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TRIANGULAR DISTRIBUTION AND PERT METHOD VS. PAYOFF MATRIX FOR DECISION-MAKING SUPPORT IN RISK ANALYSIS OF CONSTRUCTION BIDDING: A CASE STUDY

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Abstract. *Decision-making in construction bidding represents a complex process due to the present risk. Risk or uncertainty cannot be ignored and should be treated as a constituent of decision-making. The paper aims to emphasize the importance of probability theory by comparing insufficiently applicable methods in practical bidding. The triangular distribution and the PERT method belong to three-point estimate techniques, while payoff matrices represent a multi-criteria approach. Also, selected methods belong to quantitative techniques for risk cost analysis. Still, the risk costs determination of the unit costs and the total costs of bids is often based on an intuitive approach. Therefore, compared results of the triangular distribution, PERT method, and payoff matrix techniques (minimin, minimax, expected monetary value, and expected opportunity loss) indicate the significance of risk costs estimating in tendering. The analysis of the results showed some overlaps in risk costs values obtained by the PERT method and expected monetary value technique. Those are due to the specificity of the chosen practical example and cannot be adopted as a rule. This means that selected methods and techniques are very useful for all bid estimation. The paper proved the complexity of decision-making, where the primary goal is to award a contract.*

Key words: *bidding, risk, cost, unit cost, total cost, bid price*

1. INTRODUCTION

Bidding represents a complex process in the construction industry due to a significant number of factors that affect uncertainty and risks in decision-making. The uncertainty consideration in cost estimation benefits all parties involved in a tendering and contract realization. Cost risk and profit estimating must be considered in terms of the contract's specificities and the types of bill of quantities. Unit price contracts are the most commonly used in developing countries. However, all construction contracts based on unit prices have

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some parts with lump-sum items [1] - [3] Risk assessment in the bidding phase can be performed by quantitative and qualitative techniques. Quantitative techniques do not analyze risks mathematically in assessing the probability of occurrence. Instead, those use a professional approach to arbitrate the likelihood and impact on cost risk [0]. In addition, quantitative risk analysis provides a cost for each bill of quantity items. Also, it uses different techniques for cost risk applying in the bidding phase. Quantitative risk analysis in construction project management consists of three basic types [4]:

- technical performance analysis
- schedule risk analysis
- cost risk analysis

Therefore, the subject of this research is the comparative cost risk analysis that investigates the risk related to the unit cost of items from the bill of quantity. The comparative analysis was performed with the triangular distribution method, PERT method, and payoff matrix.

Construction contracts for projects like highways, railways, and other infrastructure are mostly complex and based on unit prices. This means each item from the bill of quantity is assigned with a specific unit price. The unit price consists of unit cost and profit. Besides, unit costs contain labor costs, material costs, construction machinery costs, and indirect costs. The unit cost estimation precedes the harmonization of construction norms, labor costs, and construction machinery costs per hour. Also, construction norms harmonization is in correlation with the project's specificity. Besides, this process directly affects unit costs and potential risks. Thus, construction norms and costs per hour were denoted as follows:

CNL - labor norm; hour per unit (h / (m^2, m^3, t, ...))
CNM - material norm; hour per unit (h / (m^2, m^3, t, ...))
$CNCM$ - construction machinery norm; hour per unit (h / (m^2, m^3, t, ...))
CHL - labor cost; monetary per hour (€ / h)
CHM - material cost; monetary per unit (€ / (m^2, m^3, t, ...))
$CHCM$ - construction machinery cost; monetary per hour (€ / h)
IF - indirect cost factor (project + company)

It is important to emphasize that capacity - q - is inversely proportional to the norm, i.e., q = 1/CN [5]. Based on the above mentioned, item unit costs - most likely - in the bill of quantity according to Eq. 1 are:

$$UC_i = (CNL_i \times CHL_i + CHM_i + CNCM_i \times CHCM_i) \times (1 + IF_i), \quad (€ / (m^1, m^2, m^3, t,...)) \quad (1)$$

According to Eq. 2, the total cost of an item is:

$$C_i = UC_i \times Q_i \qquad i = 1, 2, ..., n \tag{2}$$

Where:
Q_i – Quantity of an item (m^1, m^2, m^3, t,...)
n – Number of the bid items

According to Eq. 3, the total bid costs - BC - are:

$$BC = \sum_{i=1}^{n} C_i \tag{3}$$

According to Eq. 4, the final bid price - BP - consists of the total costs, increased by the profit rate - P -, and VAT.

$$BP = BC + P + VAT \tag{4}$$

The bidders can choose different profit rates of bid items after unit costs estimation. Also, an equal profit rate can be added to each item or group of items.

