UNIVERSITY OF NIŠ
The scientific journal **FACTA UNIVERSITATIS**
Series: **Mechanics, Automatic Control and Robotics** Vol.2, No 9, 1999 pp. 877 - 886
Editor of series: Katica (Stevanovi}) Hedrih, e-mail: katica@masfak.masfak.ni.ac.yu
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http:// ni.ac.yu/Facta

COMBINED TORSIONAL – LATERAL VIBRATION OF BEAMS UNDER VEHICULAR LOADING. I: FORMULATION AND SOLUTION TECHNIQUES

UDC 531 531.17 534.12 620.10

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Abstract. *The present study deals with the problem of the combined torsional-lateral vibration of beams with open monosymmetric cross-section under the effect of a moving vehicle. The formulation presented is applied on a simply supported beam excluding damping, but without particular additional mathematical difficulties it can be also used for the relevant dynamic analysis of continuous beam structures including energy dissipation. After examining in detail the free vibration of the beam, the moving vehicle is represented not only as a constant load moving with constant velocity across the span, but as a two-mass spring vehicle model and the corresponding forced motions are dealt with, using modal analysis in conjunction with approximate integration procedures and numerical schemes.*

1. INTRODUCTION

The linearized as well as nonlinear vibration analysis of beams or beam-like structural elements has been and continues to be the subject of numerous researches, since it embraces a wide class of problems with immense importance in engineering science. Depending on the assumptions adopted, the type of analysis used, the kind of the loading or excitation and the overall beam characteristics, a variety of different approaches have been reported in the literature and a great number of both theoretical and experimental findings are related to beam dynamics. In particular, studies dealing with the 3D motions of beams have revealed the dominant role of nonlinear modal coupling due to the exchange of energy between torsional and (in-plane and out-of-plane) flexural motions [3-5,7], leading to interesting primary and combination resonance phenomena [6,20], not

Received September 8, 1999

predictable by linear theory. Moreover, vehicle-induced vibration of bridges and other structures that can be simulated as beams and the effect of various parameters, such as suspension design, vehicle weight and velocity, damping, matching between bridge and vehicle natural frequencies, deck roughness etc., on the dynamic behavior of such structures have been extensively investigated by a great number of researchers [2, 9-12, 14-19]. The whole matter will undoubtedly remain a major topic for future scientific research, due to the fact that continuing developments in design technology and application of new materials with improved quality enable the construction of lighter and more slender structures, vulnerable to dynamic and especially moving loads.

To the knowledge of the authors however, only a limited number of works refer to the flexural-torsional vibration of beams under vehicular loading, starting from the pioneer work of Heilig [13]. At this point one must quote the paper by Chatterjee et al [2], where a more detailed reference on the foregoing problem can be found.

The present work deals with the derivation of equations of the combined lateral-torsional motion of a simply supported monosymmetric open cross-section beam, under vehicular loading and presents solution techniques, based on modal analysis and approximate numerical schemes; damping is not accounted for in the whole procedure, but the proposed methodology can be easily extended to multi-span beams including energy dissipation, without the need to overcome severe mathematical obstacles. Numerical results based on this theoretical formulation and corresponding discussion will be presented in a companion paper due.

2. Mathematical formulation

2.1. Free vibration

Let us consider at first the free torsional-lateral vibration of a simply supported beam, made of a homogeneous, linearly elastic material with an open cross-section and only one symmetry axis, as shown in Fig. 1, where also the structure geometry and the corresponding sign convention are depicted. The differential equations governing the motions under consideration, excluding damping, are given by the following set [3, 4, 10, 13]:

$$\begin{cases} EJ_y w'''' + m\ddot{w} = 0 \\ EJ_z \upsilon'''' + EJ_z z_M \vartheta'''' + m\ddot{\upsilon} = 0 \\ EC_S \vartheta'''' + EJ_z z_M \upsilon'''' - GJ_d \vartheta'' + \Theta_M \ddot{\vartheta} = 0 \end{cases} \tag{1}$$

where the prime denotes differentiation with respect to x, while the dot with respect to time t. The cross-sectional and material properties as well as displacement components involved in the above equations are defined as:

J_y, J_z moments of inertia with respect to the principal axes y and z, J_d Saint-Venant torsional moment of inertia, E and G elasticity and shear modulus respectively, υ and w the deformation of gravity center S along axes y and z, Θ_M the polar moment of inertia of the cross-section mass, m is the beam's mass per unit length, θ the rotation of the cross-section and C_S the warping coefficient with respect to S.

