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ON THE MOTION OF SUSPENSIONS WITH NONSYMMETRIC STRESS TENSOR IN NONLOCAL THEORY

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Abstract. *The present paper considers the suspension as a two-component mixture, and the following fields have been defined: speed (of the suspension), microrotation velocity and concentration distribution. The balance equations are given for the mixture as a whole, whereby the nonlocal effects are included through the constitutive equations. The expression obtained for the speed of suspension has been prepared for a graphical presentation, and subsequently graphically represented and compared with the corresponding expressions for the case of the classical suspension theory (the local theory, the symmetric stress tensor), and with the suspension having a nonsymmetric stress tensor in the local theory.*

1. INTRODUCTION

It is well known that the influence of the corresponding field of distant points on the reference point is eliminated, by using the local effect principle in the continuum mechanics. By abandoning this principle, there opens a possibility of describing a wide class of phenomena, which is the reason why the nonlocal continuum theory is still the subject of a large number of papers. The recent use of the nonlocal theory, particularly in the fracture mechanics [1], has shown that some phenomena can be predicted by means of nonlocal effects.

This paper represents a continuation of paper [2], and has been motivated by the actual problem of the motion of suspensions in cold drying and separation devices. Considering suspension as a two-component mixture, the paper defines the following fields: velocity (of the suspension), microrotation velocity, and concentration distribution. The balance equations are given for the mixture as a whole, whereby the nonlocal effects are included through the constitutive equations.

The expression obtained for the speed of suspension has been prepared for a graphical

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presentation, and subsequently graphically represented and compared with the corresponding expressions for the case of the classical suspension theory (the local theory, the symmetric stress tensor), and with the suspension having a nonsymmetric stress tensor in the local theory.

2. MOTION OF THE SUSPENSION BETWEEN TWO PARALLEL PLATES

Let as consider a stationary motion of an incompressible suspension between two parallel planes which are placed at a mutual distance of $2h$. Let the axis Ox of the chosen Descartes rectangular coordinate system coincide with the median, the axis Oy be perpendicular to the plane, and the axis Oz perpendicular to the direction of movement. Regarding the purpose of this investigation, we shall neglect the volume forces effects, the temperature, and the nonlocal residue in the mass balance equations, the momentum and the moment of momentum.

$$\upsilon_{i,i} = 0, \tag{1}$$

$$\rho \frac{\mathrm{d}\upsilon_i}{\mathrm{d}t} = t_{ij,j}, \tag{2}$$

$$\rho \frac{\mathrm{d}(I\nu_i)}{\mathrm{d}t} = \varepsilon_{ijk} t_{kj} + m_{ij,j}\ , \tag{3}$$

$$\rho \frac{\mathrm{d}c}{\mathrm{d}t} = -J^p_{i,i}\ , \tag{4}$$

where ρ - suspension density, υ_i - suspension velocity, t_{ij} - nonsymmetrical stress tensor, I - moment of inertion, ν_i - microrotation velocity (microrotational spin), ε_{ijk} - Ricci's tensor, m_{ij} - couple stress tensor, c - concentration, J_i^p - diffused mass flux. With $\frac{\mathrm{d}}{\mathrm{d}t}$ we denote the material derivate, and with comma "," the partial derivate. For the motion of the velocity field under consideration, the microrotation velocity and the diffused mass flux are given, whereby $0 \le y \le h$, since $\upsilon(-y) = \upsilon(y)$.

$$\begin{aligned} &\upsilon_x = \upsilon(y), \quad \upsilon_y = \upsilon_z = 0 \\ &\nu_x = \nu(y), \quad \nu_x = \nu_z = 0 \\ &J_x^p = J(y), \quad J_y^p = J_z^p = 0 \end{aligned} \tag{5}$$

From the consecutive equation for the stress tensor [3]

$$\begin{aligned} t_{ij} &= (-\pi+\lambda\upsilon_{r,r})\delta_{kl} + ,+\mu(\upsilon_{k,l}+\upsilon_{l,k}) + k(\upsilon_{l,k} - \varepsilon_{klr}\nu_r) + \\ &+ \int_V [\sigma(|x'-x|)\delta_{kl} + \lambda'(|x'-x|)\upsilon_{r,r}(|x'|)\delta_{kl} + \\ &+ 2\mu'(|x'-x|)d_{kl}(|x'|) + k'(\upsilon_{l,k}(|x'|) - \varepsilon_{klr}\nu_r(|x'|))]dV(|x'|) \end{aligned} \tag{6}$$

where λ and μ are the viscosity coefficients of the clasical theory and k is the viscosity coefficient of the miclopolar theory, while λ' , μ' and k' are the nonlocal viscosity moduli.

