UNIVERSITY OF NIŠ
The scientific journal **FACTA UNIVERSITATIS**
Series: **Mechanics, Automatic Control and Robotics** Vol.2, No 7-2, 1997 pp. 465 - 470
Editor of series: Katica (Stevanovi}) Hedrih, e-mail: katica@masfak.masfak.ni.ac.yu
Address: Univerzitetski trg 2, 18000 Niš, YU, Tel: (018) 547-095, Fax: (018)-547-950
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THE INFLUENCE OF THE TRANSVERSAL DIMENSIONS ON THE PROPAGATION VELOCITY OF THE LONGITUDINAL WAVE-LENGTHS IN AN AXISSYMETRICAL BODY

UDC: 681.11.032.11; 624.044; 539.3

Katica (Stevanović) Hedrih, Predrag Kozić, Ratko Pavlović

Faculty of Mechanical Engineering, University of Niš, YU

Abstract. *The velocity of the longitudinal waves in a cylindrical hollow bar is obtained as the velocity of infinite wave-length of a function of three parameters: Poisson's coefficient, relation of external and internal radius and wave-length (ratio). This function is given for the lowest frequency within the field of the geometrical parameters value which are physically important for applications. Geometrical interpretations of the numerical experiment of the influence of the transversal dimensions on the propagation velocity of the longitudinal wave-length in a cylindrical hollow bar is presented.*

When we provoke (cause) a perturbation of equilibrium state (natural - eigen state) of elastic body which is free of external forces actions, the motion-flicker (twinkle) of the body particles must be arised. That motion-flicker (twinkle) is transferred through the body as a wave processes. In every particle (points) of the elastic medium, the same state of perturbation must be arised, but with different phase delay. The motion-flicker (twinkle) of the body particles can be provoked in the case when elastic body is in the natural - eigen unstressed state as well as when the body is in the prestressed state. Elastic bodies with natural rigidity on the strain can twinkle in both ways. We mark with $\vec{u}$ the vector of displacements of the elastic body point. If we introduce the preposition that we observe only small deformations of elastic body and eliminate the translation which corresponds to the rigid body motion, the stationary part of the derivative in time of the displacement vector can be neglected, and for partial equation of the wave motion of the deformable elastic body we can adopt Lame's partial differential vector equation in the form:

Received September 16, 1997

$$\nu\Delta\vec{u} + (\lambda + \nu)\, grad\ div\vec{u} + \vec{F}_V' = \rho\ddot{\vec{u}} \tag{1}$$

If we neglect the volume (mass) force $\vec{F}_V'$, and take in consideration that are:

$$\nu = \mathrm{G} = \frac{\mathrm{E}}{2(1+\mu)}, \quad \lambda = \frac{\mu E}{(1+\mu)(1-2\mu)}, \tag{2}$$

$$grad\ div\vec{u} = rot\ rot\vec{u} + \nabla^2\vec{u}$$

and that strain volume is:

$$\varepsilon_V = div\vec{u} = \frac{1}{r}\frac{\partial(ru_r)}{\partial r} + \frac{1}{r}\frac{\partial u_\varphi}{\partial\varphi} + \frac{\partial u_z}{\partial z} \tag{3}$$

the Lame equation (1) obtain following form:

$$(\lambda + 2\nu) grad\ div\vec{u} - \nu\ rot\ rot\vec{u} = \rho\ddot{\vec{u}} \tag{4}$$

This problem will be solved using polar-cylindrical coordinate s, r, φ, z , and with u_r, u_φ and u_z are note components of the vector displacement $\vec{u}$. After introducing expressions for $rotrot\vec{u}$ and $graddiv\vec{u}$ in the last equation (4) we can rewrite this equation in the scalar form by tree equations:

$$\begin{aligned}
&(\lambda + 2\nu)\frac{\partial\varepsilon_V}{\partial r} + 2\nu\left(\frac{\partial\omega_\varphi}{\partial z} - \frac{1}{r}\frac{\partial\omega_r}{\partial\varphi}\right) = \rho\frac{\partial^2 u_r}{\partial t^2} \\
&(\lambda + 2\nu)\frac{1}{r}\frac{\partial\varepsilon_V}{\partial r} + 2\nu\left(\frac{\partial\omega_z}{\partial r} - \frac{\partial\omega_r}{\partial z}\right) = \rho\frac{\partial^2 u_\varphi}{\partial t^2} \\
&(\lambda + 2\nu)\frac{\partial\varepsilon_V}{\partial z} + 2\nu\frac{1}{r}\left(\frac{\partial\omega_r}{\partial\varphi} - \frac{\partial(r\omega_\varphi)}{\partial r}\right) = \rho\frac{\partial^2 u_z}{\partial t^2}
\end{aligned} \tag{5}$$

where ω_r , ω_φ and ω_z are components of the vector rotation.

