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REVIEW OF A BOOK
PREPRINCPLES OF MECHANICS
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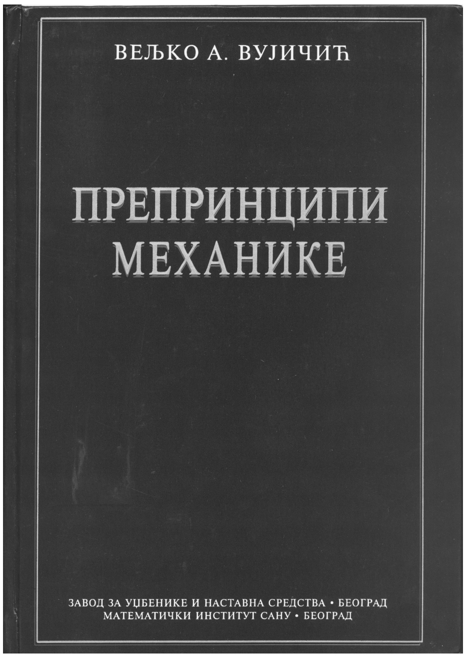

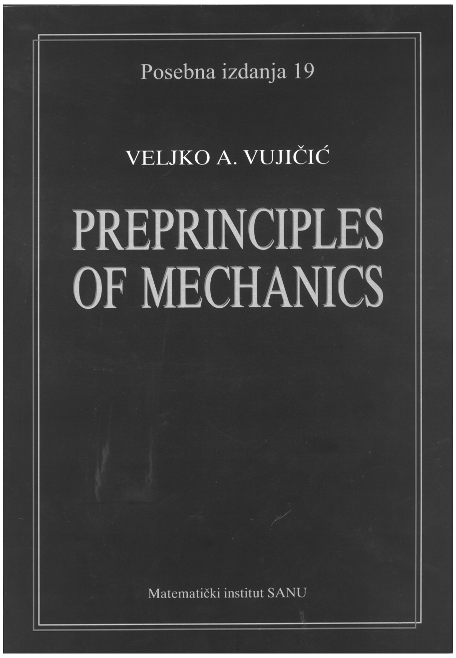

AFTERWORD

The title of this monograph as well as the selection of the given contents for each of its sections can be regarded as an introduction to more comprehensive works in the field of mechanics. What is, mostly and briefly, given here, though, in the author's opinion, are only essential assertions of one theory of motion and interaction of bodies. Not a priori assertion, but inherited, existing and acquired knowledge was the starting point. The acquired knowledge suppressed some inherited and existing logical and mathematical standards thus making relative the accuracy of the most accurate natural science in the mathematical sense. This can particularly be seen in the following relations with variable constraints:

Standard	Modification
Velocity	
$v = \frac{\partial r}{\partial q}\dot{q} + \frac{\partial r}{\partial t}$	$v = \frac{\partial r}{\partial q}\dot{q} + \frac{\partial r}{\partial q^0}\dot{q}^0$
$\dot{q} = (\dot{q}^1, \dots, \dot{q}^n)^T$	$\dot{q} = (\dot{q}^0, \dot{q}^1, \dots, \dot{q}^n)^T$
Motion impulse	
$p_i = a_{ij}\dot{q}^j + b_i$	$p_i = a_{ij}\dot{q}^j + a_{i0}\dot{q}^0$
$p_0 =: -H$	$p_0 = a_{0j}\dot{q}^j + a_{00}\dot{q}^0$
Acceleration	
$a^i = \frac{D\dot{q}^i}{dt}$	$a^i = \frac{D\dot{q}^i}{dt},\ a^0 = \frac{D\dot{q}^0}{dt}$
Forces	
Q	$Q,\ Q_0$

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Work

$$W(Q) = \int_c Q dq \qquad W = \int_s (Q dq + Q_0 dq^0)$$

Variational principles

$$\delta \int_{t_0}^{t_1} E_k dt = 0, \ \delta \int_{t_0}^{t_1} L dt = 0 \qquad \delta \int_{t_0}^{t_1} [W(Q) - W(I)]\, dt = 0$$

Kinetic energy

$$E_k = \frac{1}{2} a_{ij} \dot{q}^i \dot{q}^j + b_i \dot{q}^i + c \qquad E_k := -W(I) = \frac{1}{2} a_{\alpha\beta} \dot{q}^\alpha \dot{q}^\beta$$

$$|a_{ij}|_n^n \qquad |a_{\alpha\beta}|_{n+1}^{n+1}$$

Dieferential equations of motion

$$a_{ij} \frac{D\dot{q}^j}{dt} = 0 \qquad \begin{aligned} a_{ij} \frac{D\dot{q}^j}{dt} + a_{i0} \frac{D\dot{q}^0}{dt} &= Q_i \\ a_{0j} \frac{D\dot{q}^j}{dt} + a_{00} \frac{D\dot{q}^0}{dt} &= Q_0 \end{aligned}$$

or

$$\frac{D}{dt}\left(\frac{\partial L}{\partial \dot{q}^i}\right) = 0 \qquad \begin{aligned} \frac{D}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}^i}\right) &= 0 \\ \frac{D}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}^0}\right) &= 0 \end{aligned}$$

