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UNIVERSAL SOLUTIONS OF THE INCOMPRESSIBLE LAMINAR TEMPERATURE BOUNDARY LAYER ON A ROTATING SURFACE

UDC: 532.51; 536.22

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Abstract. *In the paper the governing equations of the incompressible laminar temperature boundary layer on the rotating surface are universalised using the improved method of Loitskianskii. The influence of rotation in the treated problem is separated by developing and linearising of all universal functions on rotational parameter* Ω*. Two different boundary conditions are concerned - adiabatic surface and constant wall temperature case. Universal solutions in one-parametric approximation are obtained by numerical integration. All calculated universal functions are presented graphically and analysed, so that the influence of rotation can be distincted.*

1. INTRODUCTION

Development of modern computational technology in recent years offers a solution of numerous problems in fluid mechanics. By direct numeric integration of the governing equations, solutions of many certain problems can be obtained easily. The problem with the solutions obtained in this manner is their particularity-they are connected to particular problem and even slight variance of governing parameters demands a completely new numerical treatment of the problem concerned. On the other side, more theoretical approach to the problem concerned leads to the solution which covers, at least, group or class of the particular cases. In the laminar boundary layer theory, method of Lotskianskii [1], improved by Saljnikov [2] and his school, seems to be most recent and very promising approach for theoretical treatment of the problem concerned.

Problem of the dynamic boundary layer on the rotating surface, Fig. 1, was very interesting due to its practical application in turbomachines. Papers of the Jungclaus [3], [6], Li [4], Glauert [5] and Saljnikov, Djordjević [7], testifying great interest for the

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treated problem, are the results of discussion on the problem of linearisation of the influence of rotation. Using improved univesalisation method of Lotskianskii [1], [2], solutions for the dynamic laminar boundary layer on the rotating surface were obtained and applied to different practical problems in Saljnikov, Pavlovi}[8].

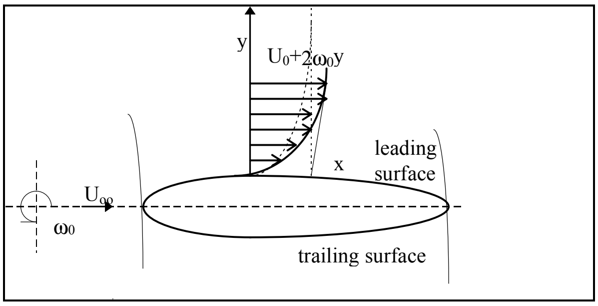

Fig. 1. Boundary layer on the rotating surface

Concerning the problem of cooling of the centrifugal impellers, temperature boundary layer on the rotating surface seems to be of certain practical interest. In this case, different classes of the problem are to be recognised, due to different boundary conditions. In this paper two of them, adiabatic and isothermal ones, are concerned as most significant for practical application.

2. Universal equations of the dynamic boundary layer

In case were influence of the rotation is present in the stream field, governing equations of the dynamic boundary layer can be derived in the following form:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}-\nu\frac{\partial^2 T}{\partial y^2}=\left(U\frac{\partial U}{\partial x}+V\frac{\partial U}{\partial y}\right)_{y\to\infty} . \tag{1}$$

with corresponding boundary conditions:

$$y=0:\ \ u=v=0\,;\ \ y\to\infty:\ \ u=U_0\pm 2\omega_0 y\,. \tag{2}$$

Treating the problem of the flow around the rotating surface, it is assumed that the potential flow velocity can be defined as:

$$U(x,y,\omega_0)=U_0(x,0)\pm 2\omega_0 y\,;\qquad V=V(x,y)\,.$$

In this way, this usual assumption reduces the influence of the rotation to the shear flow model.

