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A NEW APPROACH TO THE EVALUATION OF FRICTION AND SEPARATION LOSSES IN DUCTS

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Abstract. *Extended Bernoulli equation and energy equation for a finite fluid volume are derived in the paper and applied for fluid flow in ducts. It is shown that both friction and separation losses can be uniquely expressed in terms of the dissipation function. Consequently, they can be exactly evaluated in all cases in which the dissipation function is known, like in laminar and turbulent flow in pipes. In complicated 3-D flows, usually present in most of separation losses, method presented accurately predicts the structure of losses and enables a relatively simple fitting with experiments, as it is shown in the paper on the example of flow through branched ducts.*

1. INSTEAD OF AN INTRODUCTION

Sir William Hawthorne in: Some unanswered questions in Fluid Mechanics, by L.M. Trefethen and R.L. Panton [1]: "What are the loss mechanisms in secondary flow? For example: What is the source of the losses when secondary flows are present, e.g. in the flow around a bend? What fraction of the secondary-flow losses arise from increased shear stresses in the main body of the fluid compared to the losses associated with changes in the wall boundary layers resulting from the secondary flow? It is surprising that after so many years of work by many people on secondary flows there is inadequate understanding of how the increased losses are actually caused".

Indeed, 3-D flow structure in ducts caused by the change of flow direction and/or cross-sectional area of the duct is usually so perplex that its evaluation is not only scarcely amenable to the application of some analytical methods, but also represents a true challenge even for most powerful computers of today. Hence, an engineer faced with the associated friction and separation losses in ducts is, as a rule, directed to utilize

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several empirical formulas existing in the literature, which express the influence of the losses in ducts through an overall effect - total pressure drop. Very often these formulas are not reliable enough, or at least they do not cover the desired range of interest of the governing parameters, so that their application may necessarily lead to serious errors in engineering calculations.

This paper does not pretend to provide the reader with full answers to the questions posed by Sir V. Hawthorne. It aims at throwing some more light on the nature of friction and separation losses in ducts only. The analysis performed relies upon full and exact form of the governing equations and their approximate integration over cross-sectional areas of the duct. It is shown that both friction and separation losses are closely related to the dissipation function and can be readily evaluated whenever this function is known, like for example in the case of fully developed laminar or turbulent flow in straight pipes. Applied to the flow in branched pipes the method proposed accurately predicts the corresponding losses and enables a relatively simple fitting to experimental data.

2. EXTENDED BERNOULLI EQUATION FOR A FINITE FLUID VOLUME

Extended Bernoulli equation, or the equation of conservation of mechanical energy is the key equation for studying friction and separation losses in ducts. In literature it is usually derived in the form for a stream filament and then intuitively applied for a stream tube, or for a duct with rigid and unpermeable walls, by introducing some conveniently defined average values of physical quantities over the cross-sectional areas of the duct/tube. Here, we will derive the extended Bernoulli equation for an arbitrary, finite volume of fluid, and for the flow in a duct as a special case, in a more rigorous way. Our starting point will be the momentum conservation equation written for an individual fluid particale in the form:

$$\rho \frac{D\vec{v}}{Dt} = \rho \vec{F} - grad\, p + Div\, \vec{\tau}_n , \tag{1}$$

where ρ, p, $\vec{v}$ and $\vec{F}$ are density, pressure, velocity vector and body force vector, respectively, D/Dt is the material derivative and $\vec{\tau}_n = \vec{\tau}_x \cos\alpha + \vec{\tau}_y \cos\beta + \vec{\tau}_z \cos\gamma$ is the viscosity stress vector (α, β and γ are angles between an arbitrarily oriented normal and the Cartesian coordinates x, y and z, respectively). Symbol *Div*:

$$Div\, \vec{\tau}_n = \frac{\partial \vec{\tau}_x}{\partial x} + \frac{\partial \vec{\tau}_y}{\partial y} + \frac{\partial \vec{\tau}_z}{\partial z} .$$

