_c \end{pmatrix} = \begin{pmatrix} a_i \\ HS_{11}(q) \end{pmatrix} M_a, \; q_i = (1 \ \ h^{i\mathrm{T}})\begin{pmatrix} \overline{x}_i \\ s_c \end{pmatrix} \tag{8}$$

where $a_i = (A_{11}^{-1})_i S_{11}(q)$, $(A_{11}^{-1})_i$ is the *i*-th row of the matrix A_{11}^{-1}, h^i is the *i*-th column of the matrix H.

2.4. Constructing of modally physical models of partial motions [4]

We consider the diagonal $(n-6)\times(n-6)$ - matrix H^i with diagonal consisting of the components of the column vector h^i, i.e., $H^i = \mathrm{diag}(h_{1i}, h_{2i},\dots,h_{(n-6)i})$. Now it is possible to formulate

Proposition 2 [4]. The nonsingular transformation $\begin{pmatrix} \bar{x}_i \\ s_c^i \end{pmatrix} = \begin{pmatrix} 1 & O_{n-6} \\ 0 & (H^i)^{-1} \end{pmatrix} \begin{pmatrix} \bar{x}_i \\ z^i \end{pmatrix}$ allows to receive the modally physical models of partial motions [5] in the form

$$\begin{pmatrix} \ddot{\bar{x}}_i \\ \ddot{z}^i \end{pmatrix} + \begin{pmatrix} 0 & O_{(1\times(n-6))} \\ O_{((n-6)\times 1)} & \Omega \end{pmatrix} \begin{pmatrix} \bar{x}_i \\ z^i \end{pmatrix} = \begin{pmatrix} a_i \\ K^i \end{pmatrix} M_a, \quad q_i = \bar{x}_i + \tilde{x}_i, \quad \tilde{x}_i = \sum_{j=1}^{n-6} z_j^i, \tag{9}$$

where $K^i = L^{i\mathrm{T}} H^i H S_{11}(q)$.

The modal-physical model (MPM) (9) takes into account disturbances to coordinate q_i $(i = \overline{1,\ 6})$ from the spatial interconnected motions of all other coordinates q_j $(j = \overline{1, n},\ \ j \neq i)$. Sometimes such an interconnection so small that it is possible to neglect it. If so then the modal-physical model (MPM) (9) becomes much simpler. In [5] MPM for one angle coordinate without interconnected disturbances takes the form

$$\begin{aligned} &(a) \quad \ddot{\bar{x}} = m(u),\ \ m(u) = M(u) I^{-1}; \\ &(b_i) \quad \ddot{\tilde{x}}_i + \tilde{\omega}_i^2 \tilde{x}_i = \tilde{k}_i m(u),\ \ i = \overline{1, n}\ ; \\ &(c) \quad x = \bar{x} + \tilde{x},\ \ \tilde{x} = \sum_{i=1}^{n} \tilde{x}_i. \end{aligned} \tag{10}$$

In MPM (10) x is a measured and controlled angular coordinate ($x = q_i,\ i = \overline{1,3}$); $\bar{x}$ is a coordinate of “rigid” motion; $\tilde{x}$ is an additional angle motion of the main body in view of connected bodies vibration; I is a central spacecraft moment of inertia; $\tilde{\omega}_i,\ i = \overline{1, n}$ are eigenfrequencies of flexible oscillations; $\tilde{k}_i$ are coefficients of excitability for flexible modes; ($\mu_i^m = \tilde{k}_i m(u) \omega_i^{-2}$ is an extent of the exciting for *i*-th mode); $M(u)$ is a control acting; $u = u(x, \dot{x}, t)$ is a control law.

3. Some peculiarities of an interconnection of the FS orientation system with the structure vibrations at relay-operated controller

One of the principal problems of the FS orientation is the essential interconnection of the control system with the structure vibrations. This interconnection becomes especially strongly at applying relay or discrete control and at a boundedness of the information about the flexible object's state vector.

As a rule, the regime of stabilization is the main one (both with regard to the duration and with regard to its importance) over the course of the FS active life. It requires of reliable and economic functioning of the control system. Because of this and as on-board computer is used in the control system nowadays widely the stabilization is realized on the base of discrete systems with discontinuous control action.

In this regime as steady-state motion is stable limit cycle $\Gamma = \Gamma[\bar{u}(z,\nu)]$ (with the period $\tau_\Gamma = \tau_\Gamma(\nu)$). It is realized at nonlinear control algorithms $\bar{u} = \bar{u}[z(x),\nu]$ [7]. This cycle represents small admissible oscillations of controlled coordinate $(|x|,\,|\dot{x}|) \le \varepsilon_x$. The line over $\bar{u}$ indicates that in the synthesis of control algorithms the elasticity of the construction is not taken into account.

