

SUBSPACE-BASED FREQUENCY ANALYSIS OF A SMART ACOUSTIC STRUCTURE

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Abstract. *The aim of this paper is to perform a frequency analysis for a smart acoustic structure with integrated piezoelectric material based on the model obtained through the subspace identification procedure. In this way the relevant eigenfrequencies can be identified and used in the subsequent structural control design phases in order to avoid resonant states. Structural model is obtained in the state-space form, which enables a straightforward comparison of the identification results with numerical modeling results. Subspace identification procedure is performed for an acoustic enclosure consisting of a piezoelectric plate surrounded by an acoustic box. An experimental setup with the acoustic box and the dSPACE system was used for the identification. Experimentally identified frequency responses show a good agreement with the frequency responses obtained from the finite element model.*

Key words: *subspace-based system identification, acoustic box*

1. INTRODUCTION

The solution of problems in active structural acoustic control requires appropriate modeling and design tools, which result in reliable models capable of representing the relevant properties of structures under investigation. Although many powerful tools for numerical modeling have been developed in the recent years, e.g. finite element modeling approach [2-4], the need for verification and improvement of such models is still strong and mostly based on experimental methods. An approach which enables verification of the numerically obtained models of smart structures (including acoustic effects as well) on one hand, and a reliable model as a starting point for the controller design and further development purposes on the other hand is based on the subspace identification, which will be presented here.

In this paper the subspace identification is used to obtain experimentally the model of the piezoelectric mechanical structure influenced by the surrounding acoustic fluid in the

state-space form. The method of the identification is general – it applies to a wide range of model identification problems [5-11]. Based on the measurements of the input and output signals a state-space model should be determined. The identification algorithm uses the subspace-based approach. The solution of the identification problem results in a state-space model, which can be aimed at verification of FEM models and comparison in order to draw out the conclusions regarding the reliability of the models obtained using both modeling methods as well as regarding the minimal model orders which meet the required performances of the frequency responses.

2. SUBSPACE-BASED SYSTEM IDENTIFICATION

Subspace identification is based on sampled input/output measurement data. The resulting state-space model is identified in a discrete-time state-space form, where in a general case a deterministic-stochastic form of a discrete-time state-space model has the following form:

$$\begin{aligned} \mathbf{x}[k+1] &= \mathbf{\Phi}\mathbf{x}[k] + \mathbf{\Gamma}\mathbf{u}[k] + \mathbf{w}[k] \\ \mathbf{y}[k] &= \mathbf{C}\mathbf{x}[k] + \mathbf{D}\mathbf{u}[k] + \mathbf{v}[k] \end{aligned} \quad (1)$$

with discrete-time state and control matrices $\mathbf{\Phi}$ and $\mathbf{\Gamma}$, and the process and the measurement noise $\mathbf{w}[k]$ and $\mathbf{v}[k]$, respectively. The process noise and the measurement noise vector sequences $\mathbf{w}[k]$ and $\mathbf{v}[k]$ are white noise with zero mean and with covariance matrix:

$$\mathcal{E}\left\{\begin{bmatrix} \mathbf{w}[i] \\ \mathbf{v}[j] \end{bmatrix} \begin{bmatrix} \mathbf{w}[i]^T & \mathbf{v}[j]^T \end{bmatrix}\right\} = \begin{bmatrix} \mathbf{Q} & \mathbf{S} \\ \mathbf{S}^T & \mathbf{R} \end{bmatrix} \quad (2)$$

The general deterministic-stochastic problem of the subspace identification is to determine the order n of the unknown system and the system matrices $\mathbf{\Phi} \in \mathbb{R}^{n \times n}$, $\mathbf{\Gamma} \in \mathbb{R}^{n \times m}$, $\mathbf{C} \in \mathbb{R}^{l \times n}$, $\mathbf{D} \in \mathbb{R}^{l \times m}$ as well as the covariance matrices $\mathbf{Q} \in \mathbb{R}^{n \times n}$, $\mathbf{S} \in \mathbb{R}^{n \times l}$, $\mathbf{R} \in \mathbb{R}^{l \times l}$ of the noise sequences $\mathbf{w}[k]$ and $\mathbf{v}[k]$. Subsequent derivations regard the pure deterministic case considered in [1].

