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RATCHETTING PHENOMENON AT LOW STRAIN RATES FOR AISI 316H STAINLESS STEEL

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Abstract. *The paper deals with 3D viscoplastic strain of a rectangular block made of AISI 316H austenitic stainless steel. One of its sides is loaded by constant normal stress whereas two lateral side surfaces are acted upon by harmonically variable shear stress. it is experimentally observed that such temporal variation induces progressive but saturated increase of axial strain in the direction of tension stress components. The strain rate is of the order of 0.001 [s^{-1}]. The problem is treated by two constitutive models:*
a) Chaboche`s model with incorporated evolution equations for back stress and equivalent flow stress (its 8 material constants are take from [6]) and
b) the model explained in [15], based on tensor representations where plastic stretching is second order polynomial in stress and linear in plastic strain (having 6 material constants). Comparison with experiments has shown superioity of the second model.

1. INTRODUCTION

Steel mantel of nuclear reactors composed of austenitic stainless steels is exposed during its exploitation to time dependent stress-strain histories.

As a consequence, three typical types of response could appear:

- Incremental collapse is characteriyed by an increase of the mean stress curve up to fracture. Deviations around this curve are steady changing with unperturbed frequency.
- Low cycle fatigue takes case with a constant mean stress and the other features are the same as in the previous case of behaviour.
- Elastic vibrations (*shakedown*) appearing as a consequence of the corresponding decrease of amplitudes of plastic strain vibrations. Hence, behaviour of the material body is such that stress tensor enters elastic region i.e. interior of the yield surface in the stress space.

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The case of plastic saturation when stress frequency in "universal" flow curve diagram (i.e. equivalent Mises stress versus accumulated plastic strain) increases is of special interest. Such a behaviour is called *ratchetting*. It has been shown to take case at multiaxial stress-strain histories especially at nuclear reactors where such histories usually appear.

The paper deals with the problem of constant normal stress and a harmonically variable shear stress. Ratchetting predictions of the models presented in papers [3] and [15] are compared. Relevant stress amplitudes and shear stress frequency are chosen in such a way to be comparable with ratchetting experiments reported in [7]. The equivalent plastic strain rate is of the order of magnitude of 10^{-3} [s^{-1}] which corresponds to low strain rates.

2. Preliminary considerations and problem statement

For finite elastoplastic strains it is commonly accepted that aside of undeformed configuration B_0 and deformed current configuration B there exists a local reference configuration of natural state elements B_n. Then, Kroners`s decomposition rule [1,2] holds in the following form:

$$\mathbf{F}_P := \mathbf{F}_E^{-1}\mathbf{F} \tag{1}$$

where are:

$\mathbf{F}$ - deformation gradient tensor,

$\mathbf{F}_E$ - the elastic distortion tensor and

$\mathbf{F}_P$ - the plastic distortion tensor,

determined by the mappings $B_0 \to B$, $B_n \to B$ and $B_0 \to B_n$ respectively. Let us apply the polar decomposition theorem on the plastic distortion tensor by means of

$$\mathbf{F}_P = \mathbf{R}_P\mathbf{U}_P = \mathbf{V}_P\mathbf{R}_P$$

where $\mathbf{R}_P^{-1} = \mathbf{R}_P^T$ holds for the plastic rotation tensor.

Then as an invariant measure of plastic strain the Hill`s logarithmic tensor

$$\boldsymbol{e}_P = \ln \mathbf{V}_P = 0.5\ln(\mathbf{F}_P\mathbf{F}_P^T) \tag{2}$$

is chosen. Its principal advantage lies in the fact that it is a deviatoric tensor. In other words, its three principal invariants read:

$$\pi_1 = tr\boldsymbol{e}_P = 0\,, \quad \pi_2 = tr\boldsymbol{e}_P^2 \neq 0\,, \quad \pi_3 = tr\boldsymbol{e}_P^3 \neq 0\,, \tag{3}$$

if plastic volume change is neglected. Then plastic stretching tensor equals:

$$D_P = \frac{1}{2}\frac{d}{dt}\begin{Bmatrix} 2\ln(1+e_P) & 0 & 0 \\ 0 & -\ln(1+e_P) & \gamma_P \\ 0 & \gamma_P & -\ln(1+e_P) \end{Bmatrix} \tag{4}$$

determined by the symmetric part of plastic "velocity gradient" tensor

$$\mathbf{L}_P = \dot{\mathbf{F}}_P \mathbf{F}_P^{-1} .$$

As usual the superimposed dot stands for material time derivative.