As usual for the incompressible flow, system (1) can be transformed introducing the stream function $\psi(x,y,\omega_0)$. System (1) can be further universalised transforming the normal co-ordinate y in dimensionless form η, as well as the stream function $\psi(x,y,\omega_0)$ in $\Phi(x,\eta,\omega_0)$, and introducing new rotational parameter $\Omega(x)$:

$$\eta = \frac{U_0^{b/2}}{S_v} y \, ; \quad \Phi(x,\eta,\Omega) = \frac{U_0^{b/2-1}}{S_v} \Psi(x,y,\omega_0) \, ; \quad \Omega = \frac{2\omega_0 S_v}{U_0^{b/2+1}} \, . \tag{3}$$

In this transformation function S_v and constants a and b are used:

$$S_v = S_v(x) = \sqrt{a\nu \int_0^x U_0^{b-1} dx}; \quad a = 0{,}4408 \quad b = 5{,}714 \, . \tag{4}$$

It is important to say that this type of transformation $\boldsymbol{y}$ of normal co-ordinate and stream function ψ, proposed by Saljnikov [2], is very convenient for application of universal function Φ in practice.

More detailed analysis of the influence of rotation, performed in [8], based on the order of magnitude of the parameter Ω, proves that the universal stream function Φ can be linearised:

$$\Phi(x,\eta,\Omega) = \Phi_0(x,\eta) + \Omega\Phi_1(x,\eta) + ... \cong \Phi_0(x,\eta) + \Omega\Phi_1(x,\eta) \, . \tag{5}$$

In this way, the influence of the rotation can be separated.

The basic transformation of the Lotskiansii's method is performed by introducing the set of parameters of the form instead the longitudinal co-ordinate:

$$f_k = U_0^{k-1} \frac{d^k U_0}{dx^k} \left(\frac{\delta_0^{**2}}{\nu} \right) k = 1,2,..\infty \, , \tag{6}$$

so that derivatives according to $\boldsymbol{x}$ can be transformed in the form:

$$\frac{\partial}{\partial x} = \frac{U_0'}{U_0 f_1} \sum_{k=1}^{\infty} \theta_k \frac{\partial}{\partial f_k}; \quad \theta_k = [k(f_1 + F) - f_1] f_k + f_{k+1} \, . \tag{7}$$

In this transformation a new universal functions are used:

$$F = 2[\zeta - (2 + H) f_1]; \quad H = \frac{A_0}{B_0} \, ; \quad \zeta = B_0 (\Phi_{0\eta\eta})_{\eta=0} \, . \tag{8}$$

Dimensionless displacement and momentum thickness are defined as:

$$A_0 = \int_0^{\infty} (1 - \Phi_{0\eta}) d\eta = \frac{U_0^{b/2}}{S_v} \delta_0^* \quad B_0 = \int_0^{\infty} \Phi_{0\eta} (1 - \Phi_{0\eta}) d\eta = \frac{U_0^{b/2}}{S_v} \delta_0^{**} \, . \tag{9}$$

Using the exposed procedure, governing equations of the dynamic laminar boundary are transformed in the universal form:

$$\Phi_{0\eta\eta\eta} + \frac{1}{2B_0^2} [aB_0^2 + (2-b) f_1] \Phi_0 \Phi_{0\eta\eta} + \frac{f_1}{B_0^2} (1 - \Phi_{0\eta}^2) = \frac{1}{B_0^2} \sum_{k=1}^{\infty} \theta_k (\Phi_{0\eta} \Phi_{0\eta f_k} - \Phi_{0\eta\eta} \Phi_{0 f_k}) \tag{10}$$

$$\Phi_{1\eta\eta\eta} + \frac{1}{2B_0^2}[aB_0^2 + (2-b)f_1](\Phi_0\Phi_{1\eta\eta} - \Phi_{0\eta}\Phi_{1\eta}) + \frac{1}{B_0^2}(aB_0^2 - bf_1)\Phi_{0\eta\eta}\Phi_1 +$$

$$+\frac{1}{2B_0^2}[aB_0^2 + (2-b)f_1](\eta - \Phi_0)_{\eta\to\infty} = \tag{11}$$

$$= \frac{1}{B_0^2}\sum_{k=1}^{\infty}\theta_k(\Phi_{0\eta f_k}\Phi_{1\eta} + \Phi_{0\eta}\Phi_{1\eta f_k} - \Phi_{0\eta\eta}\Phi_{1f_k} - \Phi_{0f_k}\Phi_{1\eta\eta} + (\Phi_{0f_k})_{\eta\to\infty})$$

Corresponding boundary conditions are:

$$\eta = 0 : \Phi_0 = \Phi_{0\eta} = \Phi_1 = \Phi_{1\eta} = 0; \quad \eta \to \infty : \Phi_{0\eta} \to 1; \quad \Phi_{1\eta} \to \eta \ . \tag{12}$$

First equation, for Φ_0, defines the basic solution for the velocity field, and the second one, for Φ_1, defines together with Ω the influence of rotation. The system is solved, as it is usual for this class of problems, in one-paramertric approximation ($f_1 \neq 1, f_2 = ... = f_k = 0$) in [8].

