UNIVERSITY OF NIŠ
The scientific journal **FACTA UNIVERSITATIS**
Series: **Mechanics, Automatic, Control and Robotics** Vol.2, No 7, 1997 pp. 315 - 324
Editor of series: Katica (Stefanović) Hedrih, email: katica@masfak.masfak.ni.ac.yu
Address: Univerzitetski trg 2, 18000 Niš, YU, Tel: (018) 547-095, Fax: (018)-24-488
http:// ni.ac.yu/Facta

VARIABLE STRUCTURE MODEL REFERENCE ADAPTIVE SYSTEMS WITH PROPORTIONAL-PLUS-INTEGRAL ACTION

UDK: 62-504

Darko Mitić

Faculty of Electronic Engineering, University of Niš, Yugoslavia

Abstract. *In this paper, an implementation of variable structure system (VSS) with proportional-plus-integral (PI) term in the synthesis of model reference adaptive control (MRAC) is presented. Two algorithms which synthesize the signal adaptation, whose integral term in the control ensures minimal value of the error between the reference model and the plant, have been considered. The stability proof of these systems is obtained via VSS theory. The performances of the proposed MRACs are evaluated and shown by means of digital simulation.*
Key words: *variable structure systems, model reference adaptive control, signal adaptation*

1. INTRODUCTION

A design of linear control laws in the case of the unstable plants, whose parameters change their values with the time in the known range, is difficult and complex problem which strikes the need of the adaptive control introduction. The aim of the control is to achieve the desired system dynamic behavior in spite of the lack of information about the exact plant parameters values. In MRAC, the technical demands are given via the corresponding dynamic of the reference model. Therefore, the basic task is to design such a control which will ensure the minimal error between the reference model and the plant outputs. That can be enabled either by using the parameter adaptation or by performing the synthesis of the signal adaptation [1]. The lack of MRAC comes from the fact that the system has bad dynamic and static characteristics if controller parameters are not adequately adjusted, or if adaptation signal does not have the adequate value.

VSSs with sliding mode are well known and studied nonlinear control systems [2] which possess the robustness in relation to external disturbances and plant parameters variations. The combination of the variable structure control and the adaptation mechanisms realizes satisfactory system responses. However, the implementation of VSS requests the knowledge of all state coordinate values which can not be measured in the

Received July 28, 1995

most cases. The use of the observers is the way to overcome this problem. Taking in the consideration the fact that the plant parameters change their values with the time, it must be taking into account during the design of the observer that the roots of its characteristic equation must be on the left side of the roots of the plant characteristic equation in the extreme case. That is why the implementation of adaptive, robust and sliding mode observers are more suitable. The particular methods for state coordinates estimation, which are based on measuring the plant inputs and outputs [3], [4], have been developed recently, as well.

In this paper, the implementation of two variable structure controllers (VSC) with PI term [5], [6], [7] in the synthesis of MRAC with the signal adaptation has been presented. The main reason of introduction of the PI term in the VSC is to improve the accuracy in the steady-state in relation to the conventional VSC. That is why the both variants of the proposed adaptive control ensure the minimal error between the reference model and the plant outputs. Also, the aforementioned introduction of the PI term in VSC provides better existence conditions of the sliding mode than it is case with the conventional VSC. However, the first control algorithm, in which the control signal is formed as a discontinuous function of the error between the reference model and the plant outputs and its derivatives, gives the better accuracy in the steady-state, while the second one, where the control signal is obtained as a discontinuous function of the plant output and its derivatives, enables the faster system response and the better elimination of the external disturbance. These differences between two proposed algorithms can be noticed clearly in Fig. 1 and Fig. 2, and on the basis of the analysis in the following sections. Since it is necessary to know the state coordinate values of the plant for the VSC realization, a simplification has been done in this case by the assumption that all state coordinates of the plant are measurable.

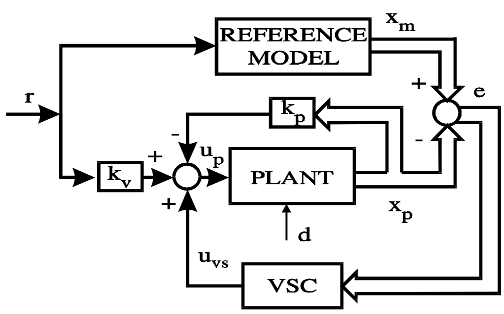

Fig. 1 Block-diagram of the first variable structure MRAC

In the Section 2, the mathematical models of the plant and the reference model in the state space are given. The design procedures of the aforementioned variable structure MRAC algorithms are presented in the Section 3. The verification of the proposed control approaches has been performed on the basis of digital simulation results which are shown in the Section 4.