Measured input and output data are organized into block Hankel matrices defined in the following form [9,13]:

$$U_{0|2i-1} \stackrel{def}{=} \begin{bmatrix} \mathbf{u}_0 & \mathbf{u}_1 & \mathbf{u}_2 & \cdots & \mathbf{u}_{j-1} \\ \mathbf{u}_1 & \mathbf{u}_2 & \mathbf{u}_3 & \cdots & \mathbf{u}_j \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \mathbf{u}_{i-1} & \mathbf{u}_i & \mathbf{u}_{i+1} & \cdots & \mathbf{u}_{i+j-2} \\ \mathbf{u}_i & \mathbf{u}_{i+1} & \mathbf{u}_{i+2} & \cdots & \mathbf{u}_{i+j-1} \\ \mathbf{u}_{i+1} & \mathbf{u}_{i+2} & \mathbf{u}_{i+3} & \cdots & \mathbf{u}_{i+j} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \mathbf{u}_{2i-1} & \mathbf{u}_{2i} & \mathbf{u}_{2i+1} & \cdots & \mathbf{u}_{2i+j-2} \end{bmatrix} \quad (3)$$

The output block Hankel matrix $Y_{0|2r-1}$ is defined in a similar way. More details on definition of the Hankel matrices and the subspace-based identification method can be found in [8-13]. The measurement data are organized in the form of the input-output relation [14]:

$$\mathbf{Y}[k] = \mathbf{\Gamma}_\alpha \mathbf{x}[k] + \mathbf{\Phi}_\alpha \mathbf{U}[k] \quad (4)$$

where $\mathbf{\Gamma}_\alpha$ represents the observability matrix for the system (1), $\mathbf{\Phi}_\alpha$ is the Toeplitz matrix [14] of impulse responses from \mathbf{u} to \mathbf{y} :

$$\mathbf{\Phi}_\alpha = \begin{bmatrix} \mathbf{D} & 0 & \cdots & \mathbf{0} \\ \mathbf{C}\mathbf{\Gamma} & \mathbf{D} & & \mathbf{0} \\ \vdots & \ddots & \ddots & \vdots \\ \mathbf{C}\mathbf{\Phi}^{\alpha-2}\mathbf{\Gamma} & \cdots & \mathbf{C}\mathbf{\Gamma} & \mathbf{D} \end{bmatrix} \quad (5)$$

and α is a specified number greater than the state dimension but much smaller than the data length. For a deterministic case [1,10] the problem is simplified to determining $\mathbf{\Gamma}_\alpha$ and $\mathbf{\Phi}_\alpha$ by computing the singular value decomposition (SVD) of \mathbf{U} in the first step:

$$\mathbf{U} = \mathbf{P}\mathbf{\Sigma}\mathbf{Q}^T = [\mathbf{P}_{u1} \quad \mathbf{P}_{u2}] [\mathbf{\Sigma}_u \quad 0] \begin{bmatrix} \mathbf{Q}_{u1}^T \\ \mathbf{Q}_{u2}^T \end{bmatrix} \quad (6)$$

If matrix \mathbf{U} has dimension $m \times n$ and rank r , then the partition in (6) is performed as follows:

$$\mathbf{P} = [\mathbf{p}_1 \quad \cdots \quad \mathbf{p}_r \mid \mathbf{p}_{r+1} \quad \cdots \quad \mathbf{p}_m] = [\mathbf{P}_{u1} \quad \mathbf{P}_{u2}] \quad (7)$$

$$\mathbf{Q} = [\mathbf{q}_1 \quad \cdots \quad \mathbf{q}_r \mid \mathbf{q}_{r+1} \quad \cdots \quad \mathbf{q}_n] = [\mathbf{Q}_{u1} \quad \mathbf{Q}_{u2}] \quad (8)$$