3. UNIVERSAL EQUATIONS OF THE TEMPERATURE BOUNDARY LAYER

Heat transfer in the laminar boundary layer is defined by the energy equation in the form:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\nu}{\Pr}\frac{\partial^2 T}{\partial y^2} + \frac{\nu}{c}\left(\frac{\partial u}{\partial y}\right)^2 . \tag{13}$$

Influence of rotation in this governing equation is indirect, through the velocity components u and v. Heat transfer process in the layer is directed by the boundary conditions. In this paper, two main types of them are presented:

- adiabatic boundary conditions:

$$y = 0 : \frac{\partial T}{\partial y} = 0 ; \quad y \to \infty : T \to 1; \ T\infty , \tag{14}$$

- isothermal boundary conditions:

$$y = 0 : T = T_w ; \quad y \to \infty : T \to 1; \quad T_\infty \ . \tag{15}$$

Different form of the boundary conditions determines corresponding form of the temperature function.

3.1 Universal equations for the temperature boundary layer for adiabatic boundary conditions

In case of the thermally isolated, adiabatic surface, temperature in the boundary layer field can be defined as:

$$T(x,y,\Pr,\omega_0) = T_\infty + \frac{U_0^2}{2c}K(x,\eta,\Pr,\Omega) \cong T_\infty + \frac{U_0^2}{2c}\left[K_0(x,\eta,\Pr,) + \Omega K_1(x,\eta,\Pr,)\right]. \tag{16}$$

It is evident from (16) that there is no surface influence on the temperature field, which is formed due to the friction in the boundary layer.

Proceeding the exposed Lotskianskii's transformation, the same as for dynamic boundary layer, a system of differential equations is obtained:

$$\frac{1}{\Pr}K_{0\eta\eta}+\frac{1}{2B_0^2}[aB_0^2+(2-b)f_1]\Phi_0K_{0\eta}-2\frac{f_1}{B_0^2}\Phi_{0\eta}K_0+2\Phi_{0\eta\eta}^2=\frac{1}{B_0^2}\sum_{k=1}^{\infty}\theta_k(\Phi_{0\eta}K_{0f_k}-\Phi_{0fk}K_{0\eta}) \tag{17}$$

$$\frac{1}{\Pr}K_{1\eta\eta}+\frac{1}{2B_0^2}[aB_0^2+(2-b)f_1](\Phi_0K_{1\eta}-\Phi_{0\eta}K_1)+\frac{1}{B_0^2}(aB_0^2-bf_1)\Phi_1K_{0\eta}-2\frac{f_1}{B_0^2}\Phi_{1\eta}K_0+4\Phi_{0\eta\eta}\Phi_{1\eta\eta}=\frac{1}{B_0^2}\sum_{k=1}^{\infty}\theta_k(\Phi_{0\eta}K_{1f_k}+\Phi_{1\eta}K_{0f_k}-\Phi_{0f_k}K_{1\eta}-\Phi_{1f_k}K_{0\eta}) \tag{18}$$

with corresponding boundary conditions:

$$\eta=0: K_{0\eta}=K_{1\eta}\,;\quad \eta\to\infty: K_0=K_1=0\,. \tag{19}$$

Table 1. Values of the universal functions f_1/B^2, K_0, K_1, $P_{0h,h=0}$, $P_{1h,h=0}$, $R_{0h,h=0}$, $R_{1h,h=0}$.