Such algorithms are called as base ones and they solve the task of control by main ("rigid") motion $\bar{x}(t)$. The control action discontinuity can be the cause of the vibrations with increasing intensity that is evaluated by the size (norm)

$$\rho_\Gamma = \|\tilde{x}\| = \frac{1}{\tau_\Gamma}\left[\int_0^{\tau_\Gamma} \tilde{x}^2(t)dt\right]^{\frac{1}{2}}. \tag{11}$$

The norm ρ_Γ can be considered as average on the period τ_Γ amplitude of multifrequency process $\tilde{x}(t) = \sum \tilde{x}_i(t)$.

Definition: *The state of controlled object* (6) *that is of limited construction* ($\omega_i^2 \neq \infty$) *is normal in known sense if its main motion is stable and satisfies with required indexes of quality and intensity of its construction vibrations* (11) *does not exceed of a prescribed value* $\rho_{\Gamma\max}$ *that is defined by the inadmissibility condition of the regulator "capture" by vibrations* [3].

It is well-known [3] that when the regulator with discrete variable of the control action level is used the vibrations are excited and amplitudes of the elastic modes $\rho_i(t) = |\tilde{x}_i(t)|$ are changed at each instant of the control action switching. In this case, as a rule, a dominant mode $\tilde{x}_d(t),\ d \in i$ is appeared. Its amplitude $\rho_d(t)$ increases more quickly in compare with the others and in some time the inequality $\rho_d(t) >> \rho_{i\neq d}(t)$ will be correct. So the summary intensity of the vibrations $\rho(t) = \sum|\tilde{x}_i(t)|$ can be estimated as $\rho(t) \approx \rho_d(t)$. In [3] it was shown that at $\overset{*}{\rho}_d \gg \mu^m$, $\mu^m \approx |m_{\bar{u}}|\tilde{k}_d\tilde{\omega}_d^{-2}$ ($m_{\bar{u}} \equiv m(\bar{u})$ is the base control) the process of the amplitude $\rho(t)$ changing can be described as follows

$$\rho(t_k) \equiv \rho_k \approx \rho_d^* + |\mu^m| \cdot \sum_{j=1}^{k} \delta_j \cos\beta_j [\operatorname{sign} \dot{m}_j(u)], \tag{12}$$

where δ_j is the coefficient of the control action level changing at the j-th switching, β_j is the corresponding phase of the dominant mode, $\dot{m}_j(u) = \dot{m}(t_j) = dm(u)/dt\big|_{t=t_j}$.

Consequently, the character of the amplitude ρ_d changing is defined by the system's state at the switching instants, that is the amplitude ρ_k can increase or decrease in accordance with the $\operatorname{sign} \dot{m}_j(u)$, and by the value of the phase $\beta_j = \beta_{(t_j)}$. As an optimal condition with respect to the phase β_j is the case when the amplitude ρ_k after the switching will be the smallest from all possible ones at prescribed direction of the control action switching $\operatorname{sign} \dot{m}_j(u)$. Optimal conditions are defined by the correlation

$$\beta_j = \begin{cases} 2\pi n & \forall \operatorname{sign} \dot{m}_j(u) = +1, \\ \pi(2n+1) & \forall \operatorname{sign} \dot{m}_j(u) = -1, \quad n = 0,1,2,\dots , \end{cases} . \tag{13}$$

The changing of the $\operatorname{sign} \dot{m}_j(u)$ to the opposite in (13) leads to the worst condition of switching and leads to the maximal increase of the amplitude ρ_k. All intermediate values of the phase $\beta_j = 2\pi n \vee \pi(2n + 1)$, $n = 0,1,2,\dots$ define either favorable (decrease of the amplitude) or unfavorable (increase of the amplitude) conditions of the switching.

Thus, the phases of the vibration amplitudes at the switching instants define the character of the oscillating processes at the FS' control. Let us note that at exceeding of the intensity $\rho(t) = |\tilde{x}(t)|$ of a critical level ρ_{cr} (mainly at the expense of the dominant mode $\tilde{x}_d$ growth) leads to the system instability (to the "capture" of the regulator by vibrations).

4. Synthesis of adaptive algorithm of the FS orientation control

Manoeuvres of the initial orientation and further stabilization at the change of the watched object are the most important regimes of the observation spacecrafts operation. Very often, these manoeuvres are realized with the help of control moments created by flywheels.

Discrete analog of linear PD-algorithm (at each interval of the digitization T_0 = const corresponding constant moment is applied to the satellite) is used usually in this case. After completion, the initial orientation or reorientation the control system realizes the process of stabilization. One of the wellknown algorithms of relay-logistic type [7] is a base algorithm for this regime. At using the algorithms of such type in the system stable limit cycle takes place that guarantees required accuracy and economics control. In any case, discontinuous character of the control actions is the cause of the excitation of the solar panels vibrations.