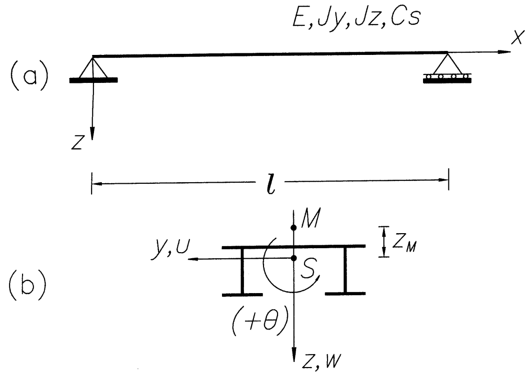

Fig. 1. Geometry, properties and sign convention of a simply supported beam with open monosymmetric cross section.

In this manner, from Eqs. (1), it is evident that the vertical eigen-vibration is independent form the horizontal flexural and torsional ones, which are related to a common modal amplitude. Thus, applying modal analysis, one may comprehensively write:

$$\begin{cases} w(x,t)=\overline{w}(x)(A_k \sin\omega_k t + B_k \cos\omega_k t) \\ \vartheta(x,t)=\overline{\vartheta}(x)(A_\sigma \sin\omega_\sigma t + B_\sigma \cos\omega_\sigma t) \\ \upsilon(x,t)=\overline{\upsilon}(x)(A_\sigma \sin\omega_\sigma t + B_\sigma \cos\omega_\sigma t) \end{cases} \tag{2}$$

Consequently, the expression of the in-plane lateral dynamic deflection w(x,t) is given by the following relation

$$\begin{cases} w(x,t)=\sum\limits_n \sin\dfrac{n\pi x}{\ell}(A_{kn}\sin\omega_{kn}t + B_{kn}\cos\omega_{kn}t) \\ \omega_{kn}^2=\dfrac{n^4\pi^4 EJ_y}{m\ell^4} \quad , \quad n=1,2,.... \end{cases} \tag{3}$$

On the other hand, combining the last two coupled equations given in (1) we get

$$\overline{\upsilon}=\frac{1}{mz_M\omega_\sigma^2}(-EC_S\overline{\vartheta}'''' + EJ_z z_M^2\overline{\vartheta}'''' + GJ_d\overline{\vartheta}'' + \Theta_M\omega_\sigma^2\overline{\vartheta}) \tag{4}$$

and since $C_M = C_S - z_M^2 J_z$

after cumbersome elaboration, we reach to an 8th order differential equation with respect to the shape function of the rotation $\theta(x,t)$, outlined below:

$$\alpha\overline{\vartheta}^{(8)} + \beta\overline{\vartheta}^{(6)} + \gamma\overline{\vartheta}^{(4)} + \delta\overline{\vartheta}^{(2)} + \varepsilon\overline{\vartheta} = 0 \tag{5}$$

where

$$\begin{cases} \alpha = -E^2 J_z C_M \\ \beta = \mathrm{EGJ}_z J_d \\ \gamma = EJ_z \Theta_M \omega_\sigma^2 + EC_M m\omega_\sigma^2 + EJ_z z_M^2 m\omega_\sigma^2 \\ \delta = -GJ_d m\omega_\sigma^2 \\ \varepsilon = -\Theta_M m\omega_\sigma^4 \end{cases} \tag{6}$$

which yields the latter characteristic algebraic equation

$$\alpha\rho^8 + \beta\rho^6 + \gamma\rho^4 + \delta\rho^2 + \varepsilon = 0 \tag{7}$$

