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OB URAVNENIÂH DVI@ENIÂ TVERDOGO TELA S VIHREVÀM SVERHPROVODÂÇIM ZAPOLNENIEM

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Резюме. *OsuÈestvlena redukciÔ uravneniy dvi`eniÔ tverdogo tela i sverhprovodÔÈey ideal†noy `idkosti, zapolnÔÓÈey ego Ìllipsoidal†nuÓ polost† k sisteme obìknovennìh differencial†nìh uravneniy kak v slu~ae, kogda telo-nositel† sover{aet uglovìe dvi`eniÔ pod deystveim vne{nih sil, tak i v slu~ae, kogda Ìti dvi`eniÔ ÔvlÔÓtsÔ izvestnìmi funkciÔmi vremeni. Ukazanì obÈie integralì plu~ennoy sistemì v oboih slu~aÔh. Ukazan slu~ay, kogda udaëtsÔ nayti to~noe prostranstvennoe re{enie kraevoy zada~i dlÔ uravneniy magnitnoy gidrodinamiki sverhprovodÔÈey `idkosti, vìra`ennoe ~erez Ìllipti~eskie funkcii vremeni i zavisÔÈie ot {esti proizvol†nìh postoÔnnìh.*

V rabote [1] O. I. BogoÔvlenskiy vpervìe vìvel uravnenie dvi`eniÔ tverdogo tela s Ìllipsoidal†noy polost†Ó, zapolnennoy sverhprovodÔÈey ideal†noy, odnorodnoy, nes`imaemoy `idkost†Ó. Pri Ìtom predpolagalos† ~to oblo~ka Ìllipsoida est† sverhprovodnik, a rotor skorosti ~astic `idkosti $\boldsymbol{v}$ i vektora naprÔ`ennosti magnitnogo polÔ $\boldsymbol{H}$ est† funkcii tol†ko vremeni $t$. (TakuÓ sistemy v dal†ney{em budem nazìvat† tverdìm telom s vihrevìm sverhprovodÔÈim zapolneniem). Im `e postroen klass re{eniy vìvedennìh uravneniy, modeliruÓÈih dinamiku vraÈeniÔ takih sverhprovodÔÈih ob†ektov, kak neytronnìe zvezdì i pul†sarì.

V predlagaemoy stat†e predlo`en drugoy put† postroeniÔ uravneniy dvi`eniÔ, voshodÔÈiy k klassi~eskomu [2, 3], ispol†zovannomy pri vìvode uravneniy dvi`eniÔ tverdogo tela s vihrevìm zapolneniem. Na Ìtom puti ispol†zovano opisanie sistemì v peremennìh, imeÓÈih, po mneniÓ avtorov, bolee Ôsnìy fizi~eskiy smìsl, i uravneniÔ magnitnoy gidrodinamiki v rotorah veli~in $\boldsymbol{v}$ i $\boldsymbol{H}$. V otli~ie ot rabotì [1], vna~ale postroenì uravneniÔ dvi`eniÔ dlÔ slu~aÔ, kogda tverdoe telo sover{aet

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zadannìe uglovìe dvi`eniÔ. Ukazanì eë pervìe integralì. Otdel†no izu~en slu~ay, kogda tverdoe telo sover{aet ravnomernìe vraÈeniÔ ili pokoitsÔ. Pokazano, ~to v Ìtom slu~ae mo`no vìpisat† re{enie izu~aemìh uravneniy v kvadraturah, opisìvaÓÈee netrivial†noe prostranstvennoe te~enie sverhprovodÔÈey `idkosti v Ìllipsoidal†noy polosti i izmenenie vektora naprÔ`ennosti magnitnogo polÔ $\boldsymbol{H}$.