where \mathbf{p}_i are the left singular vectors of \mathbf{U} [17]. It can be shown that they are eigenvectors of $\mathbf{U}\mathbf{U}^T$. Vectors \mathbf{q}_i are the right singular vectors of \mathbf{U} . It can be shown that they are eigenvectors of $\mathbf{U}^T\mathbf{U}$. Multiplying (4) by \mathbf{Q}_{u2} , matrix $\mathbf{\Gamma}_\alpha$ can be determined from a SVD of $\mathbf{Y}\mathbf{Q}_{u2}$. Then matrix \mathbf{C} is obtained as the first row (in a sense of a block-row) of the observability matrix $\mathbf{\Gamma}_\alpha$, and matrix $\mathbf{\Phi}$ is calculated from: $\mathbf{\Gamma}_\alpha = \bar{\mathbf{\Gamma}}_\alpha \mathbf{\Phi}$ applying pseudo inverse, where $\bar{\mathbf{\Gamma}}_\alpha$ is obtained by dropping the last row of $\mathbf{\Gamma}_\alpha$. Matrix $\mathbf{\Gamma}_\alpha$ represents the matrix obtained by dropping the first row of $\mathbf{\Gamma}_\alpha$. For the calculation of $\mathbf{\Gamma}$ and \mathbf{D} matrices, (4) is multiplied by the pseudo inverse of \mathbf{U} on the right and by \mathbf{P}_{u2}^T from (6) on the left. Thus the equation is reduced to:

$$\mathbf{P}_{u2}^T \mathbf{Y} \mathbf{U}^{-1} = \mathbf{P}_{u2}^T \mathbf{\Phi}_\alpha \quad (9)$$

After rearranging, (9) can be solved for $\mathbf{\Gamma}$ and \mathbf{D} using the least squares, see (5). In this way the system parameters in the form of state-space matrices of the model (1) are identified using the subspace-based identification method.

3. EXPERIMENTAL RIG WITH THE ACOUSTIC BOX

Described procedure for the subspace identification was implemented and tested using the experimental setup with the smart plate with piezoelectric patches and the acoustic box with the air as acoustic fluid within it. The aim of the experiment is the identification of the state-space model using the subspace method and its comparison with the numerically developed FEM based state-space model [3,12].

The acoustic box setup consists of a clamped aluminium plate with fifteen piezoelectric patches attached to its inner surface (Fig. 1) and of the wooden box comprising the acoustic fluid – air. The side of the box opposite to the aluminium plate is open. Dimensions of the acoustic box are represented in Fig. 2.

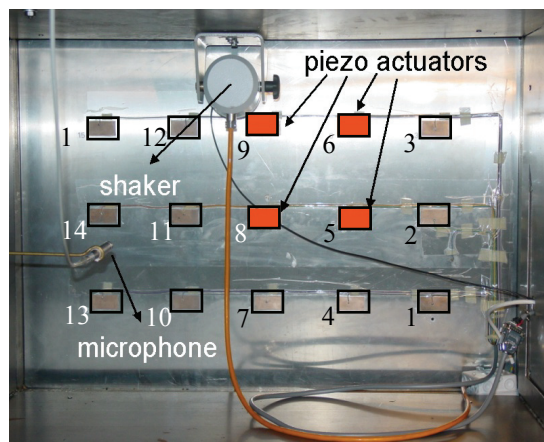


Fig. 1 Inner side of the plate with attached piezo patches

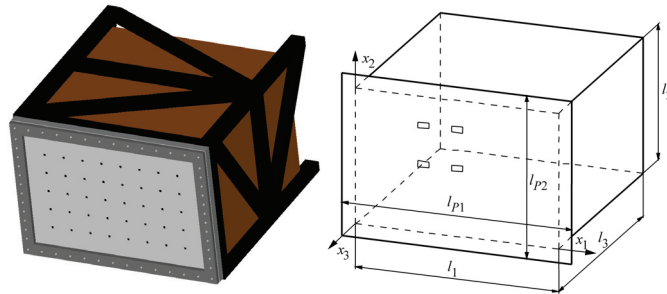


Fig. 2 Acoustic box with dimensions

Dimensions of the plate are: $l_{p1}=1020\text{mm}$, $l_{p2}=720\text{mm}$, $h=4\text{mm}$, dimensions of the cavity: $l_1=900\text{mm}$, $l_2=600\text{mm}$, $l_3=1250\text{mm}$, dimensions of the patches: $50\text{mm}\times 25\text{mm}\times 0.2\text{mm}$.

The plate is excited using a shaker driven by computer generated random noise signals. The impedance head placed on the top of the rig connected to the shaker measures

the shaker force signal. The sound pressure at the predefined point of the acoustic box ($x_1=650\text{mm}$, $x_2=300\text{mm}$, $x_3=525\text{ mm}$) is measured using a microphone. Acoustic box is placed in the sound low-reflection room in order to eliminate environmental influences during the measurements. The scheme of the experimental rig with all included measurement devices for experimental determining of the frequency responses and for the experimental subspace-based model identification is shown in Fig. 3.