In the sequel, the expression valid for the shape function of the rotation $\overline{\vartheta}(x)$ is in series form as follows:

$$\overline{\vartheta}(x) = \sum_i \mathrm{k}_i e^{a_i x}[\sin(b_i x) + \cos(b_i x)] + \sum_\lambda k_\lambda e^{r_\lambda x} \tag{8}$$

where $a_i \pm \mathrm{j}b_i$ and $\pm r_\lambda$ are the complex conjugate and real roots of (7) respectively, with k_i and k_λ being appropriate coefficients sought, provided that $i + \lambda = 8$. The boundary conditions associated with Eq.(5) are

$$\begin{cases} \overline{\vartheta}(0) = \overline{\vartheta}(\ell) = \overline{\vartheta}''(0) = \overline{\vartheta}''(\ell) = 0 \\ \overline{\upsilon}(0) = \overline{\upsilon}(\ell) = \overline{\upsilon}''(0) = \overline{\upsilon}''(\ell) = 0 \end{cases} \tag{9}$$

Substituting expressions (8) into Eq.(7) and taking into account the aforementioned boundary conditions, we reach to a linear homogeneous system of eight equations with respect to coefficients kq ($q = 1 \div 8$). For a nontrivial solution the corresponding determinant is set equal to zero, leading to the so-called frequency equation. Thus, one may write

$$\begin{cases} \overline{\vartheta}(x,t) = \sum_n \Psi_n (A_{\sigma n} \sin \omega_{\sigma n} t + B_{\sigma n} \cos \omega_{\sigma n} t) \\ \overline{\upsilon}(x,t) = \sum_n Z_n (A_{\sigma n} \sin \omega_{\sigma n} t + B_{\sigma n} \cos \omega_{\sigma n} t) \end{cases} \tag{10}$$

where Ψ_n and Z_n are the shape functions of the rotation and out-of-plane deflection respectively, to be analytically determined or at least properly approximated.

In as much as, it can be rather easily proven that the orthogonality conditions governing the free motion considered are as follows:

$$\begin{cases} \int_0^\ell X_n X_m dx = 0 \ \text{ for } n \neq m \\ \Theta_M \int_0^\ell \Psi_n \Psi_m dx = 0 \ \text{ for } n \neq m \end{cases} \tag{11}$$

2.2. Forced motion under a constant moving load acting eccentrically

For this particular problem, schematically depicted in Fig. 2, the corresponding equations of motion take the form

$$\begin{cases} EJ_y w'''' + m\ddot{w} = P_z \delta(x-\alpha) \\ EJ_z \upsilon'''' + EJ_z z_M \vartheta'''' + m\ddot{\upsilon} = 0 \\ EC_S \vartheta'''' + EJ_z z_M \upsilon'''' - GJ_d \vartheta'' + \Theta_M \ddot{\vartheta} = eP_z \delta(x-\alpha) \end{cases} \quad \text{where } \alpha = vt \text{ (load position)} \quad (12)$$

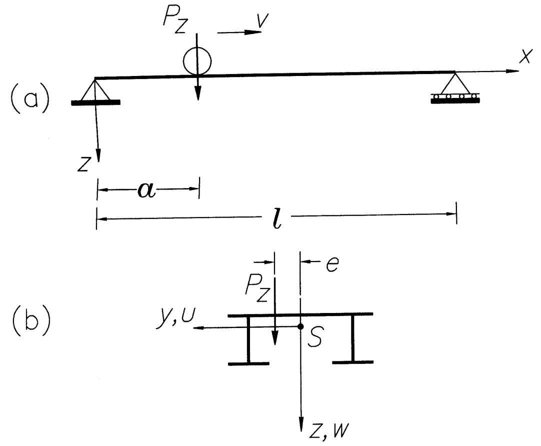

Fig. 2. The simply supported beam of Fig. 1, under the passage of a constant load moving with constant speed across the span eccentrically.

and the desired solution can be written in modal form as

$$\begin{cases} w(x,t) = \sum_n X_n(x) T_n(t) \quad , \quad X_n(x) = \sin\dfrac{n\pi x}{\ell} \\ \vartheta(x,t) = \sum_n \Psi_n(x) \Phi_n(t) \\ \upsilon(x,t) = \sum_n Z_n(x) \Phi_n(t) \end{cases} \quad (13)$$

where the amplitudes $T_n(t)$ and $\Phi_n(t)$ are the solution of the system

$$\begin{cases} \ddot{T}_n(t) + \omega_{kn}^2 T_n(t) = \dfrac{2P}{m\ell} \sin\dfrac{n\pi vt}{\ell} \\ \ddot{\Phi}_n(t) + \omega_{\sigma n}^2 \Phi_n(t) = \dfrac{eP_z}{m\int_0^\ell Z_n^2 dx + \Theta_M \int_0^\ell \Psi_n^2 dx} \Psi_n(vt) \end{cases} \quad (14)$$