1. **Постановка задачи.** Pust† tverdoe telo sover{aet vraÈenie s zadannoy uglovoy skorost†Ó $\omega(t)$ vokrug centra mass sistemì telo-`idkost†. DlÔ uproÈeniÔ izlo`eniÔ polagaem, ~to centr mass sistemì telo-`idkost†, to~ka $O$, sovpadaet s centrom polosti Ìllipsoida. UravneniÔm dvi`eniÔ magnitnoy hidrodinamiki v `estko svÔzannoy s tverdìm telom podvi`noy sisteme koordinat $Ox_1x_2x_3$, osi kotory sovpadaÓt s glavnìmi osÔmi Ìllipsoida, mo`no pridat† sleduÓÈuÓ formu:

$$\frac{d\boldsymbol{v}}{dt}+\omega\times v=-\frac{1}{\rho}grad\ p+\frac{1}{4\pi\rho}(rot\,\boldsymbol{H}\times\boldsymbol{H})\ , \tag{1.1}$$

$$\frac{\partial\boldsymbol{H}}{\partial t}=rot(\boldsymbol{u}\times\boldsymbol{H})\ , \tag{1.2}$$

$$div\,\boldsymbol{v}=0\ ,\quad div\,\boldsymbol{H}=\boldsymbol{0}\ , \tag{1.3}$$

Zdes† $\boldsymbol{H}(H_1, H_2, H_3)$, $\omega(\omega_1, \omega_2, \omega_3)$ i $\boldsymbol{v}(v_1, v_2, v_3)$ - sootvetstvenno vektorì naprÔ`ennosti magnitnogo polÔ, vektor uglovoy skorosti tverdogo tela i vektor absolÓtnoy skorosti ~astic `idkosti s komponentami v podvi`noy sisteme koordinat; $\boldsymbol{u}=\boldsymbol{v}-\omega\times\boldsymbol{r}$ - vektor otnositel†noy skorosti ~astic `idkosti; $\rho=const$ - plotnost† `idkosti, $p$-davlenie, $\boldsymbol{r}(x_1, x_2, x_3)$ - radius vektor ~astic `idkosti otnositel†no podvi`noy sistemì koordinat. Zdes† i dalee dlÔ lÓbogo vektora $\boldsymbol{a}$ imeem $\dfrac{d\boldsymbol{a}}{dt}=\dfrac{\partial\boldsymbol{a}}{\partial t}+(\boldsymbol{u}\cdot\nabla)\boldsymbol{a}$.

Grani~nìe usloviÔ dlÔ peremennìh $\boldsymbol{v}$ i $\boldsymbol{H}$ opredelenì obì~nìm obrazom

$$\boldsymbol{u}\cdot\boldsymbol{n}\big|_s=(\boldsymbol{v}-\omega\times\boldsymbol{r})_s=0\ ,\quad \boldsymbol{H}\cdot\boldsymbol{n}\big|_s=0\ , \tag{1.4}$$

gde $\boldsymbol{n}$ - vektor normali k poverhnosti Ìllipsoida S.

V dal†ney{em ponadobÔtsÔ uravneniÔ dlÔ vihrey $\boldsymbol{v}$ i $\boldsymbol{H}$, kotorìe mo`no podu~it† putem preobrazovaniy uravneniy (1.1)-(1.3):

$$\frac{d\Omega_*}{dt}+\omega\times\Omega_*=(\Omega_*\cdot\nabla)\boldsymbol{v}-\frac{1}{4\pi\rho}[(\boldsymbol{I}_*\cdot\nabla)\boldsymbol{H}-(\boldsymbol{H}\cdot\nabla)\boldsymbol{I}_*]\ , \tag{1.5}$$

$$\frac{dI_*}{dt}+\omega\times I_*=-(I_*\cdot\nabla)v+(\Omega_*\cdot\nabla)H+2\Omega_*\times I_*-\Delta(v\times H)+(H\cdot\nabla)\Omega_*+H\times rot\Omega_*-v\times rotI_* \tag{1.6}$$

gde $\Omega_*=rot\,\boldsymbol{v}$, $I_*=rot\,\boldsymbol{H}$.

Otmetim, ~to pri $\boldsymbol{H}\equiv 0$ uravnenie (1.5) perehodit v uravneniÔ Gel†mgol†ca [4].