or

$$\begin{aligned} \dot{q}^i &= \frac{\partial H}{\partial p_i}, \ \dot{q}^0 = 1 \\ \dot{p}_i &= -\frac{\partial H}{\partial q^i}, \ \dot{p}_0 = -\frac{\partial H}{\partial t} \\ p_0 &= -H \end{aligned} \qquad \begin{aligned} \dot{q}^i &= \frac{\partial E}{\partial p_i}, \ \dot{q}^0 = \frac{\partial E}{\partial p_0} \\ \dot{p}_i &= \frac{\partial E}{\partial q^i}, \ \dot{p}_0 = \frac{\partial E}{\partial q^0} \\ p_0 &\neq -H \neq E \end{aligned}$$

Regarding these and similar comparisons the author has been posed some important and logical questions at various scientific conferences, namely questions like "Do you find assertions made so far in the standard mechanics erroneous?" or "Assuming that your assertions are correct, how do you explain that they have not been noticed in practice?"

Avoiding the word "erroneous" the author has replied that the assertions made in this theory are better and more thorough. From Aristotle and Galileo, that is, Newton, the contention has been accepted that the body moves uniformly under the action of constant force. When Newton wrote his first axiom or law that the body moves uniformly or rectilinearly in absence of forces, philosophy considered and assessed that Aristotle's view was erroneous. Such a rough assessment was not given by Newton; neither did Einstein state that the proposition about rectilinear motion was erroneous; instead, Einstein found a more complete and finer statement that "rectilinear motion does not spring from experience either logically or experimentally". Example (E5.7) simply gives an answer to the second question - though such object of mechanics is taken into consideration without including axial forces; therefore, there is always a possibility of displacing one end in practice; in other words, it has been more than just noticed in practice. In this monograph the mathematical knowledge that can be applied to the theory about motion of body is extended; and thus, some other views of particular attributes of motion appear. The innovations with respect to describing known and accepted relations are stressed in more details. Thus, for instance, the concept of the *material point* is differentiated in details from the concept of the *particle* or *covariant integration* from *standard integration* of differential equations of the rigid body's rotary motion. It has been shown that the model of material point can be used to develop the theory applicable to all mechanical objects.

The section on **Preprinciples** that precedes the core of the book gives an explicit determination of the starting conjunction in mechanics as well as its basic concepts such as *mass, distance* and *time*; this defines its domain of research by means of three disjunctive sets of real numbers and pencils of three oriented vectors; the concept of geometrical spaces is abandoned, unlike that of the body volume; the possibility of two particle's coincidence is excluded, namely, the fact that, in the geometrical sense, differs the concept of the particle from both the material and the geometrical point while, at the same time, makes the "law of non-penetration" redundant. The possibility of determining motion is accepted in advance, while the accuracy is made relative by available knowledge of the relevant natural parameters about some moment of rest. The knowledge about motion and rest of the body in mechanics, described by mathematical relations in various coordinate systems, is made relative - by the precondition of invariance that the natural attributes of motion do not depend upon the formal way of description. Therefore, the preprinciples objectify the subject of the theoretical mechanics while, at the same time, they make relative its general knowledge; they are accompanying corrector and verificator of all the assertions of the body motion theory.

The first section dealing with the **Basic Definitions** introduces and defines only four concepts by means of which it is possible to elaborate further one theory of the body motion. In accordance with the preprinciples, it was necessary in the beginning to open up the problem of selecting base oriented vectors, invariable in time. Unlike the velocity definition by means of the boundary values of distances, what is avoided in the velocity definition is the boundary transfer of one vector to another and thus the standard definition of velocity is accepted as a natural derivative of velocity with respect to time. In describing *motion impulse* the importance of the inertia tensor and of its difference from the geometrical metric tensor is especially stressed. This definition, just like the others, remains in the whole later theory which excludes from the present discussion the motion impulse as negative energy (Hamilton's function), that is, work of the forces. The term "motion impulse" is used instead of "impulse" in order to stress its difference from the forces' impulse. The definition of the *inertia force* determines a dimension of the force in general which later becomes prominent at the introduction and dimensioning of various dynamic parameters, as well as formulating the laws of dynamics.

The second section of the **Laws of Dynamics** gives to the concept of the "law" a unique meaning of the force's determinant; this makes it considerably different from the concept of Newton's laws; it is due to it that the concept of *law* in mechanics is strictly differentiated from the concepts of *principles* and *theorems*. The dominant place in this section belongs to the law of constraints by which it is stressed that the constraint between material points or particles can be abstracted by forces, that is, that the constraints are sources of the forces' origins, so that the mathematical or mental relation implied in the concept of the constraint should necessarily be distinguished from the motion of mechanically and objectively existing constraint.

In the part entitled *On Mutual Attraction Force* formula (2.21) is derived, from which the Newton's law of gravity follows for some particular conjunctions. By dropping determinants of other forces, that is, of the laws of dynamics (for the sake of brevity), the newly-introduced concept of the law of dynamics is not brought into doubt.