yielding according to Duhamel the following purely analytical expressions

$$\begin{cases} T_n(t) = \dfrac{2P_z}{m\ell\omega_{kn}} \displaystyle\int_0^t \sin\frac{n\pi v\tau}{\ell} \sin\omega_{kn}(t-\tau)d\tau \\ \Phi_n(t) = \dfrac{eP_z}{\omega_{\sigma n}\left(m\displaystyle\int_0^\ell Z_n^2 dx + \Theta_M \int_0^\ell \Psi_n^2 dx\right)} \displaystyle\int_0^t \Psi_n(v\tau)\sin\omega_{\sigma n}(t-\tau)d\tau \end{cases} \tag{15}$$

2.2. Forced motion under a two-mass spring vehicle model acting eccentrically

Using the geometrical relations resulting from the configuration, which can be perceived from Fig. 3, one may write

$$M\ddot{z} = -k(z - w_{PA}) \tag{16}$$

and

$$\begin{cases} P_z = Mg - M\ddot{z} + m_0 g - m_0 \ddot{w}_{PA} \\ w_{PA} = w(\alpha) + e\vartheta(\alpha) \end{cases} \tag{17}$$

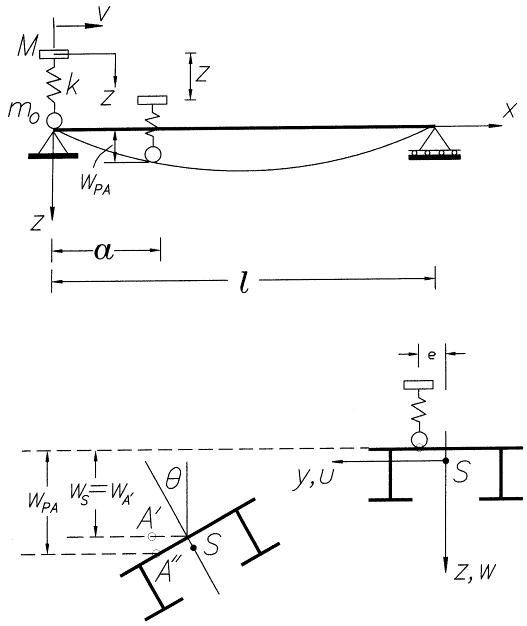

Fig. 3. The simply supported beam of Fig. 1, acted upon eccentrically by a two-mass vehicle model moving with constant velocity.

Seeking in the same manner a series solution as in the former problem, given in (13) and after some elaboration it is found that the formulation cannot lead to an analytical solution; this is due to vibration interactions of strongly nonlinear nature, although a mainly linearized approach is adopted. Moreover it is evident that

$$\begin{cases} \ddot{z} + \omega_p^2 z = \omega_p^2 [\mathrm{w}(\alpha) + \mathrm{e}\vartheta(\alpha)] \\ \omega_p^2 = k / M \end{cases} \tag{18a}$$

leading to

$$\ddot{z} + \omega_p^2 z = \omega_p^2 \sum_n \{X_n(\alpha) T_n(t) + e\Psi_n(\alpha)\Phi_n(t)\} \tag{18b}$$

Considering a solution of the form

$$z = \sum_n \Pi_n(t) \tag{19}$$

we get

$$\ddot{\Pi}_n + \omega_p^2 \Pi_n = \omega_p^2 [X_n(\alpha) T_n(t) + e\Psi_n(\alpha)\Phi_n(t)] \tag{20}$$