f_1	f_1/B^2	K_0	K_1	$P_{0h,h=0}$	$P_{1h,h=0}$	$R_{0h,h=0}$	$R_{1h,h=0}$
0.08	0.074539	0.817390	7.821811	0.20102*	2.12854*	-0.33973*	-0.16996*
0.07	0.067019	0.820111	7.175391	0.199364	2.159058	-0.373033	-0.175706
0.06	0.058380	0.822383	6.958377	0.195463	2.218936	-0.431602	0.183550
0.05	0.049168	0.824406	6.828812	0.191116	2.280160	-0.490140	-0.188715
0.04	0.396150	0.826246	6.748304	0.186501	2.344898	-0.553401	-0.192232
0.03	0.029852	0.827937	6.700626	0.181698	2.415065	-0.624457	-0.194558
0.02	0.019960	0.829501	6.677598	0.176741	2.492830	-0.706456	-0.195934
0.01	0.009995	0.830951	6.674908	0.171639	2.581008	-0.803351	-0.196494
0.00	0.000000	0.832306	6.689703	0.166385	2.683479	-0.920745	-0.196305
-0.01	-0.009998	0.833749	6.717603	0.161012	2.802922	-1.065005	-0.195316
-0.02	-0.019974	0.835079	6.761048	0.155417	2.949616	-1.249412	-0.193597
-0.03	-0.029913	0.836357	6.820701	0.149567	3.137368	-1.494156	-0.191096
-0.04	-0.039800	0.837592	6.900826	0.143365	3.394037	-1.838177	0.187687
-0.05	-0.049630	0.838798	7.006824	0.136655	3.772642	-2.358476	-0.183140
-0.06	-0.059408	0.839992	7.151063	0.129143	4.406331	-3.246088	-0.177016
-0.07	-0.069156	0.841197	7.365977	0.120129	5.754858	-5.158254	-0.168265
-0.08	-0.078745	0.842293	7.887519	0.106609	11.723161	-13.61884	-0.152583

* Values for f=0.075, f_1/B^2=0.070982

The equations obtained, are not completely universal, since they depend on Prantl number for the treated fluid. Basic temperature field in the boundary layer is defined by the first equation, while the second one determines, together with parameter Ω, the influence of rotation.

The system (17-19) is solved in one-parametric approximation, ($f_1 \neq 1, f_2 = ... = f_k = 0$), as for dynamic boundary layer, for $Pr = 1$, and for $Pr = 0.72$ (for air). Discretisation of the governing equations is performed using finite difference method, implicit scheme. The solutions for Φ_0 and Φ_1 for the dynamic boundary layer, obtained in [8], were used. Numerical integration is performed for ($0 \leq \eta \leq 0$) with step $\Delta\eta = 0.1$ and ($-0.08 \leq f_1 \leq 0.08$) and step $\Delta f_1 = 0.05$. Solutions are given in the Tab. 1 and on the Fig. 2 and 3.

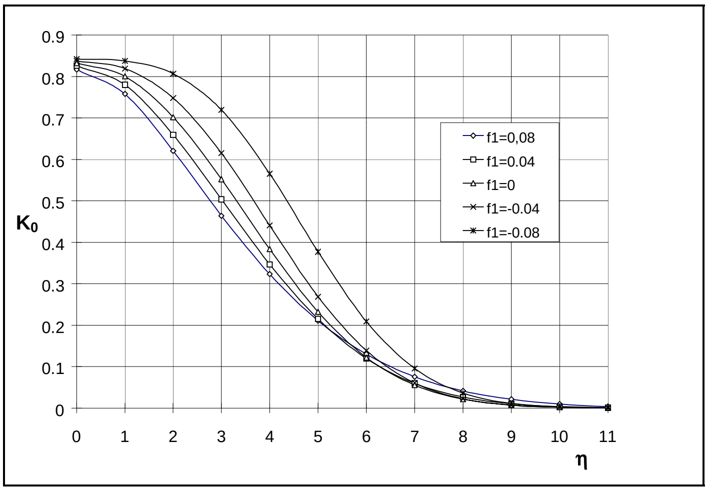

Figure 2. Profiles of universal function K_0, for different values of parameter f_1