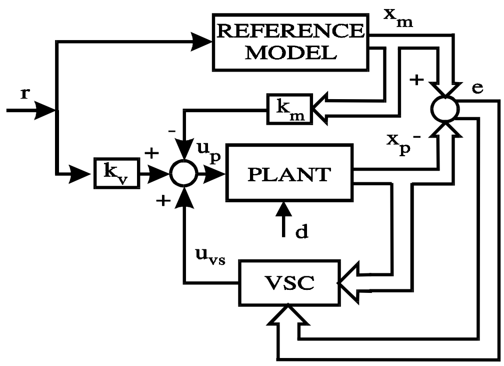

Fig.2 Block-diagram of the second variable structure MRAC

2. Problem statement

The mathematical models, given in the controllable canonical (companion) form, of the n-th order single-input-single-output (SISO) plant and reference model are presented by the following equations:

$$\frac{d\mathbf{x_p}}{dt} = \mathbf{A_p}\mathbf{x_p} + \mathbf{b_p}u_p + \mathbf{d}\,, \tag{1}$$

$$\frac{d\mathbf{x_m}}{dt} = \mathbf{A_m}\mathbf{x_m} + \mathbf{b_m}r\,. \tag{2}$$

respectively, where $\mathbf{x_p}=[x_{p_1}, x_{p_2}, ..., x_{p_n}]^T$ and $\mathbf{x_m}=[x_{m_1}, x_{m_2}, ..., x_{m_n}]^T$ represent the corresponding state coordinate vectors of the plant and the reference model, respectively; $\mathbf{d}=[0\ 0\ 0...d]^T_{nx1}$ is a vector of external disturbances; $\mathbf{A_p}$ and $\mathbf{A_m}$ are evaluation matrices of the plant and the reference model, respectively; $\mathbf{b_p}$ and $\mathbf{b_m}$ are the corresponding control vectors.

The matrix $\mathbf{A_p}$ and the vector $\mathbf{b_p}$ are given by $\mathbf{A_p} = \mathbf{A_p^o} \pm \Delta\mathbf{A_p}$ and $\mathbf{b_p} = \mathbf{b_p^o} \pm \Delta\mathbf{b_p}$, where $\mathbf{A_p^o}$ and $\mathbf{b_p^o}$ are the nominal evaluation matrix and control vector and $\Delta\mathbf{A_p}$ and $\Delta\mathbf{b_p}$ represent the corresponding matrix and vector which define the uncertainty of the plant. It is assumed that all elements of $\Delta\mathbf{A_p}$ and $\Delta\mathbf{b_p}$ are non-negative.

The goal is to form the control which will prescribe in advance the transient process of the error between the reference model and the plant outputs $e_1 = x_{m_1} - x_{p_1}$, where e_1 is the element of the vector $\mathbf{e}=[e_1, e_2, ..., e_n]^T$ defined as:

$$\mathbf{e} = \mathbf{x_m} - \mathbf{x_p}\ , \tag{3}$$

and which will force this error e_1 to tend zero asymptotically. In further text, two approaches in solving this problem are given.

3. CONTROLLER DESIGN PROCEDURES

3.1 First adaptive algorithm

In this case, the control u_p is formed according to Fig. 1, as:

$$u_p = k_v r + u_{vs} - \mathbf{k_p x_p} \;, \tag{4}$$

where: k_v is the gain, r is the reference input signal, u_{vs} is the variable structure control, $\mathbf{k_p}$ is the gain vector of the plant state feedback.