2. **Редукция уравнений (1.1) - (1.3) к системе обыкновенных дифференциалъных уравнений при заданном $\omega(t)$.** V klassi~eskom slu~ae, kogda $\boldsymbol{H} \equiv 0$, v rabotah [2,3] bìlo pokazano, ~to re{enie uravneniy (1.1), (1.3) s pervìm grani~nìm usloviem (1.4) mo`no nayti v vide

$$v = grad\,\Phi + \Omega(t) \times r\ , \tag{2.1}$$

gde garmoni~eskaÔ funkciÔ $\Phi(x_1, x_2, x_3)$ opredelÔetsÔ sleduÓÈim obrazom [5]

$$\Phi = \varepsilon_1(\omega_1 - \Omega_1)x_2x_3 + \varepsilon_2(\omega_2 - \Omega_2)x_3x_1 + \varepsilon_3(\omega_3 - \Omega_3)x_1x_2\ , \tag{2.2}$$

$\varepsilon_1 = (c_2^2 - c_3^2)/(c_2^2 + c_3^2)$ (123), c_1, c_2, c_3, - poluosi polosti-Ìllipsoida. Tam `e [5] bìli privedenì differencial†nìe uravneniÔ dlÔ opredeleniÔ zavisÔÈih tol†ko ot vremeni proekcii vektora $\Omega(\Omega_1, \Omega_2, \Omega_3)$ na podvi`nìe osi $Ox_1x_2x_3$.

Budem iskat† re{enie uravneniy (1.1)-(1.3) s grani~nìmi usloviÔmi (1.4) dlÔ $\boldsymbol{v}$ v vide (2.1) s Φ v vide (2.2), a $\boldsymbol{H}$ v sleduÓÈem vide

$$\boldsymbol{H} = \frac{2\pi}{c}\left[grad\,\tilde{\Phi} + \boldsymbol{I}(t) \times \boldsymbol{r}\right], \tag{2.3}$$

gde

$$\tilde{\Phi} = -\varepsilon_1 I_1 x_2 x_3 - \varepsilon_2 I_2 x_3 x_1 - \varepsilon_3 I_3 x_1 x_2\ , \tag{2.4}$$

c-skorost† sveta. Mno`itel† $\dfrac{2\pi}{c}$-v vìra`enii (2.3) vìbran s cel†Ó pridat† Ôsnìy fizi~eskiy smìsl vektoru $\boldsymbol{I}(t)$. Deystvitel†no, tak kak iz (2.3) sleduet ~to $I = \dfrac{c}{4\pi} rot\,\boldsymbol{H}$, to $\boldsymbol{I}(t)$ est† vektor plotnosti Ìlektri~eskogo toka, protekaÓÈego ~erez `idkost†.

Neposredstvennoy proverkoy mo`no ubedit†sÔ, ~to funkcii $\boldsymbol{v}$ i $\boldsymbol{H}$, zadannìe takim obrazom, udovletvorÔÓt grani~nìm usloviÔm (1.4) i uravneniÓ (1.3).

DlÔ opredeleniÔ funkciy $\Omega(t)$ i $\boldsymbol{I}(t)$ vospol†zuemsÔ uravneniÔmi dlÔ vihrey (1.5) i (1.6), u~itìvaÔ, ~to v izu~aemoy situacii

$$\Omega_* = 2\Omega(t)\ ,\quad \boldsymbol{I}_* = \frac{4\pi}{c}\boldsymbol{I}(\mathrm{t})\ , \tag{2.5}$$

a $\boldsymbol{v}$ i $\boldsymbol{H}$ est† lineynìe funkcii x_1, x_2, x_3, vì~islÔemìe po formulam (2.1)-(2.4).

S u~etom (2.1), (2.3), (2.5) uravneniÔ preobrazyÓtsÔ v obìknovennìe differencial†nìe uravneniÔ dlÔ opredeleniÔ komponent vektorov $\Omega(t)$ i $\boldsymbol{I}(t)$:

$$\frac{\mathrm{d}\Omega_1}{\mathrm{d}t} = (1+\varepsilon_3)\omega_3\Omega_2 - (1-\varepsilon_2)\omega_2\Omega_3 - (\varepsilon_2+\varepsilon_3)\Omega_2\Omega_3 + \frac{\pi}{\rho c^2}(\varepsilon_2+\varepsilon_3)I_2I_3, \tag{2.6}$$

$$\frac{\mathrm{d}I_1}{\mathrm{d}t} = (1+\varepsilon_2\varepsilon_3)(\omega_3 I_2 - \omega_2 I_3 + \Omega_2 I_3 - \Omega_3 I_2)\ ,\quad (123) \tag{2.7}$$

posle integrirovaniÔ kotorìh nahodim re{enie zada~i (2.1)-(2.4).