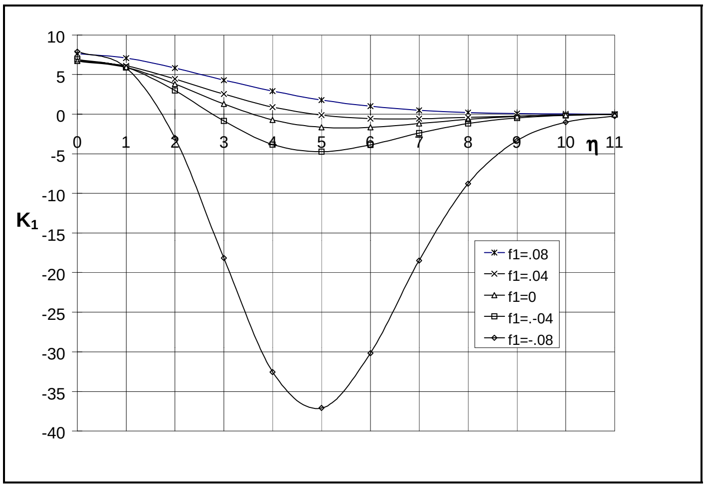

Figure 3. Profiles of universal function K_1, for different values of parameter f_1

Analysing the results obtained, for the basic solution - function K_0, the influence of the friction in the boundary layer is obvious. Temperature in the boundary layer rises from the temperature in the outer flow to the eigen-value on the wall. Influence of the function K_1, defining the influence of rotation, varies with the boundary layer development. Near the stagnation point, function is completely positive, which means that rotation contributes to the increase of temperature. Approaching the separation point, profiles become negative in the outer part, which means that rotation provokes cooling of the outer parts of the boundary layer.

3.2 Universal equations for the temperature boundary layer for isothermal boundary conditions

When the surface temperature is constant, i.e. in the isothermal case, temperature in the boundary layer field is defined by:

$$\begin{aligned}T(x,y,\mathrm{Pr},\omega_0) &= T_\infty + \frac{U_0^2}{2c}P(x,\eta,\mathrm{Pr},\Omega) + (T_w - T_\infty)R(x,\eta,\mathrm{Pr},\Omega) \cong \\ &\cong T_\infty + \frac{U_0^2}{2c}[P_0(x,\eta,\mathrm{Pr},) + \Omega P_1(x,\eta,\mathrm{Pr},)] + (T_w - T_\infty)[R_0(x,\eta,\mathrm{Pr},) + \Omega R_1(x,\eta,\mathrm{Pr},)]\end{aligned} \quad .(20)$$

In this case, the influence of friction, defined by the functions P_0, and P_1, is superposed with the influence of constant wall temperature, defined by the functions R_0 and R_1.

The same procedure of universalisation as for the previous case leads to the system of differential equations:

$$\begin{aligned}&\frac{1}{\mathrm{Pr}}P_{0\eta\eta} + \frac{1}{2B_0^2}[aB_0^2 + (2-b)f_1]\Phi_0 P_{0\eta} - 2\frac{f_1}{B_0^2}\Phi_{0\eta}P_0 + 2\Phi_{0\eta\eta}^2 = \\ &= \frac{1}{B_0^2}\sum_{k=1}^{\infty}\theta_k(\Phi_{0\eta}P_{0f_k} - \Phi_{0fk}P_{0\eta}),\end{aligned} \quad (21)$$

$$\begin{aligned}&\frac{1}{\mathrm{Pr}}P_{1\eta\eta} + \frac{1}{2B_0^2}[aB_0^2 + (2-b)f_1](\Phi_0 P_{1\eta} - \Phi_{0\eta}P_1) + \frac{1}{B_0^2}(aB_0^2 - bf_1)\Phi_1 P_{0\eta} - \\ &- 2\frac{f_1}{B_0^2}\Phi_{1\eta}P_0 + 4\Phi_{0\eta\eta}\Phi_{1\eta\eta} = \frac{1}{B_0^2}\sum_{k=1}^{\infty}\theta_k(\Phi_{0\eta}P_{1f_k} + \Phi_{1\eta}P_{0f_k} - \Phi_{0f_k}P_{1\eta} - \Phi_{1f_k}P_{0\eta}),\end{aligned} \quad (22)$$

$$\frac{1}{\mathrm{Pr}}R_{0\eta\eta} + \frac{1}{2B_0^2}[aB_0^2 + (2-b)f_1]\Phi_0 R_{0\eta} = \frac{1}{B_0^2}\sum_{k=1}^{\infty}\theta_k(\Phi_{0\eta}R_{0f_k} - \Phi_{0fk}R_{0\eta}), \quad (23)$$