If the flow is turbulent $\vec{\tau}_n$ is the sum of viscous and turbulent stresses. Equation (1) can be multiplied by $\vec{v}$ in the form of a scalar product and integrated over a finite volume of fluid *V*. Some of the terms appearing at that can be converted into surface integrals over the area *A* surroundng *V* by using the definition of material derivative, Gauss formula and Reynolds transport theorem (s. [2]). At that, if the continuity equation is used in the form:

$$\frac{D\rho}{Dt} + \rho\, div\, \vec{v} = 0 ,$$

and if the flow is assumed steady and body forces conservative, the transformation formulas will read:

$$\int_V \rho \frac{D}{Dt}\left(\frac{v^2}{2}\right) dV = \frac{D}{Dt}\int_V \rho \frac{v^2}{2} dV = \int_A \rho \frac{v^2}{2}(\vec{v}\cdot\vec{n})\, dA\,,$$

$$\int_V \rho \vec{F}\cdot\vec{v} dV = \int_V \rho \frac{Du}{Dt} dV = \int_A \rho u(\vec{v}\cdot\vec{n})\, dA\,,$$

$$\int_A p(\vec{v}\cdot\vec{n}) dA = \int_V div(p\vec{v}) dV = \int_V \vec{v}\cdot grad\, p dV + \int_V p\rho \frac{D}{Dt}\left(\frac{1}{\rho}\right) dV\,,$$

$$\int_A \vec{\tau}_n\cdot\vec{v} dA = \int_V \left[\frac{\partial(\vec{\tau}_x\cdot\vec{v})}{\partial x} + ...\right] dV = \int_V (\vec{v}\cdot Div\,\vec{\tau}_n + \Phi)\, dV\,,$$

where $\vec{n}$ is the outer normal to A, u is the potential of conservative body forces, and:

$$\Phi = \vec{\tau}_x\cdot\frac{\partial\vec{v}}{\partial x} + \vec{\tau}_y\cdot\frac{\partial\vec{v}}{\partial y} + \vec{\tau}_z\cdot\frac{\partial\vec{v}}{\partial z} > 0 \tag{2}$$

is so-called dissipation function which is always, as well known [3], positive. Emerging equation has the following form:

$$\int_A \left[\left(\rho\frac{v^2}{2} + p - \rho u\right)\vec{v}\cdot\vec{n} - \vec{\tau}_n\cdot\vec{v}\right] dA + \int_A \left[\Phi - p\rho\frac{D}{Dt}\left(\frac{1}{\rho}\right)\right] dV = 0\,, \tag{3}$$

and represents the desired extended Bernoulli equation for a finite fluid volume of arbitrary form. It is noteworthy that this equation equally applies to both incompressible and compressible fluids.

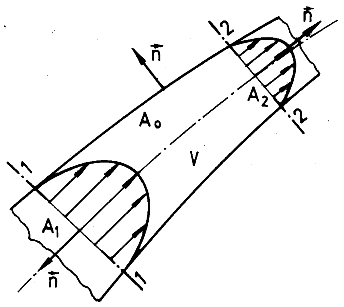

Fig. 1. Flow in a duct.

We will now apply this equation for the special case in which V is the volume contained in a duct between its two cross sections 1-1 and 2-2, and consequently A consists of A_1, A_2 and A_0, as in Fig. 1. At that, we will suppose that the flow in 1-1 and 2-2 attains the form of a fully developed laminar or turbulent flow corresponding to certain Reynolds number. As known, there will be only one velocity component - orthogonal to the cross section in this case, and pressure and density variations over the cross section will be negligible, so that some of the terms in (3) can be readily evaluated by introducing average values of different physical quantities, including the Coriolis coefficient α (s. for example [4]), according to the following scheme:

$$\int_A \rho\frac{v^2}{2}(\vec{v}\cdot\vec{n}) dA = \rho_2\alpha_2\frac{v_2^3}{2}A_2 - \rho_1\alpha_1\frac{v_1^3}{2}A_1\,,$$

$$\int_A p(\vec{v}\cdot\vec{n})dA = p_2 v_2 A_2 - p_1 v_1 A_1 \,,$$

$$\int_A \rho u(\vec{v}\cdot\vec{n})dA = \rho_2 u_2 v_2 A_2 - \rho_1 u_1 v_1 A_1 \,,$$

where all physical quantities with indices 1 or 2 refer to their average values in A_1 and A_2, respectively. As far as the term $\vec{\tau}_n \cdot \vec{v}$ is concerned it will disappear on A_0 in a duct due to non-slip boundary condition there (in a stream tube this term will have a non-zero value!). On A_1 and A_2 obviously only normal stresses contribute to its value. However, it is known that in fully developed laminar or turbulent flow these stresses are negligible with respect to the pressure, so that finally equation (3) for a duct attains the following form:

$$\dot{m}\left(\frac{1}{2}\alpha_1 v_1^2 + \frac{p_1}{\rho_1} - u_1\right) = \dot{m}\left(\frac{1}{2}\alpha_2 v_2^2 + \frac{p_2}{\rho_2} - u_2\right) + \int_V \left[\Phi - p\rho\frac{D}{Dt}\left(\frac{1}{\rho}\right)\right] dV \,, \tag{4}$$

where $\dot{m} = \rho_1 v_1 A_1 = \rho_2 v_2 A_2$ is the mass flow rate through the duct.

It will be instructive at this point to derive the corresponding form of the energy equation for the duct shown in Fig. 1 too, and to express the volume integral in (4) via the heat exchange with environment. First law of Thermodynamics for a fluid particle reads:

$$\rho\frac{De}{Dt} + p\rho\frac{D}{Dt}\frac{1}{\rho} = \Phi + \rho\dot{q} \,,$$

where e is internal energy and q is the rate of heat added per unit mass. If this equation is integrated over the volume of fluid in the duct between 1-1 and 2-2 exactly in the same way as in the previous case, we will obtain:

$$\dot{m}(e_2 - e_1) = \int_V \left[\Phi - p\rho\frac{D}{Dt}\left(\frac{1}{\rho}\right)\right] dV + \int_V \rho\dot{q}\, dV \,.$$

Combining this result with (4) one can obviously get:

$$\dot{m}\left(\frac{1}{2}\alpha_1 v_1^2 + h_1 - u_1\right) = \dot{m}\left(\frac{1}{2}\alpha_2 v_2^2 + h_2 - u_2\right) - \int_V \rho\dot{q}\, dV \,, \tag{5}$$

where $h = e + p/\rho$ is the enthalpy.

Extended Bernoulli and energy equations can be derived in exactly the same manner for flow in branched pipes too. Such a pipe in which direction of flow is from the cross section 1-1 toward the sections 2-2 and 3-3 is depicted in Fig. 2. Extended Bernoulli equation will have the following form:

$$\dot{m}_1\left(\frac{1}{2}\alpha_1 v_1^2 + \frac{p_1}{\rho_1} - u_1\right) = \dot{m}_2\left(\frac{1}{2}\alpha_2 v_2^2 + \frac{p_2}{\rho_2} - u_2\right) + \\ + \dot{m}_3\left(\frac{1}{2}\alpha_3 v_3^2 + \frac{p_3}{\rho_3} - u_3\right) + \int_V \left[\Phi - p\rho\frac{D}{Dt}\left(\frac{1}{\rho}\right)\right] dV \,,$$

while the energy equation reads:

$$\dot{m}_1\left(\frac{1}{2}\alpha_1 v_1^2 + h_1 - u_1\right) = \dot{m}_2\left(\frac{1}{2}\alpha_2 v_2^2 + h_2 - u_2\right) + \\ + \dot{m}_3\left(\frac{1}{2}\alpha_3 v_3^2 + h_3 - u_3\right) - \int_V \rho q \, dV$$

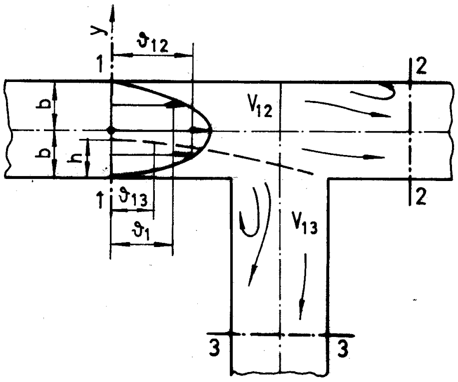

Fig. 2. Structure of flow in a branched duct.

In these equations $\dot{m}_1$, $\dot{m}_2$ and $\dot{m}_3$ are mass flow rates through the corresponding cross sections, and due to continuity equation: $\dot{m}_1 = \dot{m}_2 + \dot{m}_3$. For some other flow directions through the branch the corresponding forms of the above equations can be routinely written down.