Pod~erknem, ~to v Ìtom re{enii $\boldsymbol{v}$ i $\boldsymbol{H}$ ÔvlÔÓtsÔ lineynìmi funkciÔmi koordinat. Takie re{eniÔ v klasi~eskom slu~ae ($\boldsymbol{H} \equiv 0$) Puankare nazval prostìmi [6]. Sohranim takoe opredelenie i v obÈem slu~ae.

3. **Об общих интегралах уравнений (2.6), (2.7).** Otmetim, ~to sistema (2.6), (2.7) dopuskaet dva obÈih integrala pri lÓbom zadannom vektore $\omega(t)$:

$$V_1 = \frac{I_1^2}{c_1^2(c_2^2+c_3^2)^2} + \frac{I_2^2}{c_2^2(c_3^2+c_1^2)^2} + \frac{I_3^2}{c_3^2(c_1^2+c_2^2)^2} = const\,, \tag{3.1}$$

$$V_2 = \frac{I_1\Omega_1}{c_1^2(c_2^2+c_3^2)} + \frac{I_2\Omega_2}{c_2^2(c_3^2+c_1^2)} + \frac{I_3\Omega_3}{c_3^2(c_1^2+c_2^2)} = const. \tag{3.2}$$

V slu~ae, esli soder`aÈee Ìllipsoidal†nuÓ polost† tverdoe telo sover{aet ravnomernìe vraÈeniÔ (ω=*const*), uravneniÔ (2.6), (2.7) dopuskaÓt eÈe odin obÈiy integral - integral Ìnergii:

$$V_3 = \sum_{i=1}^{3} b_i\left(\Omega_i^2 - 2\Omega_i\omega_i + \frac{\pi}{\rho c^2} I_i^2\right) = \text{const}, \tag{3.3}$$

gde $b_1 = \dfrac{2c_2^2c_3^2}{b^2(c_2^2+c_3^2)}$ (123), $\quad b^2 = c_1^2 + c_2^2 + c_3^2$.

Esli telo pokoitsÔ ($\omega = 0$), to te `e uravneniÔ dopuskaÓt eÈe odin obÈiy integral:

$$V_4 = \sum_{i=1}^{3} c_i^{-2}\Omega_i^2 - \frac{\pi}{\rho c^2}\left[\frac{c_1^2 I_1^2}{(c_2^2+c_3^2)^2} + \frac{c_2^2 I_2^2}{(c_3^2+c_1^2)^2} + \frac{c_3^2 I_3^2}{(c_1^2+c_2^2)^2}\right] = const, \tag{3.4}$$

obobÈaÓÈiy integral Gel†mgol†ca [4] postoÔnstva intensivnosti vihrÔ ~astic `idkosti v klassi~eskom slu~ae. Nebezìnteresno otmetit† i sleduÓÈee obstoÔtel†stvo: mo`no pokazat†, ~to uravneniÔ (2.6), (2.7) pri $\omega = 0$ lineynìm preobrazovaniem privodÔtsÔ k ~astnomu slu~aÓ uravniniy Kirhgofa [7], dlÔ vìpolnenì usloviÔ Kleb{a, obespe~ivaÓÈie suÈestvovanie dopolnitel†nogo integrala (3.4).

Õto ozna~aet, ~to uravneniÔ (2.6), (2.7) mogut bìt† prointegrirovanì, poskol†ku integriruÓÈiy mno`itel† Âkobi v Ìtom slu~ae raven edinice.

EÈe odin integriruemìy slu~ay voznikaet, kogda polost† simmetri~na ($c_1 = c_2$) i $\omega_1 = \omega_2 = 0$, poslol†ku pri Ìtom tret†e uravnenie iz (2.6) prinimaet vid $\dot{\Omega}_3 = 0$, otkuda sleduet eÈe odin obÈiy integral

$$V_5 = \Omega_3 = const, \tag{3.5}$$

Nebezìnteresno otmetit†, ~to esli eÈe polo`it† $\omega_3 = 0$, to on ne ÔvlÔtsÔ dopolnitel†nìm, poskol†ku mo`no pokazat†, ~to v Ìtom slu~ae integralì, opredelÔemìe formulami (3.1)-(3.5), funkcional†no zavisimì.