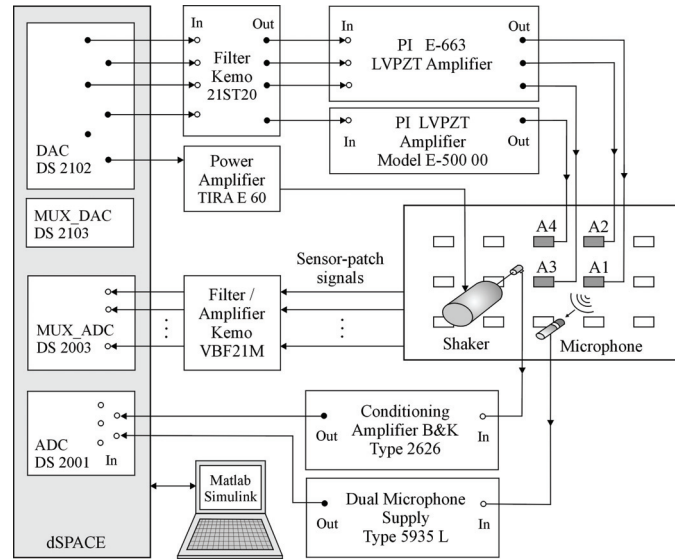


Fig. 3 Experimental rig for the frequency response determining and the state-space model identification

Excitation signals for the shaker are generated by the computer linked to the dSPACE system with ADC and DAC boards shown in Fig. 3. Random noise excitation signal from the DAC board DS 2102 (range $\pm 5\text{V}$) of the dSPACE system is amplified by the Power Amplifier TIRA E 60 and led to the shaker. Excitation force exerted by the shaker is measured using the force gauge Impedance Head B&K Type 8001, the signal of which is fed to the dSPACE ADC board DS 2001 (range $\pm 5\text{V}$) via the Conditioning Amplifier B&K Type 2626, which calibrates the force signal. The conditioning amplifier outputs 0.1 V per measured unit, in this case per electric charge produced by the impedance head of the sensitivity 369 pC/N, which is also set on the conditioning amplifier. Therefore the output of the conditioning amplifier of 0.1V corresponds to the shaker force of 1N. The frequency range set on the conditioning amplifier is between 1 Hz and 1 kHz.

Another signal acquired by the ADC board DS 2001 of the dSPACE system is the air pressure signal on the microphone. The microphone signal is amplified with the gain 10dB through the Dual Microphone Supply Type 5935L. The sensitivity of the microphone is 50.8 mV/Pa.

Selected piezo patches are used as sensors. Their voltage signals are filtered in the predefined frequency range of 1kHz and amplified with the gain of 12dB through the Filter/Amplifier Kemo VBF21M. Signal acquisition of the sensor-patches is performed

on the MUX_ADC board DS 2003 (range $\pm 5V$) of the dSPACE system using the input channels of this board.

For the identification of the MIMO models of the acoustic box the measurement of the excitation signals from the selected actuator-patches is required. The piezo patches numerated as 5, 6, 8 and 9 (Fig. 1) were selected as actuators for the purpose of numerical FEM modeling. For the sake of the consistency between the numeric and experimental (identified) model, the same patches 5, 6, 8, 9 are used respectively as actuators A1, A2, A3, A4 during the data acquisition for the model verification and identification purposes. Random noise signals generated by the computer are output through the DAC board DS2102 (range $\pm 5V$) of the dSPACE. They are first filtered through the low-pass filter Kemo 21ST20 with the cut-off frequency of 1 kHz and afterwards amplified through the piezo amplifiers PI E-663 LVPZT (for the first three channels/actuators) and PI LVPZT Model E-500 00 (for the fourth channel/actuator) with the gain 10 and offset 50V. For the controller implementation purposes the actuator signals of the predefined controller can be fed to the piezo actuators in the similar way.

4. EXPERIMENTAL RESULTS

Using the previously described experimental setup, the experimental frequency analysis of the smart acoustic box and the subspace identification were performed. The frequency analysis was aimed at obtaining the frequency response functions which provide information on the eigenfrequencies. In this way the verification of the numerically calculated eigenfrequencies can be performed.

The frequency responses of the acoustic box are determined experimentally and on the basis of the numerical FEM model [12]. Calculated eigenfrequencies of the modally truncated FEM based state-space model for the first five structural (index w) and acoustic modes (index a) are listed in the Table 1.