The model of the entire system in the error state space, defined by (3), is given by the following equation:

$$\frac{d\mathbf{e}}{dt} = \mathbf{A_m e} - \mathbf{b_p} u_{vs} + \mathbf{f} \;, \tag{5}$$

at which $\mathbf{f}$ is the disturbance vector defined by:

$$\mathbf{f} = \mathbf{f_o} + \Delta\mathbf{f} \;, \tag{6}$$

$$\mathbf{f_o} = (\mathbf{A_m} - \mathbf{A_p^o} + \mathbf{b_p^o k_p})\mathbf{x_p} + (\mathbf{b_m} - \mathbf{b_p^o} k_v) r \;, \tag{7}$$

$$\Delta\mathbf{f} = (\pm\Delta\mathbf{b_p k_p} \mp \Delta\mathbf{A_p})\mathbf{x_p} \mp \Delta\mathbf{b_p} k_v r - \mathbf{d} \;, \tag{8}$$

where: $\mathbf{f_o}$ is the average disturbance vector which corresponds to the nominal values of the plant parameters contained in the elements of the matrix $\mathbf{A_p^o}$ and the vector $\mathbf{b_p^o}$, $\Delta\mathbf{f}$ is the part of the vector $\mathbf{f}$ which originates from the plant parameters uncertainty ($\Delta\mathbf{A_p}$, $\Delta\mathbf{b_p}$) and from the action of the external disturbances $\mathbf{d}$.

The gain k_v and the gain vector $\mathbf{k_p}$ of the plant state feedback are calculated so that the average disturbance vector $\mathbf{f_o}$, which is given by the relation (7), is equal to zero vector. The sufficient existence conditions of the solution for the gain k_v and the gain vector $\mathbf{k_p}$ are respectively given by the following relations [1]:

$$(\mathbf{b_p^o}(\mathbf{b_p^{oT} b_p^o})^{-1}\mathbf{b_p^{oT}} - \mathbf{I})\mathbf{b_m} = \mathbf{0} \;, \tag{9}$$

$$(\mathbf{b_p^o}(\mathbf{b_p^{oT} b_p^o})^{-1}\mathbf{b_p^{oT}} - \mathbf{I})(\mathbf{A_m} - \mathbf{A_p^o}) = \mathbf{0} \;. \tag{10}$$

The proof of the conditions (9) and (10) is presented in the *Appendix A*.

If the equations (9) and (10) are fulfilled, the corresponding values of k_v and vector $\mathbf{k_p}$ are calculated then according to:

$$k_v = (\mathbf{b_p^{oT} b_p^o})^{-1}\mathbf{b_p^{oT} b_m} \;, \tag{11}$$

$$\mathbf{k_p} = (\mathbf{b_p^{oT} b_p^o})^{-1}\mathbf{b_p^{oT}}(\mathbf{A_p^o} - \mathbf{A_m}) \;. \tag{12}$$

VSC realizes the control law in the form of:

$$u_{vs} = \sum_{i=1}^{n-1} \Psi_i e_i + \frac{1}{T_i}\int_{-\infty}^{t} \Psi_1 e_1 dt \;, \tag{13}$$

where: Ψ_i is the commutation function of the VSC defined by:

$$\Psi_i = \begin{cases} \alpha_i & za\ ge_i \geq 0 \\ -\alpha_i & za\ ge_i < 0 \end{cases}, \tag{14}$$

and $g = 0$ is the equation of the sliding hyper-surface described by:

$$g = \mathbf{c}^T \mathbf{e}, \quad \mathbf{c} = [c_1, c_2, ..., c_n]^T, \quad c_n = 1 . \tag{15}$$

The sufficient reaching and existence condition of the sliding mode, well known in literature as:

$$g\frac{dg}{dt} < 0 , \tag{16}$$

is deduced in this case on the following inequalities:

$$\begin{aligned} \alpha_1 &> \left| \frac{(-c_1 c_{n-1} + c_1 a_{m_n} - a_{m_1})|e_1| + \Delta f}{\underline{b_p}\left(|e_1| + \frac{1}{T_i}\int_0^t |e_1| dt\right)} \right| \\ \alpha_i &> \left| \frac{c_{i-1} - c_i c_{n-1} + c_i a_{m_n} - a_{m_i}}{\underline{b_p}} \right| \quad i = 2, ..., n-1 \end{aligned} \tag{17}$$

where: $\underline{b_p} = \min(b_p)$ and Δf is the n-th element of the vector $\mathbf{\Delta f}$ (other elements are equal to zero). The assumption has been made that the non-zero initial conditions of the integral action in the control signal, given by (13), are equal to zero. The choice of the parameters of the VSC commutation function of the first adaptive system is done according to the relation (17). The desired system dynamic is defined by the choice of the sliding hyper-surface parameters c_i. The necessary and the sufficient condition of the system stability in the sliding mode is that (15) must be Hurwitz polynomial.