We solve the equations (5) by introducing propositions that $u_\varphi = 0$, and that displacements u_r and u_z do not depend on the angle φ. In that case we have an axially symmetric problem and then the harmonic wave, inside a hollow cylinder of the inner radius R_1 and the external radius R_2, moves on the z-axis. By using the introduced assumptions we can determine components of the rotation vector:

$$\omega_r = 0,\ \omega_\varphi = \frac{1}{2}\left(\frac{\partial u_r}{\partial z} - \frac{\partial u_\varphi}{\partial\varphi}\right), \omega_z = 0, \tag{6}$$

and stress tensor components:

$$\begin{aligned}
&\sigma_{rr} = 2\nu\frac{\partial u_r}{\partial r} + \lambda\left(\frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{\partial u_z}{\partial z}\right); \quad \sigma_{\varphi\varphi} = 2\nu\frac{u_r}{r} + \lambda\left(\frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{\partial u_z}{\partial z}\right); \\
&\sigma_{zz} = 2\nu\frac{\partial u_z}{\partial z} + \lambda\left(\frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{\partial u_z}{\partial z}\right); \ \tau_{r\varphi} = 0;\ \tau_{z\varphi} = 0;\ \tau_{rz} = \nu\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right)
\end{aligned} \tag{7}$$

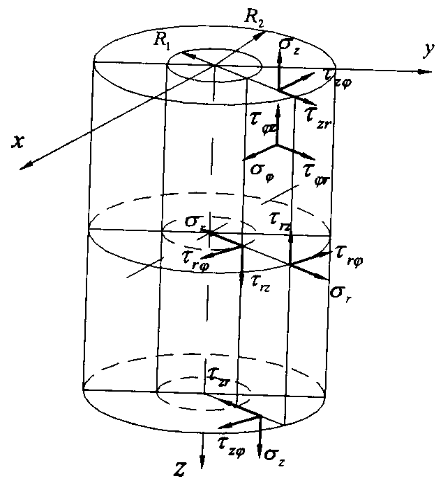

Fig 1.

By including these assumptions in the equation (5) they become two differential equations, because the second equation is identically equal to zero and after inclusion ε_V, ω_r, ω_φ, ω_z they are:

$$
\begin{aligned}
&(\lambda+2\nu)\left(\frac{\partial^2 u_r}{\partial r^2}+\frac{1}{r}\frac{\partial u_r}{\partial r}-\frac{u_r}{r^2}+\frac{\partial^2 u_z}{\partial r\partial z}\right)+\nu\left(\frac{\partial^2 u_r}{\partial z^2}-\frac{\partial^2 u_z}{\partial r\partial z}\right)=\rho\frac{\partial^2 u_r}{\partial t^2}\\
&(\lambda+2\nu)\left(\frac{\partial^2 u_r}{\partial r\partial z}+\frac{1}{r}\frac{\partial u_r}{\partial z}+\frac{\partial^2 u_z}{\partial z^2}\right)-\frac{\nu}{r}\left(\frac{\partial u_r}{\partial z}-\frac{\partial u_z}{\partial r}\right)-\nu\left(\frac{\partial^2 u_r}{\partial z\partial r}-\frac{\partial^2 u_r}{\partial r^2}\right)=\rho\frac{\partial^2 u_z}{\partial t^2}
\end{aligned}
\tag{8}
$$

As the problem is symmetric we introduce two functions $f(r,y,t)$ and $\psi(r,z,t)$. The displacement components, the rotation angle in circular direction and volume strain expressed by them are:

$$
u_r=\frac{\partial f}{\partial r}+\frac{\partial^2\psi}{\partial r\partial z}, \quad u_z=\frac{\partial f}{\partial r}-\frac{\partial^2\psi}{\partial r^2}-\frac{1}{r}\frac{\partial\psi}{\partial r}, \tag{9}
$$

$$
\omega_\varphi=\frac{1}{2}\frac{\partial}{\partial r}(\Delta\psi), \quad \varepsilon_V=\Delta f(r,z,t) \tag{10}
$$

By replacing the relations (9) and (10) into differential equations (8) they become:

$$
\frac{\partial}{\partial r}\left(\Delta f-\frac{\rho}{\lambda+2\nu}\ddot{f}\right)+\frac{\nu}{\lambda+2\nu}\frac{\partial^2}{\partial r\partial z}\left(\Delta\psi-\frac{\rho}{\nu}\ddot{\psi}\right)=0, \tag{11}
$$

$$
\frac{\partial}{\partial z}\left(\Delta f-\frac{\rho}{\lambda+2\nu}\ddot{f}\right)-\frac{\nu}{\lambda+2\nu}\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial}{\partial r}\left(\Delta\psi-\frac{\rho}{\nu}\ddot{\psi}\right)\right]=0. \tag{12}
$$