In applications that follow we will be primarily interested in incompressible flow. Extended Bernoulli equation for a single duct and a branched one, respectively, for this flow will be:

$$p_{01} - \rho u_1 = p_{02} - \rho u_2 + \frac{1}{\dot{V}} \int_V \Phi \, dV \tag{6}$$

$$\dot{V}_1(p_{01} - \rho u_1) = \dot{V}_2(p_{02} - \rho u_2) + \dot{V}_3(p_{03} - \rho u_3) \int_V \Phi \, dV \, , \tag{7}$$

where $\dot{V}$ and $\dot{V}$, $i = 1,2,3$ are the corresponding volume flow rates, and $p_0 = p + \alpha \rho v^2/2$ is a modified definition of the total pressure. Last terms on the right-hand side of these equations represent what is usually referred to as the friction or the separation loss in the duct. Obviously, the loss is immediately associated with the volume integral of the dissipation function and can be readily evaluated whenever this function is known. Even if it is not known, like in complicated 3-D flows, the revealed form of the extended Bernoulli equations (6) and (7) anables a relatively simple fitting to experimental data, as will be shown later on the example of flow in branched pipes.

3. Fully developed laminar and turbulentflow in straight pipes

It is well known [3] that the velocity profile in a fully developed laminar flow in a pipe of circular cross section has the following form of a parabola:

$$u = 2v\frac{y}{R}\left(2 - \frac{y}{R}\right),$$

where v is the average velocity, R is the radius of the pipe, and y is the distance measured from the pipe wall. Also, the dissipation function reads:

$$\Phi = \mu\left(\frac{\partial u}{\partial y}\right)^2,$$

where μ is the viscosity of the liquid. The term representing the loss due to friction in equation (6) can now be readily evaluated for a volume between two cross sections on the distance l along the pipe:

$$\frac{1}{\dot{V}}\int_V \Phi\, dV = \lambda\frac{l}{2R}\frac{l}{2}\rho v^2,$$

where $\lambda = 64/\mathrm{Re}$ is the friction coefficient and $\mathrm{Re} = 2Rv/\mu$ is the Reynolds number. The obtained result represents the well known Darcy`s formula which is usually derived in a quite diferent way in literature.

Velocity profile in a fully developed turbulent flow in pipes can be approximated, as well known, by the following expression:

$$\frac{u}{u_*} = A\ln\frac{y}{y_0},$$

in which $u_* = (\tau_w/\rho)^{1/2}$ is so-called dynamic velocity (τ_w is shear stress on the wall), $A = 2.5$ is a universal constant and the value of y_0 depends on whether the pipe behaves as a smooth or a rough one. For a hydraulicly smooth pipe:

$$\frac{\rho u_* y_0}{\mu} = e^{-B/A}, \tag{8}$$

where $B = 5.5$ is another universal constant, while for a hydraulicly fully-rough pipe:

$$\frac{y}{\delta} = e^{-B/A}, \tag{9}$$

where δ is the absolute roughness and $B = 8.5$. If, for convenience, we employ now the following expression for the dissipation function:

$$\Phi = \tau_w\left(1 - \frac{y}{R}\right)\frac{\partial u}{\partial y},$$

we will straightforwardly get:

$$\frac{1}{\dot{V}}\int_V \Phi\, dV = \frac{2lA}{Rv}\tau_w\left(\frac{\tau_w}{\rho}\right)^{1/2}\left[\ln\frac{R}{y_0}-\frac{3}{2}+O\left(\frac{y_0}{R}\right)\right]. \tag{10}$$

On the other hand, an over-all equation of balance between pressure, gravitation and friction forces for the fluid between the stations 1-1 and 2-2 in a straight pipe reads:

$$p_1 - p_2 + g\rho(z_1 - z_2) = 2l\,\tau_w/R\,,$$

where z_1 and z_2 are the heights of centers of the corresponding cross sections above an arbitrarily chosen level. If this is now combined with (6) and (10), in which the terms $O(y_0/R)$ are neglected as small, we will have:

$$\frac{A}{v}\left(\frac{\tau_w}{\rho}\right)^{1/2}\left(\ln\frac{R}{y_0}-\frac{3}{2}\right)=1\,. \tag{11}$$

Since the definition of the friction coefficient λ is:

$$\tau_w = \frac{\lambda}{8}\rho v^2\,,$$

it is now easy to obtain λ from (11) in both cases considered by taking into account (8) and (10). For smooth pipes λ depends on the Reynolds number only in an implicite fashion:

$$\lambda^{-1/2} = 2\log(\mathrm{Re}\,\lambda^{1/2}) - 0.8\,,$$

while for fully-rough pipes λ depends explicitely on the relative roughness $k = \delta/2R$ only:

$$\lambda^{-1/2} = 2\log(3.7/k)$$

Both formulas are well known and widely used in practice, but derived in literature in a quite different way.