4. **Точное решение краевой задачи (1.1) - (1.4)**. Naydem obÈee re{enie uravneniy (2.6), (2.7) pri usloviÔh:

$$c_1 = c_2 \ , \quad \omega_1 = \omega_2 = 0 \, . \tag{4.1}$$

Sistema uravniniy (2.6), (2.7) i eë pervìe integralì pri Ìtom takovì:

$$\begin{gathered}
\dot{\Omega}_1 = \omega_3\Omega_2 - \varepsilon\Omega_3^0\Omega_2 + \alpha I_3 I_2, \quad \dot{\Omega}_2 = -\omega_3\Omega_1 + \varepsilon\Omega_3^0\Omega_1 - \alpha I_3 I_1, \\
\dot{I}_1 = \omega_3 I_2 - \Omega_3^0 I_2 + I_3\Omega_2, \qquad \dot{I}_2 = -\omega_3 I_1 + \Omega_3^0 I_1 - I_3\Omega_1, \\
\dot{I}_3 = (1-\varepsilon^2)(\Omega_1 I_2 - \Omega_2 I_1),
\end{gathered} \tag{4.2}$$

$$(1-\varepsilon^2)(I_1^2 + I_2^2) + I_3^2 = I^2 = const, \tag{4.3}$$

$$(1-\varepsilon)(\Omega_1 I_1 + \Omega_2 I_2) + \Omega_3^0 I_3 = l = const, \tag{4.4}$$

$$(1-\varepsilon^2)(\Omega_1^2 + \Omega_2^2) + \alpha I_3^2 = h = const, \tag{4.5}$$

$$\Omega_3 = \Omega_3^0 = const, \tag{4.6}$$

gde $\varepsilon = (c_1^2 - c_3^2)/(c_1^2 + c_3^2) =, \quad \alpha = \pi\varepsilon/\rho c^2$.

PerehodÔ k kompleksnìm peremennìm $z, \bar{z}, w, \overline{w}$ po formulam:

$$\begin{gathered}
\Omega_1 + \mathrm{i}\Omega_2 = z e^{\mathrm{i}\omega_3 t}, \qquad \Omega_1 - \mathrm{i}\Omega_2 = \bar{z} e^{-\mathrm{i}\omega_3 t}, \\
I_1 + \mathrm{i} I_2 \Omega_2 = w e^{\mathrm{i}\omega_3 t}, \quad I_1 - \mathrm{i} I_2 \Omega_2 = \overline{w} e^{-\mathrm{i}\omega_3 t},
\end{gathered} \tag{4.7}$$

privodim uravniniÔ (4.2) i integral (4.3) - (4.6) k sleduÓÈemu vidu

$$\begin{gathered}
\dot{z} = \mathrm{i}\varepsilon\Omega_3^0 z - \mathrm{i}\alpha I_3 w, \quad \dot{\bar{z}} = -\mathrm{i}\varepsilon\Omega_3^0 \bar{z} - \mathrm{i}\alpha I_3 \overline{w}, \\
\dot{w} = \mathrm{i}\varepsilon\Omega_3^0 w - \mathrm{i} I_3 z, \qquad \dot{\overline{w}} = \mathrm{i}\varepsilon\Omega_3^0 \overline{w} - \mathrm{i} I_3 \bar{z}, \\
\dot{I}_3 = \frac{1-\varepsilon^2}{2\mathrm{i}}(\bar{z}w - z\overline{w}),
\end{gathered} \tag{4.8}$$

$$\begin{gathered}
(1-\varepsilon^2) w\overline{w} + I_3^2 = I^2 = const, \\
(1-\varepsilon)(z\overline{w} + \bar{z}w) + 2\Omega_3^0 I_3 = 2l = const, \\
(1-\varepsilon^2) z\bar{z} + \alpha I_3^2 = h = const.
\end{gathered} \tag{4.9}$$