$$\begin{aligned}&\frac{1}{\mathrm{Pr}}R_{1\eta\eta} + \frac{1}{2B_0^2}[aB_0^2 + (2-b)f_1]\Phi_0 R_{1\eta} - \frac{1}{2B_0^2}[aB_0^2 + (2+b)f_1]\Phi_{0\eta}R_1 + \\ &+ \frac{1}{B_0^2}(aB_0^2 - bf_1)\Phi_1 R_{0\eta} = \frac{1}{B_0^2}\sum_{k=1}^{\infty}\theta_k(\Phi_{0\eta}R_{1f_k} + \Phi_{1\eta}R_{0f_k} - \Phi_{0f_k}R_{1\eta} - \Phi_{1f_k}R_{0\eta}),\end{aligned} \quad (24)$$

with corresponding boundary conditions:

$$\eta = 0: P_0 = P_1 = R_1 = 0; \quad R_0 = 1; \quad n \to \infty: P_0 = P_1 = R_0 = R_1 = 0. \quad (25)$$

The dependence on Prantl number for the treated fluid means that the equations obtained (22-25), are not completely universal. Basic temperature field in the boundary layer is defined by the equations (22) and (24), while the equations (23) and (25), determine, together with parameter Ω, the influence of rotation.

As for the adiabatic case, the system (22-25) is solved in one-parametric approximation, ($f_1 \neq 1, f_2 = ... = f_k = 0$), for $Pr = 1$, and for $Pr = 0.72$ (for air). Discretisation of the governing equations is performed using finite difference method, implicit scheme. The solutions for Φ_0 and Φ_1 of the dynamic boundary layer, obtained in [8], were used. Numerical integration is done for ($0 \leq \eta \leq 0$) with step $\Delta\eta = 0.1$ and ($-0.08 \leq f_1 \leq 0.08$) and step $\Delta f_1 = 0.05$. Solutions are given in the Tab. 1 and on the Fig. 4-7.

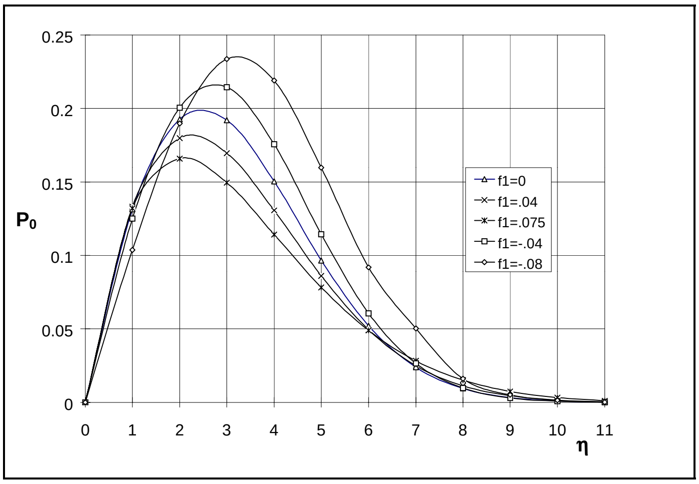

Figure 4. Profiles of universal function P_0, for different values of parameter f_1

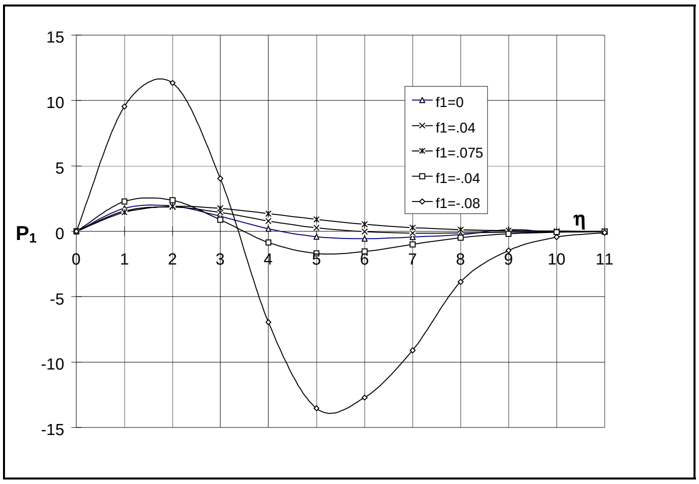

Figure 5. Profiles of universal function P_1, for different values of parameter f_1