Using once again the Duhamel solution of (20), evaluating the 1st and 2nd derivative of Πn with respect to t and taking into account the lemma of integral calculus dictating that

$$\varphi(t) = \int_0^t K(t,\tau)\theta(\tau)d\tau \Rightarrow \frac{d\varphi(t)}{dt} = K(t,\tau)\theta(t) + \int_0^t \frac{\partial K(t,\tau)}{\partial t}\theta(\tau)d\tau$$

the final product of a lengthy manipulation is the system of D.E. of motion that follows

$$\ddot{T}_n + \omega_{kn}^2 T_n = \frac{2}{m\ell} \left\{ \begin{array}{l} A_n X_n(vt) - m_0[X_n^2(vt)\ddot{T}_n(t) + eX_n(vt)\Psi_n(vt)\ddot{\Phi}_n(t)] - \\ -M[\omega_p^2 X_n^2(vt) T_n(t) - \omega_p^3 X_n(vt)\int_0^t X_n(v\tau)T_n(\tau)\sin\omega_p(t-\tau)d\tau] - \\ -M[\omega_p^2 e\Psi_n(vt)X_n(vt)\Phi_n(t) - \omega_p^3 eX_n(vt)\int_0^t \Psi_n(v\tau)\Phi_n(\tau)\sin\omega_p(t-\tau)d\tau] \end{array} \right\} \tag{21a}$$

$$\ddot{\Phi}_n + \omega_{\sigma n}^2 \Phi_n =$$

$$\frac{1}{m\int_0^\ell Z_n^2 dx + \Theta_M \int_0^\ell \Psi_n^2 dx} \left\{ \begin{array}{l} A_n \Psi_n(vt) - m_0[X_n(vt)\Psi_n(t)\ddot{T}_n(t) + e\Psi_n^2(vt)\ddot{\Phi}_n(t)] - \\ -M[\omega_p^2 X_n(vt)\Psi_n(vt)T_n(t) - \omega_p^3 \Psi_n(vt)\int_0^t X_n(v\tau)T_n(\tau)\sin\omega_p(t-\tau)d\tau] - \\ -M[\omega_p^2 e\Psi_n^2(vt)\Phi_n(t) - \omega_p^3 e\Psi_n(vt)\int_0^t \Psi_n(v\tau)\Phi_n(\tau)\sin\omega_p(t-\tau)d\tau] \end{array} \right\} \tag{21b}$$

This system can alternatively be written as

$$A_n = -\frac{2}{n\pi} g(M + m_0)(\cos n\pi - 1) \tag{22}$$

$$\ddot{\Pi}_n + \omega_p^2 \Pi_n = \omega_p^2 [X_n(\alpha) T_n(t) + e\Psi_n(\alpha)\Phi_n(t)] \tag{23}$$

$$\ddot{T}_n + \omega_{kn}^2 T_n = \frac{2}{m\,\ell} P_{zn} X_n(\alpha) \tag{24}$$

$$\ddot{\Phi}_n + \omega_{\sigma n}^2 \Phi_n = \frac{1}{m\int_0^{\ell} Z_n^2 dx + \Theta_M \int_0^{\ell} \Psi_n^2 dx} eP_{zn}\Psi_n(\alpha) \tag{25}$$

where

$$P_{zn} = A_n - M\ddot{\Pi}_n - m_0[X_n(\alpha)\ddot{T}_n + e\Psi_n(\alpha)\ddot{\Phi}_n] \tag{26}$$

and is associated to the set of initial conditions given by:

$$z = \sum \Pi_n = 0\,,\;\; \dot{z} = \sum \dot{\Pi}_n = 0\,,\;\; T_n = \dot{T}_n = 0\,,\;\; \Phi_n = \dot{\Phi}_n = 0\,. \tag{27}$$

3. Solution techniques

3.1. Free motions

From the preceding theoretical formulation it is more than perceivable that the problem of primordial concern is to determine the highest possible range of the eigenfrequencies $\omega_{\sigma n}^2$, i.e. to solve the frequency equation resulting from (7) and (8) and the use of boundary conditions (9). The analytical form of this equation and moreover the values of the coefficients of the corresponding 8th order determinant (being in fact functions of $\omega_{\sigma n}$) can only be treated via reliable symbolic mathematics software, since not only the expressions are extremely complicated and lengthy, but the elimination of not desired solution combinations (exclusion of negative real and complex roots) requires repeated manipulations of the aforementioned expressions. In addition to the above, after elimination is complete, one must then compute the value of the related 8th determinant, a task that must be performed several times, before the fundamental eigenfrequency is properly approximated, with the accuracy sought. Recapitulating, the solution technique for evaluating $\omega_{\sigma n}$ consist of the following primary steps:

a. Choose a first approximation of $\omega_{\sigma n}$ being close to the minimum of the corresponding out-of-plane flexural and torsional ones, with the motions considered uncoupled.
b. Perform symbolic manipulations and eliminations - form the 8th order system.
c. Compute the value of the determinant $\Delta(\omega_{\sigma n})$.
d. Repeat steps a, b and c using $\omega_{\sigma n} + h$, yielding $\Delta(\omega_{\sigma n} + h)$.
e. Apply the classical method of Bolzano until a root is found.