Consider now a problem of three-dimensional straining of a rectangular parallelepiped. Two of its opposite sides are acted upon by constant normal stresses whereas on two lateral sides two harmonically variable shear stresses act. The loading scheme is shown at the following figure.

Cauchy stress tensor and Piola-Kirchhoff stress tensor now read:

$$T = \begin{Bmatrix} \sigma & 0 & 0 \\ 0 & 0 & \tau \\ 0 & \tau & 0 \end{Bmatrix}, \quad \mathbf{S} = \mathbf{F}_E^{-1} \mathbf{T} \mathbf{F}_E^{-T} \approx \mathbf{T} \tag{5}$$

where

$$\sigma = const , \quad \tau = \tau_0 \sin(\omega t) .$$

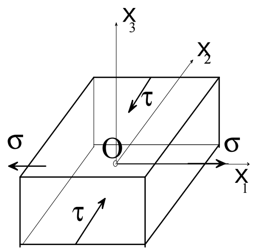

Fig. 1. Loading of the considered body

It should by underlined that the second of the above tensors is defined with respect to local reference configuration B_n and that for small elastic strains

$$\|\mathbf{F}_E\| \approx 1$$

if elastic rotations are also very small. On the other hand, the plastic distortion tensor equals

$$F_P = \begin{Bmatrix} 1 & 0 & 0 \\ 0 & 1 & \gamma_P \\ 0 & 0 & 1 \end{Bmatrix} \begin{Bmatrix} \chi_P^2 & 0 & 0 \\ 0 & \chi_P^{-1} & 0 \\ 0 & 0 & \chi_P^{-1} \end{Bmatrix} \begin{Bmatrix} \chi_P^2 & 0 & 0 \\ 0 & \chi_P^{-1} & \gamma_P \chi_P^{-1} \\ 0 & 0 & \chi_P^{-1} \end{Bmatrix}, \tag{6}$$

where notation $\chi_P^2 \equiv 1 + \boldsymbol{e}_P$ is applied. For a given stress history $\mathbf{T}(t)$ a response of the material body determined by plastic strain history is looked for. We apply additional condition on stress history

$$\frac{d}{dt} \|\mathbf{T}(t)\| \le 16 \ [\text{MPa/s}]$$

in order to keep strain rates in the low range of the order of magnitude $\le 10^{-3}$ [s^{-1}].

3. MODEL OF CHABOCHE

For a description of a viscoplastic body Chaboche .3. has generaliyed Perzyna`s model to account for an evolution of the residual stress. His evolution equations might be written as follows:

$$\dot{\boldsymbol{e}}_P = \left\langle \frac{F - D}{k} \right\rangle^n \frac{\mathbf{T}_d - \mathbf{B}_d}{F}, \quad F = \|\mathbf{T}_d - \mathbf{B}_d\|, \tag{7}$$

$$\dot{\mathbf{B}}_d = c(A\mathbf{D}_P - \mathbf{B}_d \dot{p}) - \Gamma \|\mathbf{B}_d\|^{m-1} \mathbf{B}_d, \tag{8}$$

$$\dot{p} = \sqrt{\frac{2}{3}} \|\mathbf{D}_P\|, \tag{9}$$

$$\dot{D} = b(Q - D)\dot{p}, \tag{10}$$

where:

$\dot{\boldsymbol{e}}_P \approx \mathbf{D}_P$ - the plastic strain rate (approximately equal to plastic stretching for small strains range),

$\mathbf{B}$ - the residual stress (back stress tensor),

D - a static equivalent flow stress,

p - the accumulated plastic strain scalar and

$A_c = \{k, n, c, A, \Gamma, m, b, Q\}$ - a set of material constants to be determined from experiments.

Here the traditional notations for a second tensor intensity as well as its deviatoric part are employed

$$\|\mathbf{A}\| = (tr\mathbf{A}^2)^{0.5} \equiv (\mathbf{A} : \mathbf{A})^{0.5} \quad , \quad A_d = \mathbf{A} - \frac{1}{3}\mathbf{1}tr\mathbf{A},$$

while plastic loading indicator function is determined by means of $\langle x \rangle = 0$ if $x \geq 0$ or 0 if $x < 0$.