For the vibrations damping it is possible to use suggested in [8] the principle of the phase control of single-frequency FS (FS with only one elastic mode). This principle is based on the use of the algorithm with information about flexible mode phase. Briefly, the essence of this algorithm can be explained like that.

Let $\bar{u} = f_0(y,t)$, $y \triangleq \bar{x} + \tilde{x}$, is the base algorithm which is synthesized provided that the spacecraft is rigid. Then the algorithm $u = f_1(\bar{u},\beta)$ with additional signal about the phase β will be called as an extended one. The switching instant t_j of the control action will be called as phase-controlled if it depends on not only the coordinate $y(t)$ but also the phase β. At this the direction of the control action ($\operatorname{sign} \dot{m}_j(u)$) switching is defined uniquely by the base algorithm but the switching is delayed until the phase $\beta(t)$ will be equal to its optimal value or its favorable one.

Using correlation (11) it is possible to obtain the increment of the elastic mode in the i-th period of the observation containing R switchings of the control action:

$$\Delta\rho_i \triangleq \rho_i - \rho_{i-1} \approx \left|\mu^m\right| \cdot \sum_{r=1}^{R} \delta_r \cos\beta_r [\operatorname{sign} \dot{m}_r(u)] \ . \tag{14}$$

It is obvious that for stability of the control system motion it is necessary to have optimal or favorable values of the phases β_r at least at $R/2$ points of the switching. In this case, at $\delta_r = \text{const} \ \forall\ r \in [1, R]$ the increment $\Delta\rho_j \le 0$ that guarantees damping of the vibrations.

For this algorithm using to orientation of multi-frequency FS with poorly defined parameters and at absence of the elastic modes sensors it is necessary to solve two tasks: 1) to get and realize the algorithm of the dominant mode number, 2) synthesize the subsystem of the elastic modes $\tilde{x}_i(t)$ and the object's parameters estimation, that makes it possible to calculate the current value of the phase $\beta(t)$.

4.1. The algorithm of the dominant mode number identification

The main supposition: It is known a priori the frequencies ω_i $(i = \overline{1,n})$ of the model (10) that are ordered in ascending. Algorithm of the dominant mode frequency calculation is the following:

Step 1. "Rectification" of the input signal that is obtaining the block X_a of positive (absolute) values of the MPM harmonic components sum using input data base $x_a[k] = \left|\tilde{x}_\Sigma[k]\right|,\ x_a \in X_a$.

Step 2. Obtaining the block of maximum values X_m that correspond to the "amplitudes" of rectified summary signal x_a and the block of the instants $t_m[l] \in T_m$, $l = \overline{1,L},\ \ L = \dim X_m$, l is the number of the block X_m elements (or of the block T_m). Here $t_m[l]$ are the times of maximum values $x_m[l]$.

Step 3. Obtaining the block ΔT_m. Its elements are the differences $\Delta t_m[j] = t_m[l] - t_m[l-1] \quad (l = \overline{2,L})$.

Step 4. Calculation of the average value

$$\omega_m = \frac{\pi}{L-1} \sum_{j=1}^{L-1} (\Delta t_m[j])^{-1} \ . \tag{15}$$

Step 5. Calculation of minimum absolute value of the difference between the frequency ω_m and the MPM frequencies

$$\Delta\omega = \min_i(\left|\omega_i - \omega_m\right|) \ \ (i = \overline{1,n}) \tag{16}$$

and defining the number i_m that corresponds to the minimal difference $\Delta\omega$.

Step 6. Identification of the dominant mode number according to the rule:

If the inequality

$$\frac{\Delta\omega}{\omega_m} \le \Delta\omega_p \ , \tag{17}$$

where $\Delta\omega_p$ is threshold of the frequencies ω_i and ω_m closeness, is fulfilled then as the dominant mode is ω_m and its number is $d = i_m \in [1, n]$.

If the inequality (17) is not fulfilled we consider that there is no dominant mode in the signal $\widetilde{x}_\Sigma$ and $d = 0$.

Since the absence of the dominant mode is not the indication of the oscillating component $\tilde{x}(t)$ low level it is necessary to calculate the average value $\overline{z}^m$ of the block Z_m elements. Comparing this value with its admissible one $\overline{z}^{m*}$ it is possible to estimate the intensity of the oscillating process. For example, if $\overline{z}^m \geq \overline{z}^{m*}$ the special measures must be taken for decreasing of the oscillating component intensity and for preventing the capture regulator by vibration.