Further details and more analytical presentation of the whole method and its inherent capabilities as well as a variety of numerical results will be given in the companion paper due.

3.2. Forced motions

After the evaluation of the desired range of eigenfrequencies and corresponding shape functions has been performed, there exist no mathematical difficulties in dealing with the forced motions described throughout Eqs.(13)-(15), since the solutions are purely analytic. This is not the case however for the combined torsional-lateral vibration of the beam due to the passage of a two-mass spring vehicle model acting eccentrically, because in this case the differential equations with respect to the modal amplitudes $\Phi_n(t)$, $T_n(t)$ and

$\Pi_n(t)$ are coupled and strongly non-linear, and hence analytical solutions are not accessible. Thus, one must resort to approximate numerical integration procedures and tackle the problem via a straightforward dynamic analysis. In doing this, after introducing the quantities

$$\begin{aligned}
\alpha_1 &= 1 + m_0 R X_n^2(\alpha) \\
\beta_1 &= m_0 \operatorname{Re} X_n(\alpha)\Psi_n(\alpha) \\
\gamma_1 &= R X_n(\alpha)\{A_n - M\omega_p^2[X_n(\alpha)T_n(t) + e\Psi_n(\alpha)\Phi_n - \Pi_n] - \omega_{kn}^2 T_n\} \\
\alpha_2 &= m_0 G_n e X_n(\alpha)\Psi_n(\alpha) \\
\beta_2 &= 1 + m_0 G_n e^2 \Psi_n^2(\alpha) \\
\gamma_2 &= G_n e \Psi_n(\alpha)\{A_n - M\omega_p^2[X_n(\alpha)T_n + e\Psi_n(\alpha)\Phi_n - \Pi_n] - \omega_{\sigma n}^2 \Phi_n\}
\end{aligned}$$

and consecutive substitutions and elaboration, we finally reach to the following system of 2nd order differential equations with respect to the aforementioned amplitudes.

$$\begin{cases}
\ddot{\Pi}_n(t) = \omega_p^2[X_n(vt)T_n(t) + e\Psi_n(vt)\Phi_n(t) - \Pi_n(t)] \\
\ddot{T}_n(t) = \dfrac{\gamma_1\beta_2 - \beta_1\gamma_2}{\alpha_1\beta_2 - \beta_1\alpha_2} \\
\ddot{\Phi}_n(t) = \dfrac{\alpha_1\gamma_2 - \gamma_1\alpha_2}{\alpha_1\beta_2 - \beta_1\alpha_2}
\end{cases}$$

This can be treated numerically as a system of six (6) first order O.D.E.s; the method employed herein - with results presented in the companion paper due - is the specially modified 7th order Runge-Kutta-Verner integration scheme, that produces reliable long-term results, since it combines efficiency, low programming effort, usage of minimal computer time and a very small error $O(\hbar^7)$, where $\hbar$ is the corresponding time step.

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SPREGNUTE TORZIONO-BOČNE OSCILACIJE GREDE OPTEREĆENE VOZILOM: I FORMULACIJE I TEHNIČKA REŠAVANJA

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Predstavljeni su rezultati izučavanja problema spregnutih torziono-bočnih oscilacija greda sa otvorenim monosimetričnim poprečnim presekom, pod dejstvom pokretnih vozila. Predstavljena formulacija je primenjena na prosto oslonjenu gredu, ali bez posebnih dodatnih matematičkih teškoća može biti upotrebljena za relevantna dinamičke analize kontinualnih grednih struktura uključujući i energiju disipacije. Posle detaljnog ispitivanja slobodnih oscilacija greda, pokretno vozilo je predstavljeno ne samo kao konstantno opterećenje pokretano konstantnom brzinom duž raspona, nego i kao model vozila od dve mase spojene oprugom i odgovarajućim prinudnim kretanjem, koristeći modalnu analizu sa aproksimacionim integralnim procedurama i numeričkim šemama.