One of the particular solutions of the system (11)-(12) is when the following equations are satisfied:

$$\Delta f-\frac{1}{c_1^2}\ddot{f}=0\,,\quad \Delta\psi-\frac{1}{c_1^2}\ddot{\psi}=0 \tag{13}$$

in which introduced following notations are:

$$c_1^2=\frac{\lambda+2\nu}{\rho}=\frac{\mathrm{E}(1-\mu)}{\rho(1+\mu)(1-2\mu)}\,,\quad c_2^2=\frac{\nu}{\rho}=\frac{\mathrm{E}}{2\rho(1+\mu)} \tag{14}$$

As the harmonic wave lays (propagates) along the z-axis we assume that the solution of the system (13) is:

$$f(r,z,t)=\Phi(r)e^{i(\alpha z+\omega t)}\,,\qquad \psi(r,z,t)=\Psi(r)e^{i(\alpha z+\omega t)} \tag{15}$$

By replacing the assumed solutions in the differential equations (13) they become two simple differential equations:

$$\frac{d^2\Phi(r)}{dr^2}+\frac{1}{r}\frac{d\Phi(r)}{dr}+\nu_1^2\Phi(r)=0\,,$$

$$\frac{d^2\Psi(r)}{dr^2}+\frac{1}{r}\frac{d\Psi(r)}{dr}+\nu_1^2\Psi(r)=0\,, \tag{16}$$

where are:

$$\nu_1^2=\kappa_1^2-\alpha^2\,,\quad \nu_2^2=\kappa_2^2-\alpha^2\,,\quad \kappa_1^2=\frac{\omega^2}{c_1^2}\,,\quad \kappa_2^2=\frac{\omega^2}{c_2^2}\,. \tag{17}$$

The solutions of equations (16) are given in the expressions:

$$\begin{aligned}\Phi(r)&=AJ_0(\nu_1 r)+BN_0(\nu_1 r),\nu_1>0\\ \Psi(r)&=AJ_0(\nu_2 r)+BN_0(\nu_2 r),\nu_2>0\end{aligned} \tag{18}$$

where $J_0(\nu_1 r)$, $J_0(\nu_2 r)$, $N_0(\nu_1 r)$, $N_0(\nu_2 r)$, are Bassel's special functions.

The unknown constants A, B, C and D we determine from the boundary conditions that the stresses on the internal and external boundary surfaces of the cylinder are equal to zero i.e.:

$$\sigma_{rr}\big|_{r=R_1}=\left[2\nu\frac{\partial u_r}{\partial r}+\lambda\left(\frac{\partial u_r}{\partial r}+\frac{u_r}{r}+\frac{\partial u_r}{\partial z}\right)\right]\Bigg|_{r=R_1}=0\,,$$

$$\tau_{rz}\big|_{r=R_1}=\left[\nu\left(\frac{\partial u_r}{\partial z}+\frac{\partial u_z}{\partial r}\right)\right]\Bigg|_{r=R_1}=0\,,$$

$$\sigma_{rr}\big|_{r=R_2}=\left[2\nu\frac{\partial u_r}{\partial r}+\lambda\left(\frac{\partial u_r}{\partial r}+\frac{u_r}{r}+\frac{\partial u_r}{\partial z}\right)\right]\Bigg|_{r=R_2}=0\,, \tag{19}$$

$$\tau_{rz}\big|_{r=R_2}=\left[\nu\left(\frac{\partial u_r}{\partial z}+\frac{\partial u_z}{\partial r}\right)\right]\Bigg|_{r=R_2}=0\,.$$

If, under the boundary conditions (19), we replace the displacement components u_r and u_z given in expressions (9), and as the functions $f(r,z,t)$, $\psi(r,z,t)$, $\Phi(r)$ and $\psi(r)$,are determined by expressions (15) and (18) as well as by equations (13) and (16), we obtain, for the determination of unknown constants A, B, C and D, a system of four homogenous linear equations. By making the system determinant equal to zero we obtain the following frequency equation:

$$\begin{vmatrix} J_0''(\nu_1 R_1)-\frac{\lambda\kappa_1^2}{2\nu}J_0(\nu_1 R_1) & N_0''(\nu_1 R_1)-\frac{\lambda\kappa_1^2}{2\nu}N_0(\nu_1 R_1) & iJ_0''(\nu_2 R_1) & iN_0''(\nu_2 R_1) \\ i\,J_0'(\nu_1 R_1) & i\,N_0'(\nu_1 R_1) & \frac{1}{2}\left(\frac{\kappa_2^2}{\alpha^2}-2\right)J_0'(\nu_2 R_1) & \frac{1}{2}\left(\frac{\kappa_2^2}{\alpha^2}-2\right)N_0'(\nu_2 R_1) \\ J_0''(\nu_1 R_2)-\frac{\lambda\kappa_1^2}{2\nu}J_0(\nu_1 R_2) & N_0''(\nu_1 R_2)-\frac{\lambda\kappa_1^2}{2\nu}N_0(\nu_1 R_2) & iJ_0''(\nu_2 R_2) & iN_0''(\nu_2 R_2) \\ i\,J_0'(\nu_1 R_2) & i\,N_0'(\nu_1 R_2) & \frac{1}{2}\left(\frac{\kappa_2^2}{\alpha^2}-2\right)J_0'(\nu_2 R_2) & \frac{1}{2}\left(\frac{\kappa_2^2}{\alpha^2}-2\right)N_0'(\nu_2 R_2) \end{vmatrix}=0 \qquad (20)$$

The discussion of this equation is very complicated. Therefore we shall only determine the functional dependence of the phase velocity of spreading of harmonic waves inside the cylinder $c = \omega/\alpha$ as a function of relation between the interior and exterior radius and the wave length $\lambda_1 = 2\pi/\alpha$, and for the lowest own frequency.

$$\frac{c}{c_0} = f\left(\frac{R_1}{\lambda_1}, \frac{R_2}{\lambda_1}\right), \quad c_0 = \sqrt{\frac{\mathrm{E}}{\rho}} \qquad (21)$$

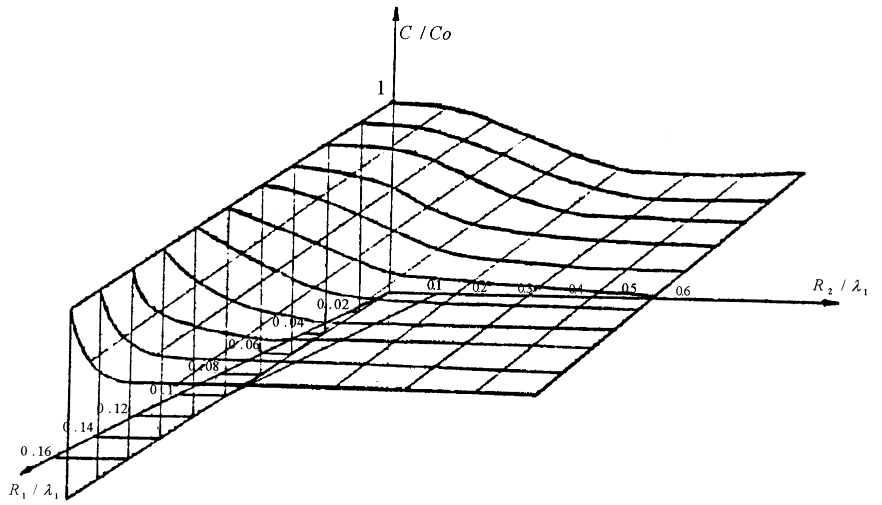

Fig. 2

Fig. 2 shows the relation $\frac{c}{c_0}$ as the function of $\frac{R_1}{\lambda_1}$ and $\frac{R_2}{\lambda_1}$ for the material **Č1531** of the elasticity module $\mathbf{E} = 2{,}973 \cdot 10^4\,\mathrm{kN/cm^2}$ of the mass density $\rho = 7850\,\mathrm{kg/m^3}$ and Poisson modules $\mu = 0{,}29$.

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UTICAJ POPREČNIH DIMENZIJA NA BRZINU PROSTIRANJA LONGITUDINALNIH TALASA U OSNO-SIMETRIČNOM ELASTIČNOM TELU

Katica (Stevanović) Hedrih, Predrag Kozić, Ratko Pavlović

Brzina longitudinalnih talasa u cilindričnom šupljem štapu određena je kao brzina beskonačnih talasa u funkciji tri promenljive: Poisson-ovog koeficijenta, odnosa unutrašnjeg i spoljašnjeg poluprečnika prema talasnoj dužini. Ova funkcija je sračunata za najnižu frekvenciju u oblasti argumenata koji imaju fizički značaj za primenu. Data je geometrijska interpretacija numeričkog eksperimenta uticaja poprečnih dimenzija na prostiranje brzine longitudinalnih talasa u cilindričnom šupljem štapu.