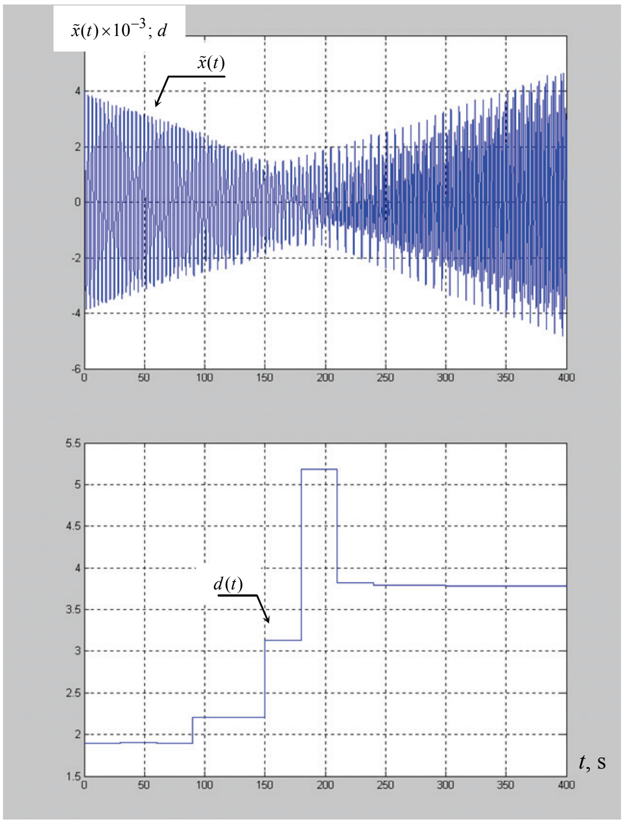

Fig. 1. The example of the dominant mode number identification.

Availability of this algorithm is illustrated by the example of simulation (Fig. 1) of multi-frequency FS ($n = 6$) motion. During the observed interval, the second dominant mode $\tilde{x}_{d=2}$ was damped and the fourth one $\tilde{x}_{d=4}$ occurs. This fact is fixed very clearly by step-like changing of the output signal $d(t)$ of the identification subsystem.

4.2. Simultaneous estimation of the FS vibration modes by Kalman filtration

For stability of motion and high quality control by FS that is multifrequency oscillating system it is necessary to have the information both about the coordinates of "rigid" motion and about its construction vibrations. Besides the FS mathematical model coefficients must be known. But usually their accurate values are unknown. Really for large space structures it is impossible to realize their full-scale ground tests.

This problem and the problem of inaccurate setting parameters of the object can be solved with the help of the method of simultaneous estimation of the parameters and modal-physical coordinates of the FS motion [9]. This method is based on the combination of the Kalman discrete filtration and the theory of the statistical hypothesizes testing.

The FS dynamics equations (10) for extended state vector can be written in vector-matrix form

$$\dot{X}(t) = f(X,u,t) + CW(t)\,, \tag{18}$$

where $X(t) = (x_1 \ldots x_{4n+2})^{\mathrm{T}}$ is current extended state vector that is needed to be estimated; $x_1 = \bar{x}$, $x_2 = \dot{\bar{x}}$, $x_{4i-1} = \tilde{x}_i$, $x_{4i} = \dot{\tilde{x}}_i$, $x_{4i+1} = \tilde{\omega}_i$, $x_{4i+2} = \tilde{k}_i$, $i = \overline{1,n}$; $u(t)$ is control function of relay class; $W(t) = (w_1 \ldots w_{3n+1})$ is vector of the object noises.

Vector-function $f(X, u, t)$ in the equation (18) is

$$f(X,u,t) = (f_1 \quad f_2 \quad \ldots \quad f_{4i-1} \quad f_{4i} \quad f_{4i+1} \quad f_{4i+2} \quad \ldots)^{\mathrm{T}}\,, \tag{19}$$

where $f_1 = x_2, f_2 = u, f_{4i-1} = x_{4i},\ f_{4i} = -x_{4i+1}^2 x_{4i-1} + x_{4i+2} u\,, f_{4i+1} = f_{4i+2} = 0,\ i = \overline{1,n}$.

The equations (18) in discrete form are the following

$$X_{k+1} = \Phi_{k+1}(Y_{k+1})X_k + \partial_{k+1}(Y_{k+1})u_k + \Psi_{k+1}(Y_{k+1})W_k;\ Y_{k+1} = M_{k+1}Y_k\,, \tag{20}$$

where $X_k = (\bar{x}_k, \dot{\bar{x}}_k, \tilde{x}_{1k}, \dot{\tilde{x}}_{1k}, \ldots, \tilde{x}_{nk}, \dot{\tilde{x}}_{nk})^T$ is the vector of coordinate, $Y_k = (\omega_{1k}, \tilde{k}_{1k}, \ldots, \omega_{nk}, \tilde{k}_{nk})^T$ is the vector of the FS parameters, W_k is the vector of the object noises.