The third section entitled **Principles of Mechanics** comprises four principles on the basis of which (meaning, of each of them) it is possible to develop the whole theory about the body motion. The *equilibrium principle* is most comprehensively described with the good arguments, though it is based on the least number of definitions and consequential determinations. It is sufficient enough to comprise all the body motions coupled with any constraints in any coordinate systems. The consequential effect of the coupled forces' moment at the system of material or dynamic points subdued to the constraints is shown. From this principle the necessity to generalize the formula of the gravitational force has followed or the need to doubt the validity of the differential equations of motion with the constraints' multipliers.

By introducing an additional definition of the concept of work the *work principle* is formulated. Unlike the vector invariant of the equilibrium principle, the work principle is expressed by means of the scalar invariant thus avoiding the difficulties in summing up the constrainted vectors. As a consequence, beside potential energy, "rheonomic pseudopotential" also appears as negative work of the constraint-changing force; that is why it is shown that kinetic energy is a negative work of inertia force. In a unique way *elementary works* upon real displacements, possible displacements as well as work upon possible variations are characterized. By introducing an additional coordinate - rheonomic coordinate - the principle of the rheonomic constraints' solidification is abandoned, so that the work principle relation is extended for one adequate addend. This was preceded by modification of the constraints' variations, as well as work of the mechanical system with rheonomic constraints.

The concept of action is defined by means of the concept of work;

the concept of action is the object of the general integral variational principle called the *principle of action*. Therefore, the statement of the action principle required six basic definitions. For such formulation of the principle and with the unique concept of variation, the classical integral variational principles appear as corollaries. Since by the preprinciple of existence time is taken as an invariable, it does not vary as such; thus, this integral principle shows itself to be invariant upon the extended configurational manifolds TM^n and T^*M^n as well as for scleronomic systems upon TM^{n+1} and T^*M^{n+1}; in other words, on the relations which are of the same shape for autonomous and non-autonomous systems. A more essential meaning of this principle is expressed in the section IV which proves the theorem on optimal control of motion.

On the basis of the first four definitions and the compulsion definition the differential variational *principle of compulsion* is expressed that, in essence, scalarizes the vector invariant of the equilibrium principle. By describing compulsion as a homogeneous quadratic form of the acceleration vector coordinate over the inertia tensor the possibility of its transformation into any coordinate system has been proved. From the principle's requirement that compulsion has the least value on actual motion, it is easy to arrive at simple scalar differential equations of motion expressed by the compulsion function.

The section on **Theorems of Mechanics**, states clearly, first of all, what is implied by the "theorem" in mechanics. By means of the natural derivative with respect to time the theorem on motion impulse change and the theorem on kinetic energy change are proved; both theorems, in accordance with to the preprinciples, have invariant sense and they differ from the accepted assertions of the analytical mechanics. This becomes obvious when using the example of the change of impulse of the rigid body's rotary motion by which the derivatives with respect to time of the inertia tensor coordinates are developed. The theorem on controllable motion and optimal motion control that comprise all the mechanical systems connect the control theory with its basic roots of the analytical mechanics.

The fifth section, namely, **Motion Determination** by Analisis and Solutions of the Relations of Motion is mostly devoted to unextended covariant integration, to the first integrals and to the covariant integrals; Poisson's' brackets are extended for rheonomic systems. A brief, but sufficiently clear description of energy integral modification is given.

The final part is the sixth section entitled **Stability of Motion and Rast** by which accuracy and validity of the differential equations of motion are assessed depending on the observed dynamic or kinetic parameters. A special emphasis is paid to the thoughts of the highly distinguished Professor Nicolai Gurevich Chetaev concerning *false modernization*, namely the thoughts that are no less actual today; besides, not only general but covariant differential equations of disturbances are presented as well as the author's general criterion of stability of the equilibrium state and of the mechanical system motion.

This book is properly referred to as a monograph since it presents one theoretical entirety based on the authors' results published in scientific journals and monographs listed in References. This theory comprises all the mechanical systems which also include rigid and deformable bodies. The author's concept of the rheonomic coordinate's application to deformable bodies has been left out. It has been shown [67], [71] that deformable bodies can be represented as a system of material points with rheonomic constraints, so that deformable medium can be modeled by (3+1)-dimensional manifolds. Such mechanics would develop upon the derived deformation tensor

$$\varepsilon_{\alpha\beta} = \begin{pmatrix} \varepsilon_{00} & \varepsilon_{01} & \varepsilon_{02} & \varepsilon_{03} \\ \varepsilon_{10} & \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{20} & \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{30} & \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{pmatrix}$$

and metrics

$$ds^2 = \varepsilon_{\alpha\beta} dx^\alpha dx^\beta, \quad \alpha, \beta = 0, 1, 2, 3.$$

This metrics has invited quoting of the examples (E7) and (E8). Even more than that, it refutes, at the end of this book, any argument trying to prove that mechanics, as a science about motion of bodies, accomplished itself a long time ago; on the contrary, it stimulates new knowledge about motion and interaction of bodies.