3.2 Second adaptive control algorithm

The control u_p of the second MRAC variant is chosen as:

$$u_p = k_v r + u_{vs} - \mathbf{k_m x_m} , \tag{18}$$

where $\mathbf{k_m}$ is the gain vector of the reference model state feedback.
By deriving (3) and substituting the equations (1) and (2) one can obtain:

$$\frac{d\mathbf{e}}{dt} = -\mathbf{A_p x_p} - \mathbf{b_p} u_{vs} + \mathbf{f} , \tag{19}$$

at which $\mathbf{f}$ is disturbance vector defined by (6), and $\mathbf{f_o}$, $\mathbf{\Delta f}$ by:

$$\mathbf{f_o} = (\mathbf{A_m} + \mathbf{b_p^o k_m})\mathbf{x_m} + (\mathbf{b_m} - \mathbf{b_p^o} k_v) r , \tag{20}$$

$$\mathbf{\Delta f} = \pm \mathbf{\Delta b_p}(\mathbf{k_m x_m} - k_v r) - \mathbf{d} \ , \tag{21}$$

where: $\mathbf{f_o}$ is the average disturbance vector which corresponds to the nominal values of the plant parameters contained in the elements of the vector $\mathbf{b_p^0}$, $\mathbf{\Delta f}$ is the part of the vector $\mathbf{f}$ which originates from the plant parameters uncertainty ($\mathbf{\Delta b_p}$) and from the action of the external disturbance $\mathbf{d}$.

The gain k_v and the gain vector $\mathbf{k_m}$ of the reference model state feedback are calculated so that the average disturbance vector $\mathbf{f_o}$, given by the relation (20), is equal to zero vector. The sufficient existence condition of the solution for the gain k_v and the gain vector $\mathbf{k_m}$ are respectively given by (9) and the following relation [1]:

$$(\mathbf{b_p^o}(\mathbf{b_p^{oT}}\mathbf{b_p^o})^{-1}\mathbf{b_p^{oT}} - \mathbf{I})\mathbf{A_m} = \mathbf{0} \ . \tag{22}$$

The proof of condition (22) is similar to the proof of the conditions (9) and (10) described in *Appendix A*. If the equations (9) and (22) are fulfilled, then the corresponding values of k_v i $\mathbf{k_m}$ are calculated according to the relation (11) and the next relation:

$$\mathbf{k_m} = -(\mathbf{b_p^{oT}}\mathbf{b_p^o})^{-1}\mathbf{b_p^{oT}}\mathbf{A_m} \ . \tag{23}$$

In this case, the variable structure control law is given in the form of:

$$u_{vs} = \sum_{i=1}^{n} \Psi_i x_{p_i} + \frac{1}{T_i}\int_{-\infty}^{t} \Psi_1 x_{p_1} dt \ , \tag{24}$$

where Ψ_i is the VSC commutation function defined as:

$$\Psi_i = \begin{cases} \alpha_i & za \ g x_{p_i} \geq 0 \\ -\alpha_i & za \ g x_{p_i} < 0 \end{cases} , \tag{25}$$

and $g = 0$ is the equation of the sliding hyper-surface described by (15).
In the case of the second adaptive control algorithm, the sufficient reaching and existence conditions of the sliding mode, carried out on the basis of the relation (16), are deduced on:

$$\alpha_1 > \left| \frac{-\underline{a_{p_1}} \, | x_{p_1} | + \Delta f}{\underline{b_p}\left(| x_{p_1} | + \frac{1}{T_i}\int_0^t | x_{p_1} | dt \right)} \right| . \tag{26}$$
$$\alpha_i > \left| \frac{c_{i-1} - \underline{a_{p_i}}}{\underline{b_p}} \right| \quad i = 2, \ldots, n$$

where: $\underline{a_{p_i}} = \min(a_{p_i})$. The non-zero initial conditions of the integral action in (24) are assumed to be equal to zero. The choice of the parameters of the VSC commutation function of the second adaptive system is done according to the relations (26).