If as the measuring instruments of the FS angular position attitude sensors and rate sensors are used then the equation of measurements in vector-matrix form will be

$$Z_{k+1} = H(Y_{k+1})X_{k+1} + V_{k+1}\,, \tag{21}$$

or in the scalar form is

$$z_1 = x_1 + \sum_{i=1}^{n} x_{4i-1} + v_1\,, \quad z_2 = x_2 + \sum_{i=1}^{n} x_{4i} + v_2\,.$$

Here $Z(t) = [z_1\ z_2]^{\mathrm{T}}$ is the vector of measurements, $V(t) = [v_1\ v_2]^{\mathrm{T}}$ is the vector of the measuring instruments noises.

For fixed vector Y the equations (20) and (21) can be reduced to the linear ones with respect to the vector X_k. In this case for simultaneous estimation of the vectors X_k and Y can be used combined algorithms that join the Kalman discrete filtration and the theory of the statistical hypothesizes testing [10].

Using as the hypothesis D_j, $j = \overline{1,l}$, the set of concrete values of the Y parameters that is $D_j = (y_1^j, \ldots, y_s^j, \ldots, y_{2n}^j)^{\mathrm{T}}$, optimal estimations of the vectors X_N and Y_N can be obtained as the solutions of the estimation equations, equations of the covariance matrices and functionals [9].

$$
\begin{aligned}
&\hat{X}_{jk+1} = \Phi_{jk+1}\hat{X}_{jk} + \partial_{jk+1}u_k + K_{jk+1}\Delta_{jk+1},\\
&\Delta_{jk+1} = Z_{k+1} - H(\Phi_{jk+1}\hat{X}_{jk} + \partial_{jk+1}u_k),\\
&K_{jk+1} = \overline{P}_{jk+1}H^T(H\overline{P}_{jk+1}H^T + R)^{-1},\\
&\overline{P}_{jk+1} = \Phi_{jk+1}P_{jk}\Phi^T_{jk+1} + \Psi_{jk+1}Q\Psi^T_{jk+1},\\
&P_{jk+1} = \overline{P}_{jk+1} - \overline{P}_{jk+1}H^T(H\overline{P}_{jk+1}H^T + R)^{-1}H\overline{P}_{jk+1},\\
&I_{jk+1} = I_{jk} + \Delta^T_{jk+1}\sum\nolimits^{-1}_{jk+1}\Delta_{jk+1} + \varepsilon_{jk+1},\ \varepsilon_{jk+1} = \ln(|\overline{P}_{jk+1}||H\overline{P}_{jk+1}H^T + R| / |P_{jk+1}|),\\
&\sum\nolimits^{-1}_{jk+1} = [E - (H\overline{P}_{jk+1}H^T + R)^{-1}H\overline{P}_{jk+1}H^T](H\overline{P}_{jk+1}H^T + R)^{-1}.
\end{aligned}
\tag{22}
$$

Testing of the simultaneous estimation algorithm (22) functioning was realized through simulation of the FS attitude control system using relay-logical algorithm [8] with "reversing".

The initial values of the state-vector X coordinates were the following: $x_1(0) = \varepsilon$, $x_2(0) = 1 \times 10^{-4}$, $x_{4i-1}(0) = 3\tilde{x}_{ci}$, $i = \overline{1,2}$, where $\tilde{x}_{ci} = m_u\tilde{k}_i / \tilde{\omega}_i^2$ is the excitability degree of the i-th mode during the time interval when control action m_u. Fundamental frequencies of two elastic modes (only two ones were taken into account) and their coefficients of excitability were: $\tilde{\omega}_1 = 2{,}05$ rad/s, $\tilde{\omega}_2 = 3{,}519$ rad/s, $\tilde{k}_1 = 1{,}441$, $\tilde{k}_2 = 1{,}72$.

As initial values of the component estimations of the state-vector X (coordinates and parameters) were chosen the following: $\hat{x}_i(0) = \hat{x}_{4i-1}(0) = \hat{x}_{4i}(0) = 0$, $i = \overline{1,2}$, $\hat{x}_5(0) = 0{,}7 \times \tilde{\omega}_1$, $\hat{x}_6(0) = 3{,}0 \times \tilde{x}_{c1}$, $\hat{x}_9(0) = 0{,}8 \times \tilde{\omega}_2$, $\hat{x}_{10} = 2{,}0 \times \tilde{k}_2$.

In Fig 2 simulation example of the process estimation of the dominant mode and its parameters is shown. It was assumed that the vector Y components are constant, that is $Y_j = \text{const}\ \forall j \in [1, l]$, for each number of the hypothesis $j = \overline{1,l}$.