O~evidnoe to`destvo

$$\left(\frac{\bar{z}w - z\overline{w}}{2\mathrm{i}}\right)^2 = -\left(\frac{\bar{z}w + z\overline{w}}{2\mathrm{i}}\right)^2 + (z\bar{z})(w\overline{w}),$$

integralì (4.9) i poslednee uravnenie sistemì (4.8) pozvolÔÓt ukazat† kvadraturu, svÔzìvÓuÓ peremennuÓ I_3 so vremenem t

$$t = \int_{I_3^0}^{I_3} \frac{\mathrm{d}\tau}{\sqrt{F(\tau)}}, \tag{4.10}$$

gde

$$F(\tau)=[(I^2-\tau^2)(h-\alpha\tau^2)-(1+\varepsilon)^2(l-\Omega_3^0\tau)^2].$$

ObraÈaÔ integral (4.10), nahodim I_3 kak Ìllipti~eskuÓ vremeni

$$I_3=f(t,h,I^2,l,\Omega_3^0,I_3^0). \qquad (4.11)$$

Kak sleduet iz integralov (4.8), stanovÔtsÔ izvestnìmi funkciÔmi vremeni i takie veli~inì:

$$\rho_1^2(t)=|z|^2=(1-\varepsilon^2)^{-1}[h-\varepsilon f^2(t)],$$
$$\rho_2^2(t)=|w|^2=(1-\varepsilon^2)^{-1}[I-f^2(t)], \qquad (4.12)$$
$$q(t)=\frac{1}{2}\left(z\overline{w}+\bar{z}w\right)=(1-\varepsilon^2)^{-1}[l-\Omega_3^0 f(t)].$$

^tobì nayti zavisimost† veli~in z, w, a sledovatel†no, Ω_1, Ω_2, I_1, I_2 ot vremeni, pereydem k trigonometri~eskoy forme kompleksnìh veli~in z i w

$$z=|z|e^{\mathrm{i}\varphi}, \quad w=|\mathrm{w}|e^{\mathrm{i}\psi}. \qquad (4.13)$$

Iz sootno{eniy (4.12) vìtekaÓt sleduÓÈie ravenstva:

$$|z|=\rho_1(t), \quad |w|=\rho_2(t), \qquad (4.14)$$

$$e^{\mathrm{i}(\varphi-\psi)}+e^{-\mathrm{i}(\varphi-\psi)}=2q(t)/\rho_1(t)\cdot\rho_2(t). \qquad (4.15)$$

Takim obrazom, ostaetsÔ nayti zavisimost† φ i ψ ot t, poskol†ku |z| i |w| opredelenì ravenstvami (4.14).

$$|\dot{z}|+\mathrm{i}\,|\mathrm{z}|\dot{\varphi}=\mathrm{i}\varepsilon\Omega_3^0\,|z|-\mathrm{i}\alpha f\,|w|e^{-\mathrm{i}(\varphi-\psi)},$$
$$|\dot{z}|-\mathrm{i}\,|\mathrm{z}|\dot{\varphi}=-\mathrm{i}\varepsilon\Omega_3^0\,|z|+\mathrm{i}\alpha f\,|w|e^{\mathrm{i}(\varphi-\psi)},$$
$$|\dot{w}|+\mathrm{i}\,|w|\dot{\psi}=\mathrm{i}\Omega_3^0\,|w|-\mathrm{i}f\,|z|e^{-\mathrm{i}(\varphi-\psi)}, \qquad (4.16)$$
$$|\dot{w}|-\mathrm{i}\,|w|\dot{\psi}=-\mathrm{i}\Omega_3^0\,|w|+\mathrm{i}f\,|z|e^{\mathrm{i}(\varphi-\psi)}.$$

Iz sootno{eniy (4.16), s u~etom ravenstva (4.15), nahodim

$$\dot{\varphi}=\varepsilon\Omega_3^0-\alpha \quad f(t)q(t)/\rho_1(t),$$
$$\dot{\psi}=\Omega_3^0- \quad f(t)q(t)/\rho_2(t). \qquad (4.17)$$