4. BRANCHED DUCTS

As inferred from Fig. 2 the flow in branched ducts consists of highly complicated 3-D flow patterns and one can not expect to have a closed form expression for the dissiapation function. Thus, experiments are inevitably necessary for the description of energy losses in branched ducts. In spite of that the method presented here predicts accurately the structure of these losses and enables a relatively simple fitting to experimental data.

For simplicity we will suppose that the tee-junction in Fig. 2 consists of plane walls. In addition to the extended Bernoulli equation (7) encompassing all three cross sections, we can devide the section 1-1 into the parts supplying sections 2-2 and 3-3, and state the corresponding Bernoulli equations for that part of 1-1 (above h in Fig. 2) that supplies 2-2 with the fluid, and for that part of 1-1 (below h) that supplies 3-3 with the fluid. These two equations have the form of the extended Bernoulli equation (6) for a single stream tube. To write these equations it is necessary to evaluate first volume flow rates $\dot{V}_2$ and

$\dot{V}_3$, corresponding average velocities v_{12} and v_{13} and coriolis coefficients α_{12} and v_{13} for given ratio *h*/*b* (s. Fig. 2). This is done in Appendix by using an empirical velocity profile for the turbulent flow in section 1-1 of the form:

$$u = u_m \left(1 - \frac{|y|}{b}\right)^m ,$$

where u_m is maximum velocity at the axis, and m is exponent that weakly depends on the Reynolds number. The equations with neglected body forces read:

$$\dot{V}_2 p_{01}^{(2)} = \dot{V}_2 p_{02} + \int_{V_{12}} \Phi\, dV$$
$$\dot{V}_3 p_{01}^{(3)} = \dot{V}_3 p_{03} + \int_{V_{13}} \Phi\, dV$$

where $p_{01}^{(2)}$ and $p_{01}^{(3)}$ are modified total pressures corresponding to the respective parts of the section 1-1.

$$p_{01}^{(2)} = p_1 + \frac{1}{2}\alpha_{12}\rho v_{12}^2 , \quad p_{01}^{(3)} = p_1 + \frac{1}{2}\alpha_{13}\rho v_{13}^2$$

If, instead, total pressure p_{01}, is introduced, we may write the previous equations into the following form:

$$\dot{V}_2 (p_{01} - p_{02}) = \int_{V_{12}} \Phi dV + \frac{1}{2}\alpha_1 \rho v_1^2 \dot{V}_2 \left(1 - \frac{\alpha_{12}}{\alpha_1}\frac{v_{12}^2}{v_1^2}\right)$$
$$\dot{V}_3 (p_{01} - p_{03}) = \int_{V_{12}} \Phi dV + \frac{1}{2}\alpha_1 \rho v_1^2 \dot{V}_3 \left(1 - \frac{\alpha_{13}}{\alpha_1}\frac{v_{13}^2}{v_1^2}\right) \quad (12)$$

and finally, if continuity equation $\dot{V}_1 = \dot{V}_2 + \dot{V}_3$ is employed, and the following notations are used:

$$q = \dot{V}_3 / \dot{V}_1 , \quad K_{12} = \frac{2}{\alpha_1 \rho v_1^2 \dot{V}_1} \int_{V_{12}} \Phi dV , \quad K_{13} = \frac{2}{\alpha_1 \rho v_1^2 \dot{V}_1} \int_{V_{13}} \Phi dV ,$$

in the form:

$$\frac{2(p_{01} - p_{02})}{\alpha_1 \rho v_1^2} \equiv \Delta_{12} = \begin{cases} \frac{K_{12}}{1-q} + 1 - \frac{1 - 0.5(2q)^{3m+1/m+1}}{1-q}, & q \le 0.5 \\ \frac{K_{12}}{1-q} + 1 - [2(1-q)]^{2m/m+1}, & q \ge 0.5 \end{cases} \quad (13)$$

$$\frac{2((p_{01} - p_{03})}{\alpha_1 \rho v_1^2} \equiv \Delta_{13} = \begin{cases} \frac{K_{13}}{q} + 1 - (2q)^{2m/m+1}, & q \le 0.5 \\ \frac{K_{13}}{q} + 1 - \frac{1 - 0.5[2(1-q)]^{3m+1/m+1}}{q}, & q \ge 0.5 \end{cases} \quad (14)$$