Table 1. Calculated eigenfrequencies of the elastic plate and of the acoustic cavity

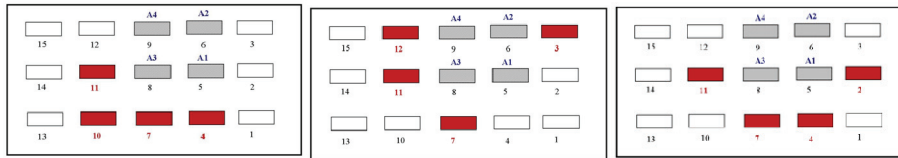
Number i	1	2	3	4	5
f_{wi} [Hz]	66.7	106.2	163.8	172.1	201.2
ω_{wi} [rad/s]	419.1	667.3	1029.2	1081.3	1264.2
f_{ai} [Hz]	68.0	200.8	204.0	278.1	291.5
ω_{ai} [rad/s]	427.3	1261.7	1281.8	1747.4	1831.6

Experimental frequency response functions (FRFs) were determined based on the selected measured input and output signals, i.e. their fast Fourier transforms (FFTs). The FRF expresses a frequency domain relationship (in terms of ratio) between a response signal (output) and a reference signal (input) of a linear time-invariant system, i.e. the ratio of their FFTs. For three different sensor-patch constellations (Table 2) all combinations of the output-to-input FRFs were determined (actuator-to-sensor, shaker-to-sensor, actuator-to-microphone, sensor-to-microphone) and compared with the corresponding frequency responses determined on the basis of the numerically obtained state-space model [12] of the order 20. In the numerical state-space model as well as for its experimental

verification and subspace identification the actuator-patch signals were considered as inputs, the shaker force signal as measurable disturbance input, and the sensor-patch and microphone signals as the measured outputs. For all three actuator/sensor constellations the patches 5, 6, 8, 9 were used as actuators. Sensor groups for the three constellations are shown in Table 2.

Table 2. Piezo patches of the three sensor constellations

	Constellation 1	Constellation 2	Constellation 3
Sensor 1	2	3	4
Sensor 2	4	7	7
Sensor 3	7	11	10
Sensor 4	11	12	11



Selected frequency responses are represented in the following figures showing a good agreement between the experimental and numerical results.

Figures 4 – 6 show the comparison of the experimental and numerical results for the single-input single-output (SISO) case when the acoustic plate is excited only by a shaker or only by a single actuator-patch and the signals are measured on the microphone or on the appropriate sensor-patches.

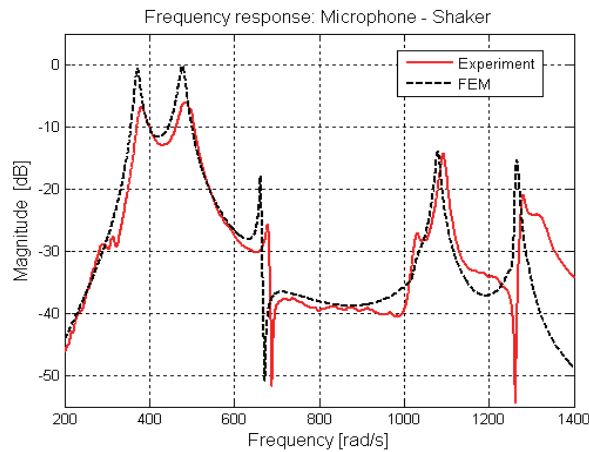


Fig. 4 Excitation by shaker (valid for all three sensor constellations)

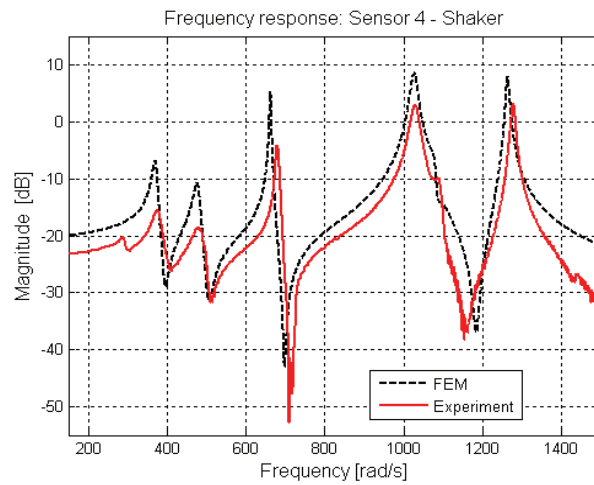


Fig. 5 Excitation by shaker (constellation 2)