Sledovatel†no, s u~etom (4.7), (4.12), (4.13), (4.14), (4.17) nahodim zavisimost† Ω_1, Ω_2, I_1 i I_2 ot vremeni t:

$$\Omega_1=\rho_1(t)\cos[\varphi(t)+\varphi_0], \quad \Omega_2=\rho_1(t)\sin[\varphi(t)+\varphi_0],$$
$$I_1=\rho_2(t)\cos[\psi(t)+\psi_0], \quad I_2=\rho_2(t)\sin[\psi(t)+\psi_0], \qquad (4.18)$$

gde

$$\varphi(t)=\varepsilon\Omega_3^0(t-t_0)-\alpha\int_{t_0}^{t}f(\tau)q(\tau)\rho_1^{-1}(\tau)\mathrm{d}\tau+\omega_3 t+\varphi_0,$$

$$\psi(t)=\Omega_3^0(t-t_0)-\int_{t_0}^{t}f(\tau)q(\tau)\rho_2^{-1}(\tau)\mathrm{d}\tau+\omega_3 t+\psi_0,$$

φ_0, ψ_0 - proizvol†nìe postoÔnìe. Takim obrazom formulì (4.11), (4.12), (4.18) daÓt obÈee re{enie uravneniy (2.6), (2.7) pri usloviÔx (4.1), zavisÔÈee ot {esti proizvol†nìh postoÔnnìh I^2, h, l, Ω_3^0, ,φ_0, ψ_0, a vmeste s sootno{eniÔmi (2.1) - (2.4) - to~noe re{enie kraevoy zada~i (1.1) - (1.4).

5. **Редикция уравнений (1.1) - (1.3) при условии (1.4) в общем случае**. Esli oblo~ka ne sover{aet zadannìe vraÈatel†nìe dvi`eniÔ i vektor $\omega(t)$ sam podle`it opredelniÓ, to dlÔ veli~in $\omega(t)$, $\Omega(t)$, $\mathbf{I}(t)$ neobhodimo ras{irit† sistemu (2.6), (2.7) uravneniÔmi, otra`aÓÈimi zakon izmeneniÔ polnogo momenta impul†sa [5]

$$A_1\frac{\mathrm{d}\omega_1}{\mathrm{d}t}+A_1'\frac{\mathrm{d}\Omega_1}{\mathrm{d}t}+(A_3-A_2)\omega_2\omega_3+A_3'\omega_2\Omega_3-A_2'\omega_3\Omega_2=L_1. \quad (123) \qquad (5.1)$$

Zdes† $\mathbf{L}(L_1, L_2, L_3)$ - moment vne{nih sil, rezul†tiruÓÈee vozdeystvie kotorìh na sistemu telo-`idkost† ravno nulÓ;

$$A_1=A_1^0+\frac{M_2}{5}\frac{(c_2^2-c_3^2)^2}{c_2^2+c_3^2}, \quad A_1'=\frac{4M_2c_2^2c_3^2}{5(c_2^2+c_3^2)}, \qquad (123)$$

gde A_i^0 (i=1,2,3) – osevìe momentì inercii tverdogo tela, M_2- massa `idkosti. Otmetim, ~to uravneniÔ (5.1) napisanì v predpolo`enii, ~to glavnìe osi tverdogo tela i poluosi polosti Ìllipsoida sovpadaÓt.

Esli `e cent polosti ne sovpadaet s centrom mass sistemì telo-`idkost† i osi $Ox_1x_2x_3$ ne ÔvlÔÓtsÔ glavnìmi dlÔ tverdogo tela, to uravneniÔ (5.1) zamenÔÓtsÔ na sleduÓÈie [5]:

$$\frac{\mathrm{d}K_1}{\mathrm{d}t}+A_1'\frac{\mathrm{d}\Omega_1}{\mathrm{d}t}+\omega_2K_3-\omega_3K_2+A_3'\omega_2\Omega_3-A_2'\omega_3\Omega_2=L_1. \quad (1,2,3) \qquad (5.2)$$

Zdes†

$$\mathrm{K}(K_1,K_2,K_3)=[\mathrm{A}+M_2(\mathrm{E}\mathrm{r}^2-\mathrm{r}\cdot\mathrm{r})+\mathrm{A}']\omega \;,$$

gde ***A*** - tenzor inercii tverdogo tela, ***A'*** - diagonal†nìy tenzor s Ìlementami A_1', A_2', A_3', ***E*** - edini~naÔ matrica, $\mathrm{r}(\mathrm{x}_1^0,\mathrm{x}_2^0,\mathrm{x}_3^0)$ - radius-vektor centra polosti v sisteme koordinat $Ox_1x_2x_3$.