At that we obviously use formulas derived in Appendix to express ratios of both average velocities and Coriolis coefficients appearing in (12) via q. Physical meaning of the quantities K_{12} and K_{13} is obvious: K_{12} is dimensionless total pressure drop between stations 1-1 and 2-2 when $q = 0$, while K_{13} is dimensionless total pressure drop between stations 1-1 and 3-3 for $q = 1$. If they were constant the fitting to numerous experiments would be elementary. However, the analysis reveals that they are not constant but depend on q. We performed the fitting to the experiments presented in [5] for relatively high values of the Reynolds number for which $m \approx 0.1$, and obtained the following formulas for K_{12} and K_{13} that we recommend here:

$$K_{12} = (1-q)[0.144 - 0.113(1-q)^{0.606}]$$
$$K_{13} = q(0.806 + 0.462\,q^{2.845}).$$

In Fig. 3 we present the dependence of Δ_{12} and Δ_{13} on q obtained both by using (13), (14) and (15), and by experiments in [5]. Also, since the term $\int_V \Phi\, dV$ appearing in (7) is simply the sum of the corresponding terms present in (12), $K_{12} + K_{13}$ represents a nondimensional over-all energy loss in the tee-junction. This sum together with K_{12} is presented in Fig. 4. Obviously, and as expected, this loss is a monotonously increasing function of q that attains the minimum for $q = 0$ and the maximum for $q = 1$.

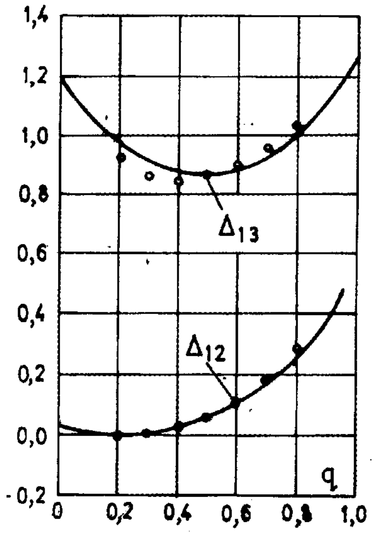

Fig. 3. Losses of mechanical energy in the tee-junction shown in Fig. 2.

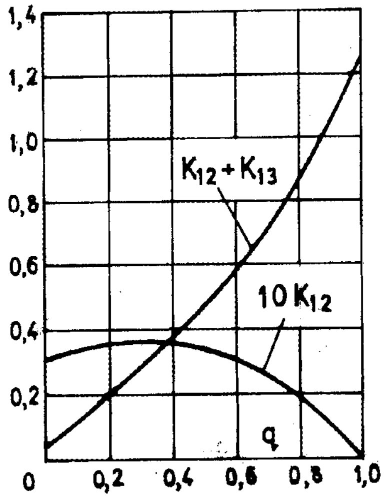

Fig. 4. Over-all losses in branched ducts.

5. Conclusions

We demonstrate in this paper how extended Bernoulli equation and energy equation can be rigorously derived for a finite fluid volume. Their application in the classical problem of flow through ducts shows that so-called friction and separation losses can be in an unique way expressed in terms of the dissipation function. Hence, they can be readily evaluated whenever this function is known, and we confirm this fact on the example of laminar and turbulent (smooth and fully-rough) flow in pipes by deriving precisely the friction law in both cases. When the method is applied to more complicated, 3-D flows for which the dissipation function cannot be obtained analytically, it still offers some insight into the structure of losses and enables a relatively easy fitting with experiments. We have shown this on the example of flow through branched ducts. It is worthwhile mentioning at this point, however, that the fitting with experiments presented in [5] have been performed for tee-junctions with rectangular mutually equal cross sections and sharp edges. Numerous experiments show however that the loss very considerably depends on all of above mentioned factors, so that actually for each type of a branched duct a particular dependence of K_{12} and K_{13} on q should be found by fitting. Independently of this inconvenience we hope to have contributed in this paper at least partially to the problem stated by Sir W. Hawthorne, cited in the Introduction.