The contractor's relation to bidding risk is still based on the harmonization of the input cost with the project's specificity. In addition, contractual clauses are a substantial part of risk management and decision-making. It means that despite the norm harmonization, and the stated unit costs, variations are still present in estimating [5]. This problem can be solved by probabilistic methods, although resistance is still present in the practice bidding. Nevertheless, potential contractors began applying optimistic and pessimistic costs in the bidding phase. It means that unit costs were intuitively estimated (optimistic, most likely, and pessimistic). Such an approach led the author to analyzes and compares unit costs by applying probabilistic estimation methods in decision-making.

Variations frequently occur within the estimated construction norms due to insufficiently detailed geomechanical parameters and uncertain availability of construction components and systems. These variations directly affect unit costs and profit (unit prices). Whereas contractors still use deterministic techniques in estimating unit costs, an application of a triangular estimation and PERT method (Three-Point Estimate) can simplify the decision-making. Payoff matrices can also simplify the decision-making under cost variations and for an altered state of nature.

Methods choice was influenced by the traditional aversion to the application of probability theory in project management. Also, cost estimation methods are chosen for comparison and analysis and do not require special software and complex staff training.

2. SELECTED LITERATURE REVIEW

A three-point estimate is a valuable technique for cost risk estimation. This estimation technique involves the cost selection based on optimistic, pessimistic, and the most likely values. Two commonly used three points estimates are based on the triangular and beta distribution (PERT) [6] - [8].

The triangular distribution is commonly used as a subjective description with limited sample data. It is based on lower limit data, mode, and upper limit data. Also, the triangular distribution possesses the possibility of choosing a confidence interval, where the upper and lower limits can be exceeded within a predefined percentage [9] - [12]. The triangular distribution technique for the cost risk analysis was proposed first by J. M. Neil (1982) [13]. Also, one of the estimation techniques suggested in the Project Management Body of Knowledge (PMI) is the three-point estimate based on the triangular and beta distribution – PERT – method. Furthermore, the triangular probability distribution was used as a substitution for the normal distribution [13].

It is important to emphasize that the area under the triangular distribution represents the probability of the cost occurrence. Therefore, decision-making for each bid item and the total cost of the bid consists of four steps [13], [14]:

- estimating optimistic cost (o), pessimistic cost (p), and most likely unit costs (m) according to Eq. 1
- probabilities of project costs
- cumulative probabilities of unit costs
- finding the bid costs

Comparing obtained costs with associated probabilities to the general expression for three-point estimation of triangular distribution can be very practical and useful (Eq. 5) [15]:

$$C_i = (o_i + p_i + m_i)/3 \tag{5}$$

The PERT is a useful technique for cost risk estimating in bidding. Although developed for the American Polaris missile program in the 1947s, this technique has found application in all scientific areas for assessment of various data. The characteristic of this technique is basing on a beta distribution and an optimistic, pessimistic, and most likely assessment (time, cost risk). Also, PERT uses the Central Limit Theorem (CLT) for estimating cost risks with associated probabilities. The cost estimate of the bill of quantities items is determined, according to Eq. 6 [15] - [17].

$$C_i = (o_i + 4 \times p_i + m_i)/6 \tag{6}$$

C_i is estimated cost; o = optimistic estimate; p = pessimistic estimate; m = most likely estimate.

The standard deviation is determined, according to Eq. 7:

$$SD_i = (p_i - o_i)/6 \tag{7}$$

SD represents the standard deviation; p = pessimistic estimate; o = optimistic estimate.