From the motion equation (2) and (6) it follows that

$$t_{yx} = (\mu+\kappa)\frac{\mathrm{d}\upsilon}{\mathrm{d}y} + kv + \int_0^y \left[(\mu' + k')\frac{\mathrm{d}\upsilon}{\mathrm{d}\eta} + k'v \right] \mathrm{d}\eta \,, \tag{7}$$

$$t_{xy} = (\mu+\kappa)\frac{\mathrm{d}\upsilon}{\mathrm{d}y} - kv + \int_0^y \left[\mu'\frac{\mathrm{d}\upsilon}{\mathrm{d}\eta} - k'v \right] \mathrm{d}\eta \ . \tag{8}$$

$$t_{yx,y} = \pi_{,x} \equiv P \ . \tag{9}$$

After the integration of equation (9) and by using of (7) it can be obtained

$$t_{xy} = (\mu+\kappa)\frac{\mathrm{d}\upsilon}{\mathrm{d}y} - kv + \int_0^y \left(\mu'\frac{\mathrm{d}\upsilon}{\mathrm{d}\eta} - k'v \right) \mathrm{d}\eta \ . \tag{10}$$

For further considerations we need the constitutive equation for the couple stress tensor [4]

$$\begin{aligned} m_{kl} &= \alpha v_{r,r}\delta_{kl} + \beta v_{k,l} + \gamma v_{l,k} + \\ &+ \int_V [\alpha'(|x'-x|)\delta_{kl} v_{r,r}(|x'|) + \beta'(|x'-x|)v_{k,l}(|x'|) + \\ &+ \gamma'(|x'-x|)v_{l,k}(|x'|)] dV(|x'|) \ , \end{aligned} \tag{11}$$

where α, β and γ are the viscosity coefficients of the micropolar theory, while α', β' and γ' are the nonlocal viscosity moduli.

By using of (3), (11) and (5) we obtain

$$\frac{\mathrm{d}\upsilon}{\mathrm{d}y} + \int_0^y \frac{k'}{k}\frac{\mathrm{d}\upsilon}{\mathrm{d}\eta}\mathrm{d}\eta = \frac{\beta}{k}\frac{\mathrm{d}^2 v}{\mathrm{d}y^2} + 2v + \frac{\beta'}{k}\frac{\mathrm{d}v}{\mathrm{d}y} - \int_0^y \frac{1}{k}\frac{\mathrm{d}\beta'}{\mathrm{d}\eta}\frac{v}{\eta} d\eta + 2\int_0^y \frac{k'}{k} v d\eta \ . \tag{12}$$

For further consideration we also need, the phenomenological equation [5]

$$\rho D(c_{,i} + k_p p_{,i}) = J_i^p + a_1 a_2 a_3 \varepsilon_{ijk} J_j^p (v_k - \Omega_k) + a_1 a_2 a_3 \varepsilon_{ijk} v_{j,k} + a_1 a_2 \varepsilon_{ijk} v_j (v_k - \Omega_k) \tag{13}$$

where D is the diffusion coefficient, a_i (i = 1, 2, 3) are the scalars characterizing the isotropic properties of the environment, c-concentration, p-hydrostatic pressure, Ω_i angular velocity of suspension, $k_p = \dfrac{a_1\gamma}{\rho D}$.

For the case of motion under consideration, the relation (13) take the form

$$\rho D\frac{\mathrm{d}c}{\mathrm{d}y} = \left(-a_1 a_2 \rho D k_p P - a_1 a_4 \frac{\mathrm{d}v}{\mathrm{d}y} \right)(v - \Omega) \ . \tag{14}$$

By applying Laplace's transforms to Eqs. (11) and (13), we obtain

$$\mu_1 + \bar{\mu}_1'(s\bar{\upsilon} - \upsilon_0) = \frac{P}{s^2} - \bar{v}(k + \bar{k}_1') \ , \tag{15}$$

$$(k + \bar{k}_1')(s\bar{\upsilon} - \upsilon_0) = 2\bar{v}(k + \bar{k}_1') + (\beta_1 + \bar{\beta}')(s^2\bar{v} - sv_0 - v_0) \ , \tag{16}$$

where the bar over a definite quantity denotes its Laplace's transformation. Assuming that $v_0 = 0$, from (15), and (16), it follows that

$$\bar{\upsilon} = \bar{v}\left(\frac{2}{s} + \frac{\beta_1 + \bar{\beta}'}{k + \bar{k}'} s \right) + \frac{\upsilon_0}{s} \ , \tag{17}$$