4. DIGITAL SIMULATION RESULTS

In order to verify the advantages of the proposed control approaches, let us consider the control of the second order plant whose model in the state space is given by (1) and the corresponding matrix and the vector are:

$$\mathbf{A_p^0} = \begin{bmatrix} 0 & 1 \\ -1 & -1.2 \end{bmatrix}, \quad \mathbf{b_p^0} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

The reference model, described by (2), is defined via the following matrix and the vector:

$$\mathbf{A_m} = \begin{bmatrix} 0 & 1 \\ -20 & -12 \end{bmatrix}, \quad \mathbf{b_m} = \begin{bmatrix} 0 \\ 20 \end{bmatrix}$$

First algorithm

By implementation of the design procedure, taking into account the changes of the plant parameters for ±50% of their nominal value, the following parameters of the control part of the system are chosen: $k_v = 0.05$, $k_p = [19, 10.08]$, $a_1 = 100$, $c_1 = 5$, $T_i = 0.05$ s. In Fig. 3, the responses of the plant and the reference model, as well as the error between the reference model and the plant outputs are shown. The elimination of the external disturbance is given in Fig. 4.

Second algorithm

By implementation of the design procedure, taking into account the changes of the plant parameters for ±50% of their nominal value, the following parameters of the control part of the system are chosen: : $k_v = 0.05$, $k_m = [20, 12]$, $a_1 = 100$, $c_1 = 5$, $T_i = 0.05$ s. In Fig. 5, the responses of the plant and the reference model, as well as the error between the reference model and plant outputs are shown. The elimination of the external disturbance is given in Fig. 6.

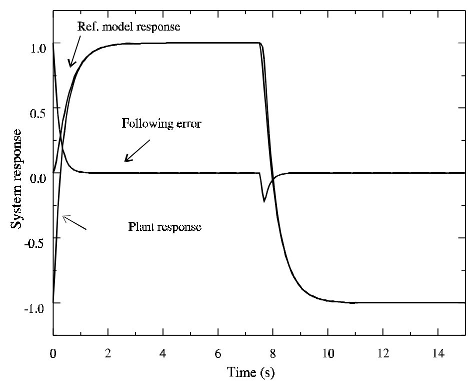

Fig. 3 Reference model and plant responses (first control algorithm)

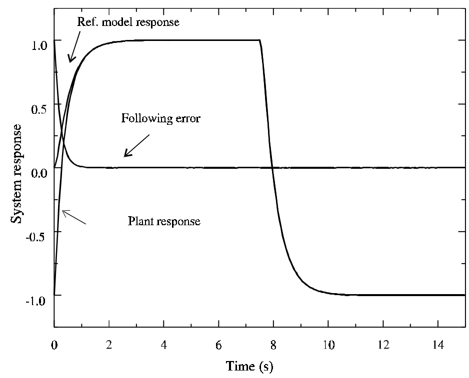

Fig. 4 Reference model and plant responses (second control algorithm)

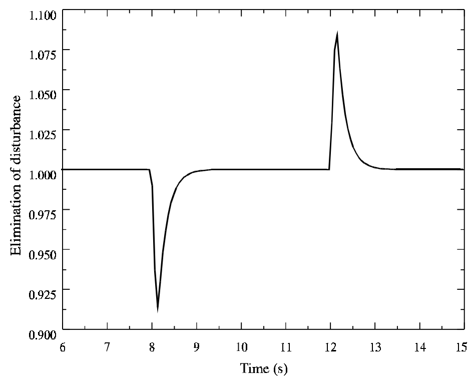

Fig. 5 Elimination of external disturbance (first control algorithm)

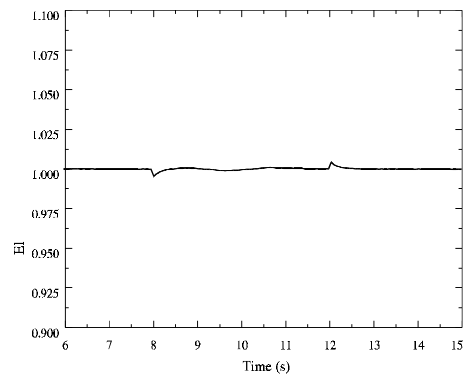

Fig. 6 Elimination of external disturbance (second control algorithm)

It is obvious that the second algorithm gives the faster system response and the better elimination of the external disturbance, unlike the first algorithm which ensures better system accuracy in the steady-state. This better accuracy of the system with the first algorithm is the consequence of the variable structure control nature, which is discontinuous function of the mentioned error and its derivatives.