Unit costs decision-making for each bid item, and the total cost of the bid consists of four steps [18] - [20]:

- decompose the project into items (bill of quantity)
- estimate the UC value and SD for each item
- calculate the BC value for the total project according to Eq. 3
- calculate the SDP value for the total project according to Eq. 8,

$$SDP = \sqrt{\sum_{i=1}^{n} SD_i} \tag{8}$$

The EC and SD values are used to convert the project estimates to confidence levels as follows:

- the confidence level for EC value +/-1.000 × SD is 68.27%
- the confidence level for EC value +/-1.150 × SD is 75.00%
- the confidence level for EC value +/-1.645 × SD is 90.00%
- the confidence level for EC value +/-2.000 × SD is 95.45%
- the confidence level for EC value +/-3.000 × SD is 99.73%

Given that multicriteria approaches are rarely used in practical cost estimation, payoff matrices can be a valuable tool for problem-solving with uncertainties [21] - [23]. This technique consists of five different strategies: finding the maximin or minimax, maximax or minimin, minimax regret, expected monetary value (EMV), and expected opportunity loss (EOL) of each item, and a total bill of quantities [24], [25]. Also, decision-making using payoff matrices consists of determining decision alternatives and states of nature [26], [27]. This technique is used in a quantitative and qualitative approach to problem-solving. Besides, in the quantitative approach, states of nature are economic, while in the qualitative decision-making, alternative weights are intuitively assigned [4].

The results of the payoff matrices provide the decision-maker with several possible choices. Also, the last step using the payoff matrix (EMV), which is based on probability, enables a more precise insight into the decision made consequences.

3. METHODOLOGY FOR ESTIMATING AND ANALYZING THE COMPARED COST RISK IN THE BIDDING PHASE

The proposed methodology has consisted of two basic's parts. Furthermore, parts are consisted of estimation of cost variations with associated probabilities according to the triangular distribution, PERT method, and Payoff matrix. Comparison and analysis of the obtained results are integral for both techniques. This can be indicated as in Fig. 1:

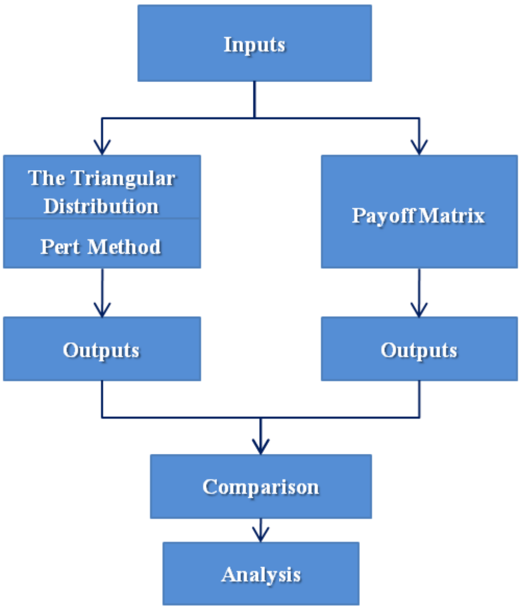


Fig. 1 General procedure of the proposed methodology

Although the triangular and beta distribution technique (PERT method) differs in terms of the most likely value, an identical number of steps are required to determine costs with associated probabilities. Respecting such specifics, the same algorithmic process of estimating item unit prices of the bill of quantities can be adopted.

A specific procedure/algorithm for the cost risk estimation under the triangular distribution and the PERT method (first part) is as shown in Fig. 2:

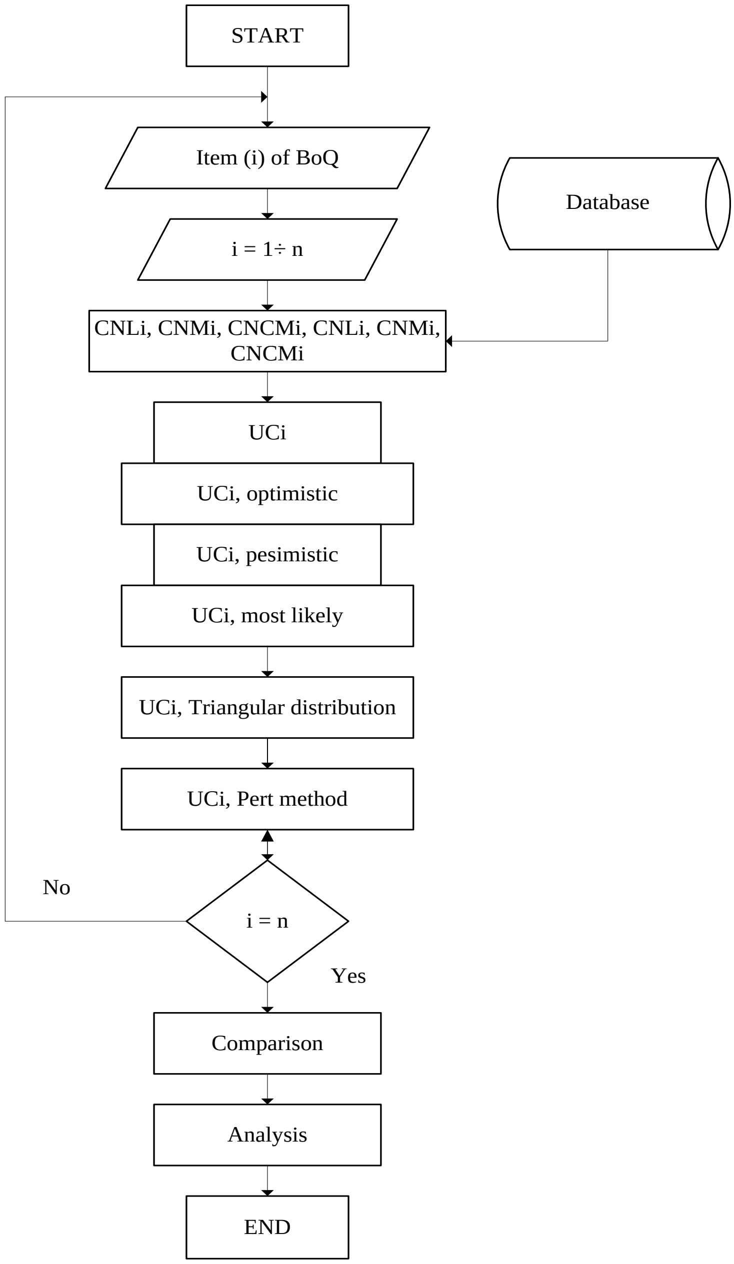


Fig. 2 Algorithm for the cost risk estimation, comparison, and analysis under the triangular distribution and the PERT method

The second part of the proposed methodology is cost risk assessment using payoff matrices. An algorithm of this technique is as shown in Figure 3:

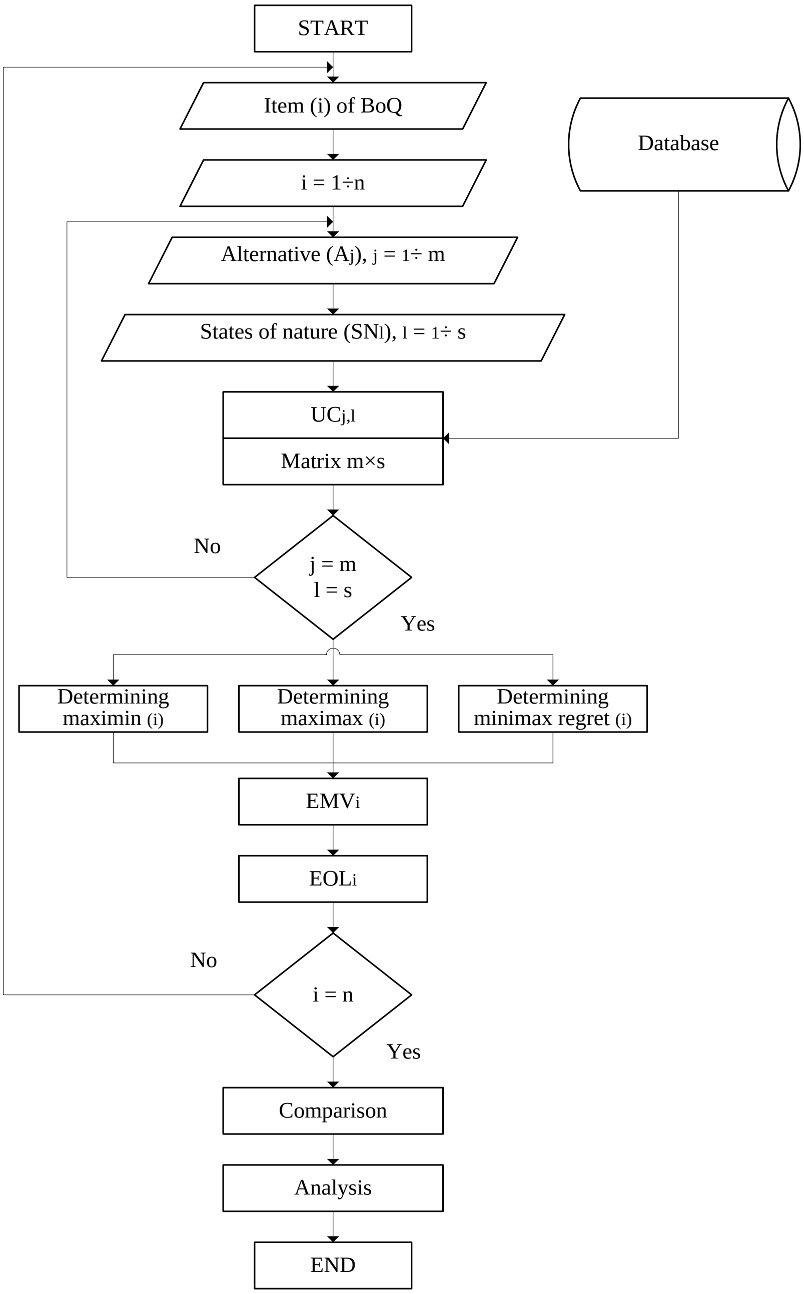


Fig. 3 Algorithm for the cost risk estimation, comparison, and analysis under the payoff matrices

The payoff matrices offer a wide range of possible solutions to the decision-maker. This feature is a consequence of the five strategies that are integral parts of the chosen technique – minimax, minimin, minimax regret, expected monetary value (EMV), and expected opportunity loss (EOL) [26], [27].

Minimax represents the lowest cost risk value of all maximum values for given alternatives and states of nature. Analogously, minimin is the lowest value among all alternative minimums. Besides, determining minimax regret represents the possible opportunity lost. Also, this cost risk estimation represents the determination of the minimum loss in the case of wrong decision-making. The final step in the payoff matrix technique is the expected monetary value (EMV). EMV is determined by summing the multiplied values of each state of nature with the probability of occurrence. This technique can be recognized as an alternative to a three-point estimation. The last step in estimating the cost risk using payoff matrices is the expected opportunity loss (EOL). Expected opportunity loss can be realized as a variation of the expected monetary value. An aim of this approach is minimizing the expected opportunity loss, rather than maximizing the expected monetary value. *Testing of the proposed methodology will be performed on a practical example from the bill of quantity in the bidding phase*.

4. ILLUSTRATIVE EXAMPLE

The proposed methodology was applied to the practical example of the pavement rehabilitation bill of quantity. Table 1 shows the items with quantities and estimated unit costs in the bidding.

Table 1 Part of the bill of quantity for asphalt pavement

Item	Text	Quantity	Unit	Unit Cost (€/m^2)			Total Cost (€)		
3	Asphalt Constructions			O	ML	P	O	ML	P
3.1	Apply AC 32 TS 50/70, thickness 12 cm	11000	m2	18.53	18.84	19.23	203,830.00	207,240.00	211,530.00
3.2	Apply AC 16 BS 25/55-55 A, thickness 6 cm	11000	m2	12.60	12.79	13.22	138,600.00	140,690.00	145,420.00
3.3	Apply AC 11 DS 25/55-55 A, thickness 4 cm	11000	m2	11.26	11.53	11.84	123,860.00	126,830.00	130,262.00
Net	Bid Sum						466,290.00	474,760.00	487,212.00

4.1 Probability Estimation by Triangular Distribution

The characteristic graph of the triangular distribution with item unit costs is as shown in Fig. 4.