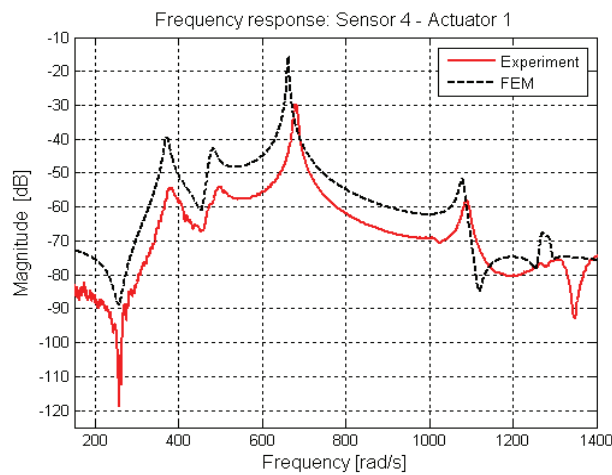


Fig. 6 Excitation by actuator-patch (constellation 3)

For the investigation of multiple-input multiple-output (MIMO) models the responses of the sensor-patches were acquired under excitation by four actuator-patches and by the shaker. Selected frequency responses obtained experimentally and on the basis of the reduced FEM model are compared in the figures 7 and 8. First the experimental FRFs (solid line) were obtained under random excitation by shaker and actuator-patches. During the experiment the excitation signals were acquired and saved using dSPACE. The same excitations were used for the simulation with the reduced FEM based state-space model and based on the obtained simulated time responses, using the FFTs, the FEM based FRFs were obtained (dashed black line).

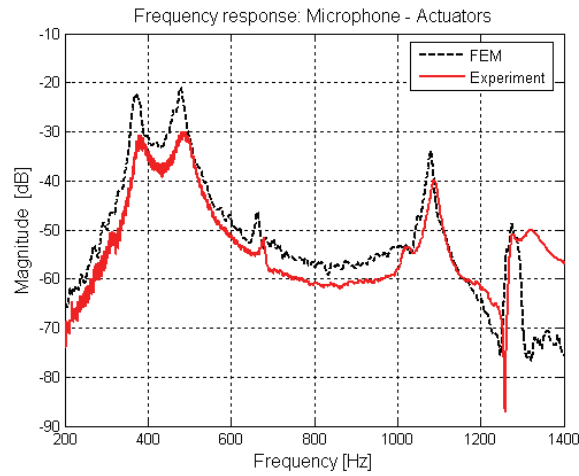


Fig. 7 Excitation by shaker and actuator-patches (constellation 1 – MIMO case)

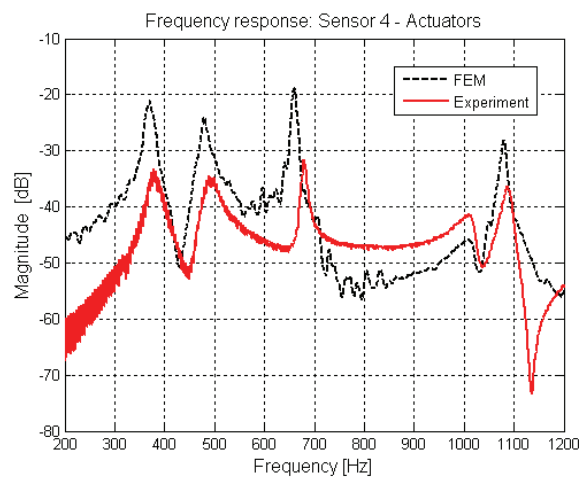


Fig. 8 Excitation by shaker and actuator-patches (constellation 1 – MIMO case)

The MIMO model subspace identification of the acoustic box was performed on the basis of the measured input/output signals using the experimental rig shown in Fig. 3 and applying the identification algorithm (described in section 2) and the auxiliary Matlab function *n4sid* [6,14]. The criterion for selection of the model order was to obtain a MIMO state-space model, from which any of the possible single output-to-input FRFs (actuator-to-sensor, actuator-to-shaker, microphone-to-sensor or microphone-to-shaker) can be derived in such a way that they correspond accurately enough to the measured

SISO FRFs. In this case *accurately enough* means that the eigenfrequencies of interest can be obviously recognized from the frequency responses obtained on the basis of the identified MIMO model. Through the iterative procedure it was found that the identified state-space model of the order $n=85$ fulfills such a condition. Lower model orders cannot provide pronounced eigenfrequencies of interest, while the higher model orders cause high computational effort without obvious improvement of the frequency response diagrams. Selected comparative FRFs are shown in figures 9 and 10.