An identification of material constants has been made in [6] on the basis of experiments for standard cyclic tension-compression test as well as cyclic torsion test. Their values for AISI 316H as reported in [6] read:

$$A_c = \{68.38, 5.8, 65, 113.33, 8.7 \cdot 10^{-10}, 1.3, 8.8, 220.45\} \tag{11}$$

For the problem presented in previous section, deviatoric residual stress (cf. also .6.) equals to:

$$B_d = \frac{1}{3} \begin{Bmatrix} 2B_L & 0 & 0 \\ 0 & -B_L & 3B_T \\ 0 & 3B_T & -B_L \end{Bmatrix}, \tag{12}$$

such that

$$F = \sqrt{\frac{2}{3}} \sqrt{(\sigma - B_L)^2 + 3(\tau - B_T)^2} \tag{13}$$

We are giong to consider a relatively small cycle number. This makes possible to neglect the material constant Γ. This simplifies the set (7)-(10) such that it becomes:

$$\dot{\varepsilon}_P = \frac{2}{3}\left\langle \frac{F-D}{k} \right\rangle^n \frac{\sigma - B_L}{F}, \quad \dot{\gamma}_P = 2\left\langle \frac{F-D}{k} \right\rangle^n \frac{\tau - B_T}{F}, \tag{7'}$$

$$\dot{B}_L = c\left(\frac{3}{2}A\dot{\varepsilon}_P - \dot{p}B_L\right), \quad \dot{B}_T = c\left(\frac{1}{2}A\dot{\gamma}_P - \dot{p}B_T\right) \tag{8'}$$

$$\dot{p} = \sqrt{\dot{\varepsilon}_P^2 + \frac{1}{3}\dot{\gamma}_P^2}\,, \tag{9'}$$

$$\dot{D} = b(Q-D)\dot{p}\,, \tag{10'}$$

where we introduced the notation $\varepsilon_P \equiv \ln(1+e_P) \approx e_P$ such that $\dot{\varepsilon}_P \approx \dot{e}_P$.

It is worthy of note that Chaboche`s model is developed for small strains only. Taking this fact into account we may simplify logarithmic strain tensor (2) into the following form:

$$\boldsymbol{e}_P = \ln \mathbf{V}_P \approx \frac{1}{2}\begin{Bmatrix} 2e_P & 0 & 0 \\ 0 & -e_P & \gamma_P \\ 0 & \gamma_P & -e_P \end{Bmatrix}. \tag{14}$$

The evolution equations (7') - (10') have been integrated numerically for a simulatd stress controlled test with normal stress $\sigma = 245$ [MPa], shear stress amplitude $\tau_0 = 75$ [MPa] and shear stress frequency $\omega = 0.5$ [rad/s] for 10 stress cycles. Such special choice of history parameters is made in order to acquire results comparable with experimental data reported in [7]. These experiments have been made with AISI 316H with constant tension of 250 [MPa], maximal shear strain of 2.6% and amplitude of shear strain equal to 0.17%. Number of experimental cycles was 10.

Result of integration of evolution equations (7') - (10') with material constants (11) and $\Gamma = 0$ are presented on the following figure. Aside of time-plastic strain components plots the so-called universal flow curve (i.e. equivalent stress versus accumulated plastic strain) as well as phase portrait of plastic strain components are given. The densification feature characteristic for ratchetting has been observed at the "universal" flow curve.

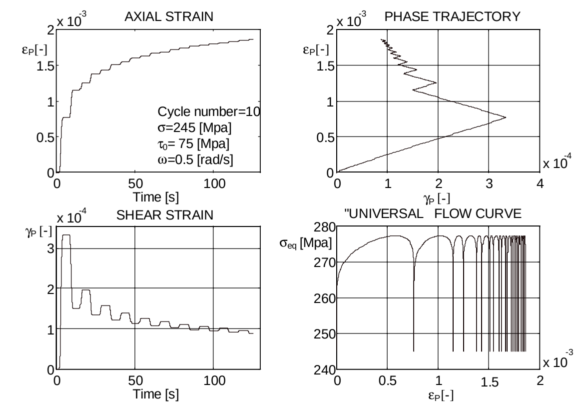

Fig. 2. Ratchetting under stress control simulated by Chaboche`s model

4. AN OVERSTRESS MODEL WITH RICE-ZIEGLER NORMALITY

As a new model let us take the generalization of Rice`s model by means of tensor representation approach as given in [15]. Rice`s model [14] by itself is based on normality of plastic strain rate tensor on a loading function $\mathbf{\Omega}$ taking account of microstructural rearrangements during plastic straining. Translated to finite strains and with Ziegler`s modification derived from the notion of least irreversible force, the evolution equation reads:

$$\mathbf{D}_P = \Lambda \frac{\partial \Omega}{\partial \mathbf{S}} \tag{15}$$

where the microstructural rearrangements are supposed to be completely determined by the plastic strain tensor given by (2).

An essential generaliyation of this model was given in [15]. In this paper the loading function was assumed to depend on the following set

$$\Omega = \Omega(\gamma), \quad \gamma \equiv (S_2, S_3, \mu_1, \mu_3) \tag{16}$$

of proper and mixed invariants of stress and plastic strain tensors

$$S_2 = tr\mathbf{S}_d^2, \quad S_3 = tr\mathbf{S}_d^3,$$

$$\mu_1 = tr\mathrm{S}_d \boldsymbol{e}_P, \quad \mu_3 = tr\mathbf{S}_d^2 \boldsymbol{e}_P. \tag{17}$$

If we take for plastic stretching a second order approximation in stress and linear approximation in plastic strain, we get the following formula

$$\Lambda^{-1}\mathbf{D}_P = (A_1 + A_3\mu_1)\mathbf{S}_d + A_2(\mathbf{S}_d^2)_d + \frac{1}{2}A_3 S_2 \boldsymbol{e}_P + A_4(\mathbf{S}_d \boldsymbol{e}_P + \boldsymbol{e}_P \mathbf{S}_d)_d \tag{18}$$

where again subscript "d" denotes the deviatoric part of a second rank tensor. Let us introduce the overstress tensor

$$\delta\mathbf{S} \equiv \mathbf{S} - \mathbf{S}^*. \tag{19}$$

Moreover, we need to know direction of S^*. Let us assume that dynamic stress $\mathbf{S}$ as well as static stress S^* have the same direction. This is justified by the hypothetical nature of the static stress namely it is just a projection of the dynamic stress tensor to the yield surface in the stress space. Hence

$$\mathbf{S}_d = \sqrt{S_2}\mathbf{M}, \quad \mathbf{S}_d^* = \sqrt{S_2^*}\mathbf{M}, \tag{20}$$

where a unit tensor $\mathbf{M}$ with the property $tr\mathbf{M}^2 = 1$ is introduced. Let us denote the initial static yield by Y_0 i.e. $Y_0^2 = S_{2_0}^*$ and a relative equivalent overstress by means of

$$\Delta = h_{eq} - h_{eq}^* \equiv \sqrt{\frac{S_2}{S_{2_0}}} - \sqrt{\frac{S_2^*}{S_{2_0}^*}}. \tag{21}$$

The scalar coefficient Λ is obtained from the condition that plastic stretching vanishes when overstress equals to zero. Then evolution equation takes its final form:

$$\mathbf{D}_P = \Delta[A_1\mathbf{S}_d + A_4(\mathbf{S}_d\boldsymbol{e}_P + \boldsymbol{e}_P\mathbf{S}_d)_d] + \Delta^2\left[A_2(\mathbf{S}_d^2)_d + A_3\left(\mu_1\mathbf{S}_d + \frac{1}{2}S_2\boldsymbol{e}_P\right)\right] \tag{22}$$

On the basis of experimental data for tension and shear of AISI 316H [16] material constants of this model are here found to read

$$A_c = \{A_1,\ A_2,\ A_3,\ A_4,\ \lambda_1,\ \lambda_2\} = \{0.01181,\ 0.01033,\ -0.008012,\ -0.01978,\ 20,\ 0.2\} \tag{23}$$

by means of Nelder-Mead method of analyzing chi-square functional. The two new constants λ_1 and λ_2 appearing in (23) arrive from the usual approximation of static equivalent flow stress:

$$h_{eq}^* = (1+\lambda_1 p)^{\lambda_2} \tag{24}$$

where p is the accumulated plastic strain. The governing set of differential equations is obtained from (22) by replacing constants (23) into it. Its integration for the given stress history produced the results shown on the following figure.