The probability of occurrence (p) of each item is determined through the ratio areas of triangles 1 and 2, to the total area under the curve. This can be indicated by Eq. 9 and Eq. 10:

Probability (3.i.1) = Area of 3.i.1 / Total Area of item 3.i (9)

Respectively,

Probability (3.i.2) = Area of 3.i.2 / Total Area of item 3.i (10)

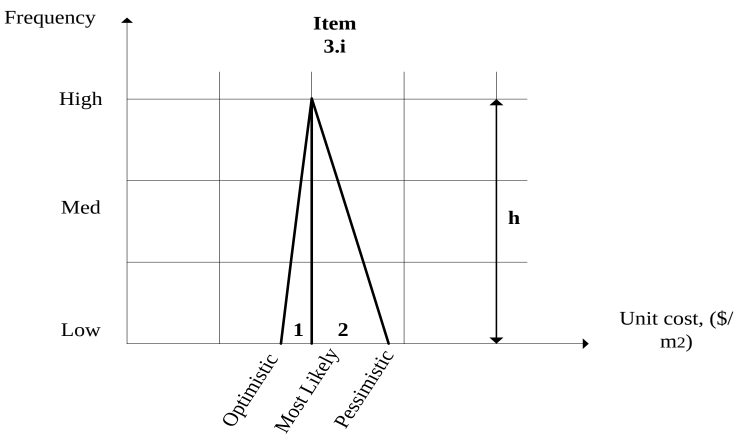


Fig. 4 Triangular distribution of the unit cost for item 3.i

The obtained values, according to Eq. 5 and Eq. 6 are as shown in Table 2:

Table 2 Probability of Occurrence

Triangle	Area	Probability of Occurrence (p)
3.1.1	0.5×(207,240.00 - 203,830.00) h = 1,705.00h	A3.1.1/(A3.1.1+A3.1.2) = 1,705.00/(1,705.00+2,145.00) = 0.44
3.1.2	0.5×(211,530.00 - 207,240.00) h = 2,145.00h	A3.1.2/(A3.1.1+A3.1.2) = 2,145.00/(1,705.00+2,145.00) = 0.56
3.2.1	0.5×(140,690.00 - 138,600.00) h = 1,045,00h	A3.2.1/(A3.2.1+A3.2.2) = 1,045.00/(1,045.00+2,365.00) = 0.31
3.2.2	0.5×(145,420.00 - 140,690.00) h = 2,365.00h	A3.2.2/(A3.2.1+A3.2.2) = 2,365.00/(1,045.00+2,365.00) = 0.69
3.3.1	0.5×(126,830.00 - 123,860.00) h = 1,485.00h	A3.3.1/(A3.2.1+A3.3.2) = 1,485.00/(1,485.00+1,716.00) = 0.46
3.3.2	0.5×(130,262.00 - 126,830.00) h = 1,716.00h	A3.3.2/(A3.2.1+A3.3.2) = 1,716.00/(1,485.00+1,716.00) = 0.54

4.1.1 Probabilities and Cumulative Probabilities of Project Cost

After finding the probabilities of occurrence for each area under the distribution curve, all possible combinations follow. Each combination is assigned by cost based on the midpoint and the corresponding probabilities of occurrence. These data are as shown in Table 3:

Table 3 Probability of Cost Occurrence for the Project

Combination of Item Zones	Cost Base on Midpoint (€)	Probability of Occurrence
A3.1.1+A3.2.1+A3.3.1	470,525.00	0.44 × 0.31 × 0.46 = 0.062744
A3.1.1+A3.2.1+A3.3.2	473,726.00	0.44 × 0.31 × 0.54 = 0.073656
A3.1.1+A3.2.2+A3.3.1	473,935.00	0.44 × 0.69 × 0.46 = 0.139656
A3.1.1+A3.2.2+A3.3.2	477,136.00	0.44 × 0.69 × 0.54 = 0.163944
A3.1.2+A3.2.1+A3.3.1	474,375.00	0.56 × 0.31 × 0.46 = 0.079856
A3.1.2+A3.2.1+A3.3.2	477,576.00	0.56 × 0.31 × 0.54 = 0.093744
A3.1.2+A3.2.2+A3.3.1	477,785.00	0.56 × 0.69 × 0.46 = 0.177744
A3.1.2+A3.2.2+A3.3.2	480,986.00	0.56 × 0.69 × 0.54 =0.208656
SUM		1.00