5. CONCLUSION

In the paper, two variants of MRAC in which the signal adaptation is implemented by using the VSS with PI action, have been presented. The control signal of the first proposed algorithm is the discontinuous function of the error between the reference model and the plant outputs and its derivatives, while the second algorithm produces the control signal as discontinuous function of the plant output and its derivatives. The integral term in the control enables high system accuracy in the steady-state and provides better existence conditions of the sliding mode than it is case with the conventional VSC. The proposed approaches have as a consequence the synthesis of adaptive systems which are robust in relation to external disturbances and the parameter variation.

APPENDIX A

In order to prove the condition (9) we start from equation $\mathbf{f_0} = \mathbf{0}$ which gives:

$$\mathbf{b_m} - \mathbf{b_p^o} k_v = \mathbf{0} \tag{A.1}$$

If we multiply the equation (A.1) from the left side by $\mathbf{b_p^o}^{\mathrm{T}}$ and if we suppose that $\mathbf{b_p^o}^{\mathrm{T}} \mathbf{b_p^o} \neq 0$, the equation:

$$(\mathbf{b_p^o}^{\mathrm{T}} \mathbf{b_p^o})^{-1} \mathbf{b_p^o}^{\mathrm{T}} \mathbf{b_m} - k_v = 0 \tag{A.2}$$

can be obtained.
Finally, the equation (9) is obtained from (A.1) and (A.2).
The proof of the condition (10) starts from the equation:

$$\mathbf{A_m} - \mathbf{A_p^o} + \mathbf{b_p^o} \mathbf{k_p} = \mathbf{0} \tag{A.3}$$

which is obtained from $\mathbf{f_0} = \mathbf{0}$.
By implementing the similar procedure as it is given above, one can get:

$$(\mathbf{b_p^o}^{\mathrm{T}} \mathbf{b_p^o})^{-1} \mathbf{b_p^o}^{\mathrm{T}} (\mathbf{A_m} - \mathbf{A_p^o}) - \mathbf{k_p} = 0 \tag{A.4}$$

Then the condition (10) is proved by virtue of (A.3) and (A.4).

REFERENCES

1. Lj. Draganović: "Adaptive Control Systems", Sarajevo, "Svjetlost", 1982. (in Serbian)
2. V. I. Utkin: " Sliding Modes and Their Application in Variable Structure Systems", Mir Publishers, Moscow, 1978
3. L. Hsu et. al.: "Analysis and design of I/O based variable structure adaptive control", *IEEE Trans. Automatic Control*, Vol. 39, No. 1, pp. 4-22, January 1994.
4. G. Ambrosino G. Celentano and F. Garofalo: "Variable structure model reference adaptive control systems", *Int. J. Control*, Vol. 39, No. 6, pp. 1339-1349, 1984.
5. D. Antić, Č. Milosavljević and G. Đorđević: "Comparative analysis of the different methods for speed control of separately excited DC-motor", Facta Universitatis, Series: Mechanics, Automatic Control and

Robotics, Vol. 1, No. 3, pp. 373-387, 1993.
6. Č. Milosavljević, D. Antić, D. Mitić and G. Đorđević: "Comparative analysis of variable structure systems with proportional-plus-integral (PI) control", Facta Universitatis, Series: Electronics and Energetics, Vol. 1, pp. 1-9, 1995.
7. D. Antić: "Variable Structure Systems with Proportional-Plus-Integral Action", Ph. D. Dissertation, Faculty of Electronic Engineering, Niš, 1994.

ADAPTIVNI SISTEMI SA REFERENTNIM MODELOM PROMENLJIVE STRUKTURE SA PROPORCIONALNI-INTEGRALNIM DELOVANJEM

Darko Mitić

U ovom radu je predstavljena primena sistema promenljive strukture (SPS) sa proporcionalno-integralnim (PI) članom u sintezi adaptivnih sistema sa referentnim modelom (ASRM). Razmatrana su dva algoritma koji sintetišu signalnu adaptaciju i čije integralno dejstvo obezbeđuje minimalnu vrednost greške između izlaza referentnog modela i objekta. Dokaz stabilnosti ovih sistema je dobijen preko teorije SPS. Osobine predloženih ASRM-a su izvedene i prikazane korišćenjem digitalne simulacije.