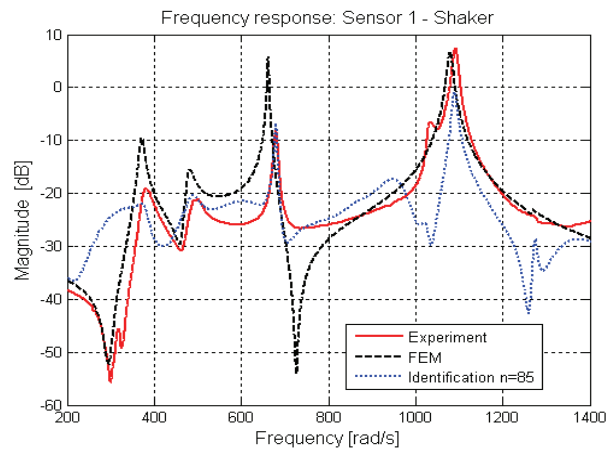


Fig. 9 FRFs *sensor 1 – shaker* determined experimentally, numerically and from the identified model (constellation 1)

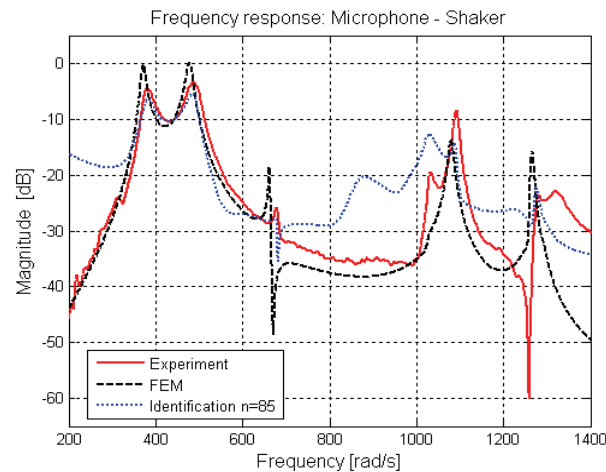


Fig. 10 FRFs *microphone – shaker* determined experimentally, numerically and from the identified model (constellation 1)

5. CONCLUSION

In this paper the subspace based identification method is used for the frequency analysis and model development of a smart acoustic box. The properties of the surrounding acoustic fluid were taken into consideration via appropriate measurement of the air pressure in the prescribed point of the acoustic box. The identification results were compared with the numerical analysis based on the FEM method. A good agreement of the compared results was shown.

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FREKVENTNA ANALIZA AKTIVNE AKUSTIČNE STRUKTURE NA BAZI IDENTIFIKACIJE METODOM PODPROSTORA

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Cilj ovog rada je identifikacija frekventnih odziva aktivne akustične strukture sa integrisanim piezo električnim materijalom u ulozi senzora a na osnovu modela dobijenog primenom metode podprostora matrica (subspace identification). Na ovaj način mogu se identifikovati relevantne sopstvene frekvencije, koje igraju bitnu ulogu u kasnijim fazama projektovanja upravljanja u cilju sprečavanja pojave rezonantnih stanja. Kao rezultat identifikacije dobija se model u prostoru stanja, čime je omogućeno upoređivanje rezultata eksperimentalne identifikacije i numeričkog modeliranja. Identifikacija metodom podprostora matrica izvršena je za aktivnu akustičnu strukturu, koja se sastoji od aluminijumske ploče sa integrisanim piezo električnim davačima – pločicama i od akustične kutije. Za identifikaciju je korišćeno eksperimentalno postrojenje sa akustičnom kutijom i dSPACE sistemom. Upoređivanje rezultata ukazuje na dobro slaganje između identifikovanih frekventnih odziva i numeričkih rezultata dobijenih metodom konačnih elemenata.

Ključne reči: Aktivne akustične strukture, frekventna analiza, identifikacija modela, metod podprostora stanja