APPENDIX

If the velocity profile: $u = u_m(1-|y|/b)^m$ is adopted to represent the turbulent flow in the cross section 1-1 (Fig. 2), the volume flow rate and the average velocity in 1-1 are, respectively:

$$\dot{V}_1 = \frac{2bu_m}{m+1}, \quad v_1 = \frac{\dot{V}_1}{2b} = \frac{u_m}{m+1}.$$

Also, the corresponding quantities referring to the parts of 1-1 above h and below h (s. Fig. 2) read, respectively:

$$\dot{V}_2 = \begin{cases} \dfrac{bu_m}{m+1}\left[2-\left(\dfrac{h}{b}\right)^{m+1}\right], & h \le b \\ \dfrac{bu_m}{m+1}\left[\left(2-\dfrac{h}{b}\right)^{m+1}\right], & h \ge b \end{cases}$$

$$v_{12} = \begin{cases} \dfrac{u_m}{2(m+1)}\dfrac{1}{1-\dfrac{1}{2}}\left[2-\left(\dfrac{h}{b}\right)^{m+1}\right], & h \le b \\ \dfrac{u_m}{2(m+1)}\dfrac{1}{1-\dfrac{1}{2}\dfrac{h}{b}}\left[\left(2-\dfrac{h}{b}\right)^{m+1}\right], & h \ge b \end{cases}$$

$$\dot{V}_3 = \begin{cases} \dfrac{bu_m}{m+1}\left(\dfrac{h}{b}\right)^{m+1}, & h \le b \\ \dfrac{2bu_m}{m+1}\left[1-\dfrac{1}{2}\left(2-\dfrac{h}{b}\right)^{m+1}\right], & h \ge b \end{cases}$$

$$v_{13}=\begin{cases}\dfrac{u_m}{m+1}\dfrac{1}{1-\dfrac{1}{2}}\left(\dfrac{h}{b}\right)^m, h\le b\\ \dfrac{2u_m}{m+1}\dfrac{b}{2}\left[1-\dfrac{1}{2}\left(2-\dfrac{h}{b}\right)^{m+1}\right], h\ge b\end{cases}.$$

Using the definition of the Coriolis coefficient (s. for example .4.), its respective values can be also readily evaluated:

$$\alpha_1=\frac{(m+1)^3}{3m+1}$$

$$\alpha_{12}=\begin{cases}\dfrac{(m+1)^3}{3m+1}\dfrac{\left(1-\dfrac{1}{2}\dfrac{h}{b}\right)^2\left[1-\dfrac{1}{2}\left(\dfrac{h}{b}\right)^{3m+1}\right]}{\left[1-\dfrac{1}{2}\left(\dfrac{h}{b}\right)^{m+1}\right]^3}, h\le b\\ \dfrac{(m+1)^3}{3m+1}, h\ge b\end{cases}$$

$$\alpha_{13}=\begin{cases}\dfrac{(m+1)^3}{3m+1}, h\le b\\ \dfrac{(m+1)^3}{4(3m+1)}\dfrac{h^2}{b^2}\dfrac{1-\dfrac{1}{2}\left(1-\dfrac{h}{b}\right)^{3m+1}}{\left[1-\dfrac{1}{2}\left(2-\dfrac{h}{b}\right)^{m+1}\right]^3}, h\ge b\end{cases}$$

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JEDAN NOVI PRISTUP IZRAČUNAVANJU GUBITAKA ENERGIJE U CEVIMA

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U ovom radu su izvedene proširena Bernulijeva jednačina i jednačina energije za konačnu zapreminu fluida i primenjene su za strujanje fluida u cevima. Pokazano je da su gubici kako usled trenja tako i usled odvajanja mogu se jedinstveno izraziti u obliku funkcije disipacije. Shodno tome, oni se mogu tačno oceniti u svim slučajevima u kojima je funkcija disipacija poznata, slično laminarnim i turbulentnim strujanjima u cevima. U složenim trodimenzionalnim strujanjima, koji se javljaju kod većine gubitaka usled odvajanja, metod koji je predstavljen, tačno predviđa strukturu gubitaka i omogućava relativno jednostavno poklapanje sa eksperimentalnim rezultatima, kako je prikazano u radu na primeru strujanja kroz cevi koje se granaju.