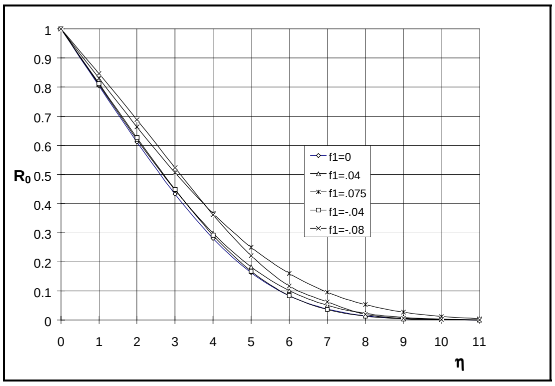

Figure 6. Profiles of universal function R_0, for different values of parameter f_1

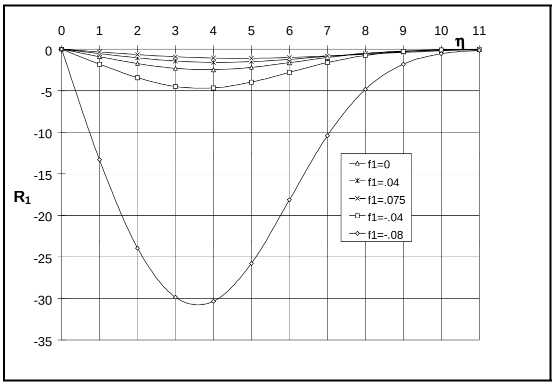

Figure 7. Profiles of universal function R_1, for different values of parameter f_1

Analysing the results for the universal function R_0, it is evident that the isothermal wall, heats the fluid so that temperature rises from the value in the outer flow to the value on the wall. The results for the universal function P_0, prove that the influence of friction in this case is maximal in the middle of the boundary layer. The values of the universal function R_1 are negative throughout the boundary layer, which signifies certain cooling effect. For the universal function P_1 it is obvious that the influence of rotation, varies with the development of the boundary layer. Near the stagnation point, function P_1 is completely positive, which means that rotation contributes to the rise of temperature. Approaching the separation point, profiles are negative in outer part, which means that the rotation provokes cooling of the outer parts of the boundary layer.

4. CONCLUSIONS

In the paper presented, the governing equations of the incompressible laminar temperature boundary layer are derived in the universal form, using the method of Loitskianskii [1], improved by Saljnikov [2]. Two different cases of the boundary conditions are concerned - adiabatic and isothermal wall. Universal temperature functions for the treated case are obtained by numerical integration and presented both numerically and graphically. Basic solutions of the temperature universal functions, K_0, P_0, and R_0, determine the basic temperature fields. The influence of the rotation on the on the temperature boundary layer is determined with dinensionless rotational parameter Ω and solutions of the suplementar universal functions K_1, P_1, and R_1. Numerical results of the universal function obtained, are very convenient for practical calculation. Connection of the universal function with the particular case, with known distribution of the potential velocity U_0 along the surface concerned, is done by means of the function f_1/B^2. Procedure for the temperature boundary layer calculation, for different particular cases (cylinder, profile etc.) will be the object of some future papers.

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UNIVERZALNA REŠENJA NESTIŠLJIVOG LAMINARNOG TEMPERATURSKOG GRANIČNOG SLOJA NA ROTIRAJUĆOJ POVRŠINI

Miloš D. Pavlović

U radu su jednačine nestišljivog laminarnog temperaturskog graničnog sloja univerzalizovane korišćenjem poboljšane metode Lojcijanskog. Uticaj obrtanja u razmatranom problemu izdvojen je razvijanjem i linearizacijom svih univerzalnih funkcija po parametru obrtanja Ω. Razmatrana su dva različita granična uslova - adiabatska površina i slučaj konstantne temperature zida. Univerzalna rešenja u jedno-parametarskom približenju dobijena su numeričkom integracijom. Sve sračunate univerzalne funkcije prikazane su grafički i analizirane, tako da se može uočiti uticaj obrtanja.