The values of the vector Y_j elements y_{ji} were chosen between their minimum and maximum possible values with selected step of digitization.

On the basis of the formula (22) the final values of the functional I_{jN} were calculated and as the most probable hypothesis was chosen one with the number ν. For this hypothesis the functional I_{jN} must comply with the condition $I_{\nu N} = \min_j I_{jN}$. The estimation $\hat{X}_{\nu N}$ that corresponds to the hypothesis with the number ν is the optimal one of the vector X_N and the parameters of this hypothesis define the estimation of vector Y.

Fig. 2 shows that at initial displacements $\hat{\tilde{\omega}}_i(0) \le 1{,}4\tilde{\omega}_i(0)$ and $\hat{\tilde{k}}_i(0) \le 3\tilde{k}_i(0)$ it is guaranteed not only admissible rate of the estimations convergence but their high accuracy. This information is sufficient for the dominant mode current phase $\beta(t)$ calculation at any instant.

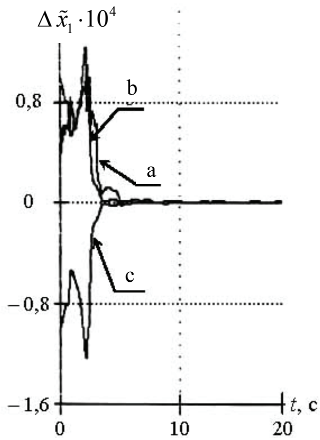

Fig. 2a The error of estimation of the dominant mode coordinate $\tilde{x}_1$:
a) $\Delta\tilde{x}_1$, b) $+2\sigma_{\tilde{x}_1}$, c) $-2\sigma_{\tilde{x}_1}$

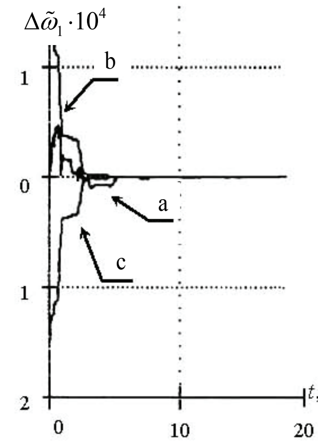

Fig. 2b The error of estimation of the dominant mode frequency $\tilde{\omega}_1$:
a) $\Delta\tilde{\omega}_1$, b) $+2\sigma_{\omega 1}$, c) $-2\sigma_{\omega 1}$

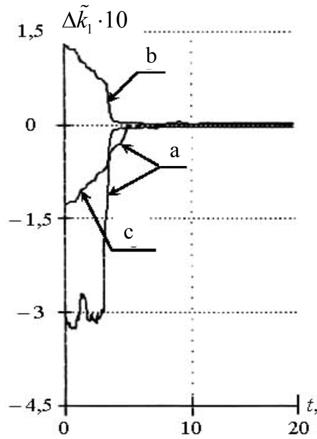

Fig. 2c The error of estimation of the dominant mode coefficient excitability $\tilde{k}_1$:
a) $\Delta\tilde{k}_1$, b) $+2\sigma_{\tilde{k}1}$, c) $-2\sigma_{\tilde{k}1}$

4.3. Adaptive system of the FS orientation control

On the basis of the dominant node number identification and calculation current value of the phase $\beta_d(t)$ the task of the FS orientation system design is solved.

Block-scheme of the adaptive orientation system, that is realized phase control of the multi-frequency FS, is shown in Fig. 3. This system can guarantee high accuracy of the reorientation, stabilization with respect to a new direction and damping of the constriction vibrations.

The z_{re} is the signal of reorientation in the block-scheme in Fig. 3. The control loop of the main ("rigid") FS motion is depicted by a dot line. It includes an additional link that realizes the time-delay of the control action switching until the phase β_d will be as optimal. The value of the phase β_d is calculated in the informational module of the time-delay switching subsystem. In this module, the time-delay $\tau = t^*_{\mathrm{sw}}(\beta_d) - t_{\mathrm{sw}}$ is calculated also. Here t^*_{sw} is the instant, when the phase β_d is equal to its optimal value, t_{sw} is the instant of the control action switching according to the base algorithm $u = u(z, t)$. So $u(t) = \bar{u}(t - \tau)$.