S u~etom (2.6) uravneniÔ (5.1) v obozna~eniÔh rabotì [8] mo`no pridat† sleduÓÈuÓ formu:

$$a_1\frac{\mathrm{d}\omega_1}{\mathrm{d}t}=(a_2-a_3)\omega_2\omega_3-\varepsilon_2\varepsilon_1b_3\omega_2\Omega_3+\varepsilon_1\varepsilon_3b_2\omega_3\Omega_2+$$
$$+b_1(\varepsilon_2+\varepsilon_3)\left(\Omega_2\Omega_3-\frac{\pi}{\rho c^2}I_2I_3\right)+L_1, \quad (123) \qquad (5.3)$$

gde $\mathbf{L}(L_1, L_2, L_3)$ - moment vne{nih sil.

Sistema (2.6), (2.7), (5.3), daÓÈaÔ re{enie izu~aemoy zada~i k kotoromu dootvetstvuÓt prostìe dvi`eniÔ, pri $\mathbf{L}\equiv 0$ s to~nost†Ó do lineynoy zamenì peremennìh i obozna~eniy, sovpadaet s uravneniÔmi, polu~ennìmi v rabote [1].

Krome obÈih integralov (3.1), (3.2), Ìta sistema dopuskaet pri L=0 eÈe dva obÈih integrala - energii

$$a_1\omega_1^2 + a_2\omega_2^2 + a_3\omega_3^2 + b_1\left(\Omega_1^2 + \frac{\pi}{\rho c^2} I_1^2\right) + b_2\left(\Omega_2^2 + \frac{\pi}{\rho c^2} I_2^2\right) + b_3\left(\Omega_3^2 + \frac{\pi}{\rho c^2} I_3^2\right) = const$$

i postoÔnstva modulÔ momenta koli~estva dvi`eniÔ sistemì

$$(a_1\omega_1 + b_1\Omega_1)^2 + (a_2\omega_2 + b_2\Omega_2)^2 + (a_3\omega_3 + b_3\Omega_3)^2 = const \ ,$$

kotorìe v drugih peremennìh i obozna~eniÔh privedenì v rabote [1].

Zame~anie. V klassi~eskom slu~ae Puankare pokazal [6], ~to esli dvi`enie bìlo v kakoy-to moment vremeni odnorodnìm vihrevìm, to ono vse vremÔ budet takovìm, esli polost† imeet formu Ìllipsoida. V izu~aemom slu~ae isistema uravneniy (2.6), (2.7) (kogda $\omega(t)$ zadana), i sistema uravneniy (2.6), (2.7), (5.3) (kogda $\omega(t)$ podle`it opredeleniÓ) pozvolÔÓt sdelat† vìvod, ~to dvi`enie `idkosti v na~al†nìy moment mo`et bìt† bezvihrevìm ($\Omega(0) = 0$), no esli $\mathbf{I}(0) \neq 0$, to v posleduÓÈie momentì vremeni `idkost† budet sover{at† odnorodnìe vihrevìe dvi`eniÔ ($\Omega(0) \neq 0$).

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O JEDNAČINAMA KRETANJA ČVRSTOG TELA S VRTLOŽNIM SUPERPROVODNIM PUNJENJEM

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Ostvareno je svođenje jednačina kretanja čvrstog tela i superprovodne idelane tečnosti koja zauzima elipsoidni prostor na sistem običnih diferencijalnih jednačina, kako u slučaju kada telo vrši ugaono kretanje pod dejstvom spoljašnjih sila, tako i u slučaju kada su ta kretanja poznate funkcije vremena. Istaknut je slučaj kada je moguće naći tačno rešenje graničnih zadataka za jednačine megnetne hidromehanike superprovodne tečnosti, izraženo preko eliptičnih vremenskih funkcija koje zavise od šest proizvoljnih konstanti.