i.e., that the expressions for the microrotation velocity and for the velocity of the movement of suspension, can be written in the form

$$\bar{\nu} = \frac{P}{3s^2(k+\bar{k}') + 2s^2(\mu+\bar{\mu}') + s^4(\beta+\bar{\beta}')\left(1+\frac{\mu+\bar{\mu}'}{k+\bar{k}'}\right)} \quad , \tag{18}$$

$$\bar{\upsilon} = \frac{2P}{s^2\left[3s(k+\bar{k}') + 2s(\mu+\bar{\mu}') + s^3(\beta+\bar{\beta}')\left(1+\frac{\mu+\bar{\mu}'}{k+\bar{k}'}\right)\right]} + \frac{P\frac{\beta+\bar{\beta}'}{k+\bar{k}'}}{s^2\left[3s(k+\bar{k}') + 2s(\mu+\bar{\mu}') + s^3(\beta+\bar{\beta}')\left(1+\frac{\mu+\bar{\mu}'}{k+\bar{k}'}\right)\right]} + \frac{\upsilon_0}{s} \quad . \tag{19}$$

For the calculation of the inverse Laplace's transformation, it is necessary to assume the form of the nonlocal viscosity moduli. Keeping in mind the nature of the nonlocal effects, let us assume that the moduli could be written in the form [2]:

$$\begin{aligned} \mu' &= \mu_0' e^{-ay} \quad , \\ k' &= k_0' e^{-by} \quad , \\ \beta' &= \beta_0' e^{-cy} \quad , \end{aligned} \tag{20}$$

By applying Laplace’s transformation, it follows from (20) that

$$\begin{aligned} \bar{\mu}' &= \mu_0' \frac{1}{s+a} \quad , \\ \bar{k}' &= k_0' \frac{1}{s+b} \quad , \\ \bar{\beta}' &= \beta_0' \frac{1}{s+c} \quad . \end{aligned} \tag{21}$$

If, besides we also assume that the couple stress is constant, the expressions (18) and (19) are reduced to

$$\bar{\nu} = \frac{P}{A_1}\left(\frac{M_1}{s} + \frac{M_2}{s^2} + \frac{M_3}{s-s_1} + \frac{M_4}{s-s_2}\right) , \tag{22}$$

$$\bar{\upsilon} = \frac{\upsilon_0}{s} + \frac{2P}{A_1}\left(\frac{M_2}{s^3} + \frac{M_1}{s^2} + \frac{m_1}{s} + \frac{m_2}{s-s_1} + \frac{m_3}{s} + \frac{m_4}{s-s_2}\right) , \tag{23}$$

where

$$\begin{aligned} & a_k = b + \frac{k_0'}{k} \ , \quad a_\mu = a + \frac{\mu_0'}{\mu} \ , \quad a_\beta = a + \frac{\beta_0'}{\beta} \ , \\ & A_1 = 3k + 2\mu \ , \quad 2B_1 = 3ka_k + 3ka + 2\mu a_\mu + 2\mu b \ , \quad C_1 = 2\mu b a_\mu + 3kaa_k \ , \\ & B = \frac{B_1}{A_1} \ , \quad C = \frac{C_1}{A_1} \ , \quad s_{1,2} = -B \pm \sqrt{B^2 - C} \ , \\ & M_1 = \frac{ab(s_1+s_2) + (a+b)s_1 s_2}{s_1{}^2 \cdot s_2{}^2} \ , \quad M_2 = \frac{ab}{s_1 s_2} \ , \\ & M_3 = (-M_1 + M_4) \ , \quad M_4 = -\frac{ab + (a+b)s_2 + s_2^2}{s_2^2(s_1 - s_2)} \ , \\ & m_1 = -m_2 = \frac{M_3}{s_1} \ , \quad m_3 = -m_4 = \frac{M_4}{s_2} \ . \end{aligned} \tag{24}$$

By applying the inverse Laplace's transformation to (23), the expression for the suspension velocity becomes

$$\upsilon = \frac{\upsilon_0}{s} + \frac{2P}{A_1}\left(\frac{1}{2}M_2 y^2 + M_1 y + m_1 + m_2 e^{s_1 y} + m_3 + m_4 e^{s_2 y}\right) . \tag{25}$$

Taking into account the boundary conditions for $y = \pm b$, and the extreme conditions

$$\upsilon(b) = 0 , \quad \left.\frac{\mathrm{d}\upsilon}{\mathrm{d}y}\right|_{y=0} = 0 , \tag{26}$$

it follows that

$$\left.\frac{\mathrm{d}\upsilon}{\mathrm{d}y}\right|_{y=0} = M_1 + m_2 s_1 + m_4 s_2 = 0 . \tag{27}$$