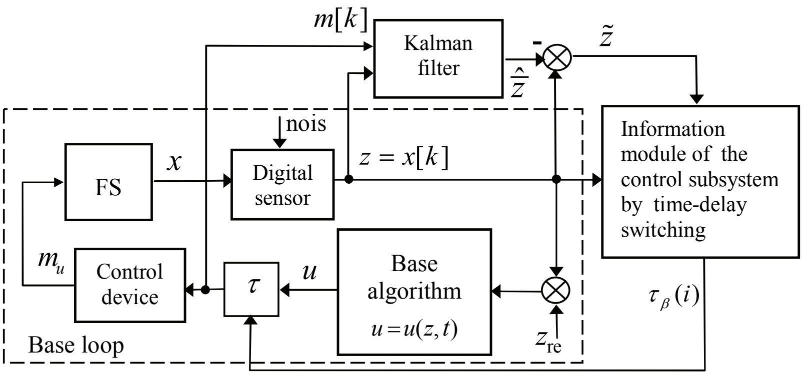

Fig. 3. Block-scheme of adaptive control system for FS.

After reorientation of the FS, the system passes in to a regime of stabilization of controlled coordinate x concerning new position of an observation axis. Thus the corresponding limit cycle is formed.

At this auto-oscillations that are represented by this stable limit cycle occur in the system. Optimal phases in the half of number of the control switching points are realized on the bases of described algorithm. The example of simulation in space MATLAB-Simulink suggested strategy of phase control is shown in Fig 4. As the object, described by equations (10) at $n = 6$, was the FS with moment of inertia $I^c_{\mathrm{FS}_6} = 10^4 kg{\cdot}m^2$. Other model parameters are in the table.

Table 1. Parameters of FS model

$i = \overline{1,6}$	1	2	3	4	5	6
$\tilde{f}_i = \tilde{\omega}_i / 2\pi$	0,07	0,1	0,15	0,5	2,8	5,2
$\tilde{k}_i$	0,17	0,03	0,015	0,01	0,004	0,002
$\tilde{x}_i(0)$	0,0005	0,00015	0,0001	0,00001	0,00005	0,00003

From the table it is obvious that at the initial stage of control the first elastic mode $\tilde{x}_1$ is the dominant one because of $\tilde{x}_1(0) \gg \tilde{x}_j(0),\ j = \overline{2,6}$, and the coefficient of excitability $\mu_1^{(1)} = \tilde{k}_1 \tilde{\omega}_1^{-2} = 0,88$ of this mode is the largest one. For clear at the initial part of the system operating the algorithm of time-delay of the control action switching was turned off (Fig 4, $t \le t_1 = 220s$).

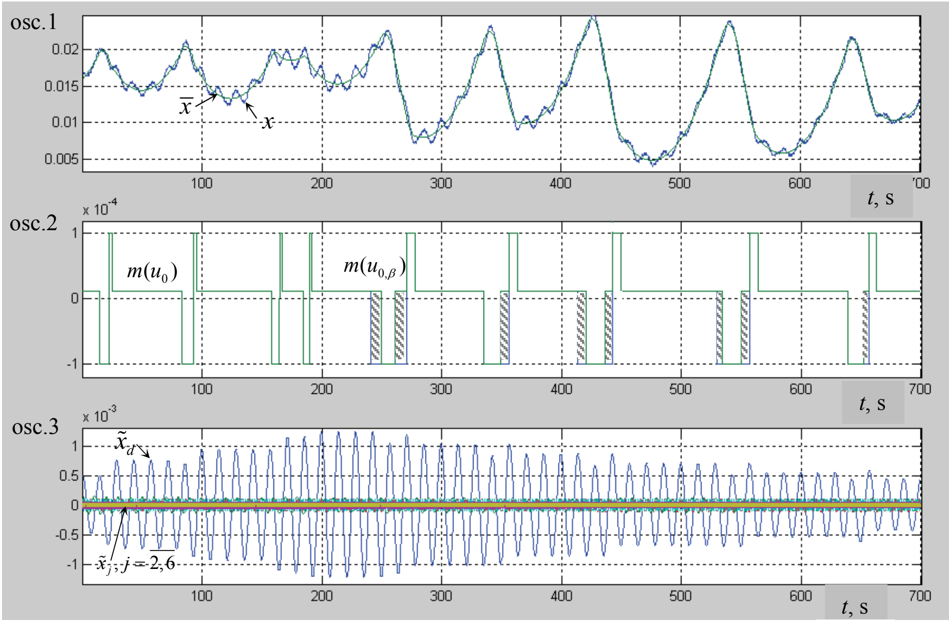

Fig. 4. Processes of FS adaptive stabilization for case of $d = 1$.

One can see from the oscillogram 3 that control action $m(u_0)$ of the stabilization system tends to increase dominant mode amplitude to value $\rho_d \approx 1{,}25 \cdot 10^{-3}$rad. This value is close to the critical one when the system motion becomes unstable. On the interval $170s \le t < 200s$ the period of limit cycle decreases sharply (osc. 1 and 2) indicating that the system approaches to the stability boundary. To avoid the capture regulator by vibrations and the motion instability at $t_1 = 220s$ the contour of phase control was switched on.