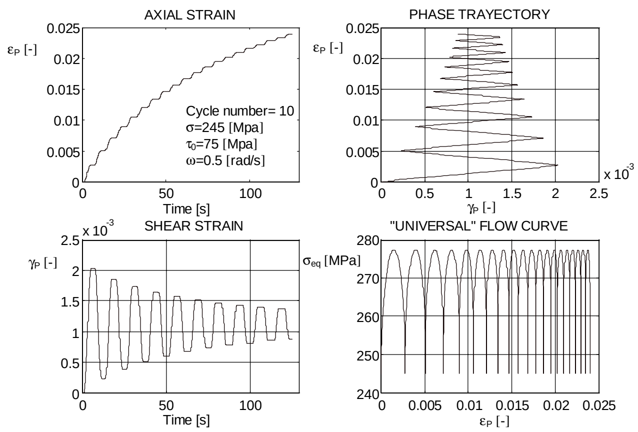

Fig. 3. Ratchetting with stress control for overstress model with generalized Rice-Ziegler normality

5. COMPARISONS AND CONCLUDING REMARKS

In papers [7, 8, 9, 10] the authors have investigated ratchetting behaviour at combined tension-torsion tests. On the other hand, the papers [12] deals with tension tests only. For the comparison purposes the paper [7] is especially convenient since its loading programme was such that after 10 cycles the maximal normal strain was 2.6% at the corresponding tension stress of 250 [MPa] and a prescribed harmonically changed shear strain whose amplitude was 0.17%. Of course, the specimens tested were made of the stainless steel AISI 316H.

Examining situation of the end point of the phase trajectory calculated by the Chaboche model (figure 2) we see that at these conditions the maximal normal strain has the value of 0.18% which means that the predicted strain is approximately 15 times smaller than the corresponding experimentally acquired strain.

On the other hand, the phase trajectory depicted on the figure 3. gives the maximal tension strain equal to 2.4%. Therefore the relative error amounts to 8%.

A short conclusion could be formulated as follows:

- Although handicapped by the absence of compression data which are essential for a good cyclic behaviour prediction, the tensor representation model [15] has been shown to cover multiaxial variable stress-strain histories in a surprisingly good way.
- An eventual inclusion of the residual stress tensor into (22) would even improve the predicting abilities of the tensor representation based viscoplasticity models. This depends on available experimental data bank and not on theoretical considerations.
- Both models for the given number of stress cycles do not forecast a commence of elastic strain vibrations i.e. the shakedown behaviour (cf. also [13]). However the model of Chaboche aside of poor predictions gives the results very near to these vibrations.

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POJAVA RAČETINGA KOD ČELIKA AISI 316H PRI MALIM BRZINAMA DEFORMACIJE

Milan Mićunović, Dejan Veljković

U radu se razmatra problem 3D viskoplastične deformacije paralelopipeda sačinjenog od nerđajućeg austenitnog čelika AISI 316H (prema SAE-oznakama). Njegove dve suprotne strane su napadnute konstantnim normalnim naponom, dok na dve suprotne bočne strane deluje harmonijski promenljiv smičući napon. Eksperimentalno je utvrđeno da takva vremenska promena napona izaziva progresivni ali zasićeni priraštaj uzdužne deformacije u pravcu dejstva normalnog napona. Korišćena su sledeća dva konstitutivna modela:

a) Chaboche-ov model sa uključenim evolucionim jednačinama za rezidualni tenzor napona i ekvivalentni napon tečenja (sa 8 materijalnih konstanata koje su uzete iz [6]),

b) model objašnjen u [15], zasnovan na tenzorskoj reprezentaciji gde je tenzor brzine plastične deformacije aproksimiran polinomom drugog stepena po naponu, a linearan je po plastičnoj deformaciji. Model sadrži 6 materijalnih konstanata koje su u ovom radu kalibrisane na osnovu eksperimenata [16].

Upoređenje sa eksperimentima je pokazalo superiornost drugog modela za opisivanje višeosnih vremenski promenljivih naponsko-deformacionih istorija.