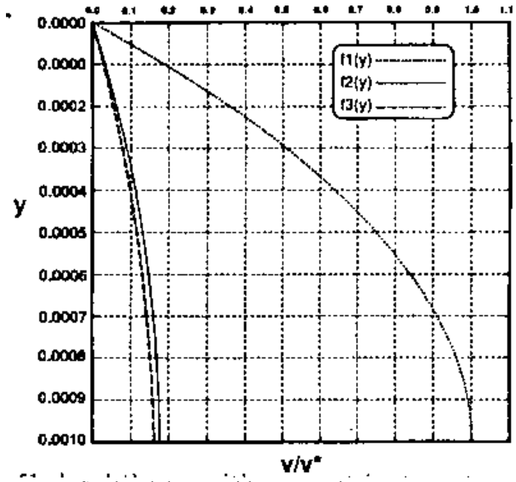
From (24)

$$M_1 + M_3 + M_4 = 0 , \tag{28}$$

and

$$\upsilon(b) = \upsilon_0 + \frac{2P}{A_1}\left(\frac{1}{2}M_2 b^2 + M_1 b + m_1 + m_2 e^{s_1 b} + m_3 + m_4 e^{s_2 b}\right) = 0 . \tag{29}$$

can be obtained.



*f*1 - local theory with symmetric stress tensor,
*f*2 - local theory with nonsymmetric stress tensor,
*f*3 - nonlocal theory with nonsymmetric stress tensor.

Figure 1. Dimensionless velosity of suspension

From (26), it follows that

$$\upsilon = -\frac{P}{A_1}[M_2(b^2 - y^2) + 2M_1(b - y) + 2m_2(e^{s_1 b} - e^{s_1 y}) + 2m_4(e^{s_2 b} - e^{s_2 y})] = 0 . \tag{30}$$

For obtaining a dimensionless form of the expression for the suspension velocity, the expression (29) is divided by $\upsilon_* = -\frac{PM_2}{A_1}b$. In that way, we obtain

$$\frac{\upsilon}{\upsilon_*} = \left(1 - \frac{y^2}{b^2}\right) + \frac{2M_1}{M_2 b}\left(1 - \frac{y}{b}\right) + \frac{2m_2}{M_2 b^2}(e^{s_1 b} - e^{s_1 y}) + \frac{2m_4}{M_2 b^2}(e^{s_2 b} - e^{s_2 y}) . \tag{31}$$

By using [5] in which the values of the micropolar fluid constants are determined experimentally and assuming that the basic phase (fluid) is to a considerable extent more present than the dispersed phase (solid particles) of suspension, the constant k can be calculated. It can be used for the determination of all the quantities present in (24).
For the motion of the suspension under consideration, for b = 0.002m, from (31) it follows that

$$\frac{\upsilon}{\upsilon_*} = (1 - 25\times 10^4 y^2) + 0{,}12(1 - 500y) + 0{,}6(e^{-0{,}19} - e^{-96{,}1y}) \ . \qquad (32)$$

The graphical presentation of the function (32) is given in Fig.1 ($f\,3$).

3. Conclusion

The aim of this paper has been to define the effects of the nonsymetric stress tensor, as well as the nonlocal effects upon the field of the velocity of motion of the suspension. The paper considers the problem of the motion of the suspension between two parallel planes. The expression obtained for the velocity in a nondimensional form (32) has been compared with the expressions valid in the local theory [6] and [7], and a graphical presentation has been given (Fig. 1).
It can be concluded from that graph that the maximum velocity in the case under consideration is lower as compared with the corresponding velocities in the local theories, which had been intuitively expected, and which should be proved experimentally.

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KRETANJE SUSPENZIJA SA NESIMETRIČNIM TENZOROM NAPONA U NELOKALNOJ TEORIJI

P. Cvetković, D. Kuzmanović, Z. Golubović, S. Marković

U ovom radu suspenzije se posmatraju kao dvokomponentne mešavine a date su sledeće oblasti: brzina (suspenzije), brzina mikrorotacije i raspodela koncentracije. Ravnotežne jednačine su date za celu mešavinu dok su nelokalni efekti uključeni u konstitutivnim jednačinama. Izrazi dobijeni za brzinu suspenzije su pripremljeni za grafičko prikazivanje i kasnije grafički pokazani i uporedjeni sa odgovarajućim izrazima za slučaj teorije klasičnih suspenzija (lokalna teorija, simetrični naponski tenzor) i sa suspenzijama koje imaju nesimetrični naponski tenzor u lokalnoj teoriji.