From the oscillograms in Fig 4 it is shown that the realization of the phase control strategy (the time-delay intervals τ_β are darken in osc. 2) leads to decreasing of the dominant mode amplitude with rather high velocity and without additional consumption of energy for control.

5. Conclusion

Suggested approach in the designing of the FS control system realizes the adaptive tuning of the base algorithm with a view to get optimal phase of the dominant mode in the instants of the control action switching. This guarantees the damping of the construction vibrations in the case of poorly defined object mathematical model without additional consumption of energy for control.

In the future it is worth while to investigate the efficiency of the suggested algorithm in the presence of two or more dominant modes of approximately equal intensity ($d = 0$). In this case the task of the mode choosing for which the phase of the control action switching is optimal will be actual.

Acknowledgement: *The work reported in this paper is a contribution to Projects 08-08-00512, funded by Russian Foundation for Basic Research, for which authors are grateful.*

REFERENCES

1. Junkins J.L., Kim Y., 1993, *Introduction to Dynamics and Control of Flexible Structures*, American Institute of Aeronautics and Astronautics, Washington, DC, (ISBN 1-56347-054-3).
2. Dynamics and control of structures in space. I-IV. 1990, 1993, 1996, 1999, *Proc. of the International conferences on dynamics and control of structures in space,* Kirk, C. (Editor). Computational mechanics publications, Southampton Boston.
3. Rutkovsky V.Yu., Sukhanov V.M., 1971, On the Stability of Relay Systems in Orientation Control of Flexible Satellites, *Proceedings of the 4-th IFAC Symposium on Automatic Control in Space*, Publishing House "Technique".
4. Zemlyakov S.D., Rutkovsky V.Yu., Sukhanov, V.M. , 2007, Computer-based derivation and transformation of spatial motion equations of a large space structure in the course of its assembly. *Computer and Systems Sciences International*. ISSN 1064-2307. V. 46, No 1. pp. 137-149.
5. Glumov V.M., Zemlyakov S.D., Rutkovsky V.Yu., Sukhanov V.M., 1998, Spatial Angular Motion of Flexible Spacecraft. The Modal Physical Model and Its Characteristics. *Automation and Remote Control.* V. 59. N 12, Part 1, pp. 1728-1738.
6. Rutkovsky V.Yu., Sukhanov V.M., 1996, Large space structure: Models, methods of study and control. I. *Automation and Remote Contro*l. Vol. 57, №7, Part 1. pp. 953-963.
7. Raushenbakh B.V. and E.N. Tokar, 1974, *Control of spacecraft orientation*. M.: Nauka. (In Russian).
8. Rutkovsky V.Yu., Sukhanov V.M., 1974, Attitude control algorithms in flexible satellites using information on the phase of elastic oscillations. *Proceedings of the 6-th IFAC Symposium on Automatic Control in Space.*
9. Ermilova T.V., Sukhanov V.M., Ermilov A.S., 2006, Simultaneous estimation of modal-physical coordinates and parameters at orientation control of large objects of space-system engineering with nonrigid construction. *Aerotronics Equipment*, № 3, pp. 58-64. (In Russian).
10. Sage A.P., Melsa J.L., 1972, *Estimation Theory with Application to Communication and Control*. N.-Y. Mc Graw-Hill.

MATEMATIČKI MODEL FLEKSIBILNOG KOSMIČKOG BRODA I FIZIČKA REALIZACIJA ADAPTIVNO-RELEJNOG URAVLJANJA ZA NJEGOVU ORIJENTACIJU

Vladislav Yu. Rutkovsky, Victor M. Sukhanov,
Stanislav D. Zemlyakov, Victor M. Glumov

U ovom radu se razmatraju kosmički brodovi sa fleksibilnom konstrukcijom. Predlažu se ugaone jednačine razlaganja pomeranja i procedura upravljanja sistemom. Istražuju se i neki problemi stabilizacije procesa dinamike ove upravljanih objekata. Uzimaju se u obzir esencijalna fleksibilnost konstrukcije, varijacije parametara matematičkih modela i razjedinjeni karakter ručne kontrole akcija sistema orijentacije. Fizička realizacija adaptivnog pristupa sistemu upravljanja obezbedjujući robustnost u odnosu na elastične oscilacije takogje se sugeriše. Koriste se Kalmanove procene elastičnog modula. Izabrani su elementi relejno-upravljacke akcije uključivanja u odnosu na uslove dominantnog modula optimalne faze.

Ključne reči: *fleksibilni kosmički brodovi, oscilacija, modeliranje, adaptivni algoritmi, kontrola releja*