Data from Table 3 need to be arranged in ascending order of costs with corresponding probabilities. This is as shown in Table 4:

Table 4 Combined Probability of Cost Occurrence for the Project

Combination of Item Zones	Cost Based on Midpoint (€)	Probability of Occurrence
A3.1.1+A3.2.1+A3.3.1	470,525.00	0.0627
A3.1.1+A3.2.1+A3.3.2	473,726.00	0.0737
A3.1.1+A3.2.2+A3.3.1	473,935.00	0.1397
A3.1.2+A3.2.1+A3.3.1	474,375.00	0.0799
A3.1.1+A3.2.2+A3.3.2	477,136.00	0.1639
A3.1.2+A3.2.1+A3.3.2	477,576.00	0.0937
A3.1.2+A3.2.2+A3.3.1	477,785.00	0.1777
A3.1.2+A3.2.2+A3.3.2	480,986.00	0.2087
SUM		1.0000

The final costs of all possible combinations vary from €470,525.00 to €480,986.00, as shown in Table 4. The further step is determining the cost frequencies and cumulative probabilities. The following data are as indicated in Table 5:

Table 5 Probability of Project Cost Occurrence

Project Cost (€)	Frequency of Occurrence	Joint Probabilities	Cumulative Probability
465,850.00 (min)	-	0.0000	0.0000
470,525.00	1	0.0627	0.0627
473,726.00	1	0.0737	0.1364
473,935.00	1	0.1397	0.2761
474,375.00	1	0.0799	0.3559
477,136.00	1	0.1639	0.5199
477,576.00	1	0.0937	0.6136
477,785.00	1	0.1777	0.7913
480,986.00	1	0.2087	1.0000
487,212.00 (max)	-	0.0000	1.0000

The final costs and the project price determination are typically based on preliminary data and a key decision on profit rate.

4.1.2 Finding the Bid Cost and Bid Price

The target cost represents the total of the most likely cost of each bid item. For the analyzed part of the bill of quantity, the total cost is €474,760.00. For the stated target cost, according to Fig. 5, the confidence level is 0.3790. In cases where the contractors cannot accept a certain level of confidence, they can set the desired level. In this case, it can be assumed that the bidder will be satisfied with the ratio of 0.75:0.25, i.e., 3:1. The 75 % confidence level results in a higher cost than the target cost. In this case, the corresponding *confidence-limit cost* is *€477,746.00*. The difference between confidence limit cost and target cost is the *contingency fund*. In this case, it is €2,986.00.

If a profit rate of 7% is assumed, the bid price is according to Eq. 11:

$$\text{Bid price} = \text{Target cost} + \text{Contingency fund} + \text{Profit} \tag{11}$$

Bid price = 474,760.00 + 2,986.00 + (474,760.00 + 2,986.00) × 0.07 = €511,188.22

4.2 Probability Estimation by PERT Method

Unit costs and total costs for the analyzed practical example, according to Eq. 6, Eq. 7, Table 1, and a confidence level of 75% are as shown in Table 6:

Table 6 Estimating total item costs and project cost by PERT method

Item	Asphalt Constructions		Unit Cost (€/m2)			Total Cost (€)		
3	Quantity	Unit	Optimistic	Most Likely	Pessimistic	Basic	Min	Max
3.1	11000.00	m2	18.53	18.84	19.23	207,386.67	205,910.83	208,862.50
3.2	11000.00	m2	12.60	12.79	13.22	141,130.00	139,822.83	142,437.17
3.3	11000.00	m2	11.26	11.53	11.84	126,907.00	125,679.95	128,134.05
Net Bid Sum						475,423.67	473,101.51	477,745.83

According to Table 6, the total costs are within an interval of minimum and maximum value (€473,101.51 ÷ €477,745.83). The total bid price is determined by adopting a profit rate of 7%, to compare with the triangular distribution technique.

Min. bid price = 473,101.51 + (473,101.51 × 0.07) = €506,218.58

While the maximum bid price is:

Max. bid price = 477,745.83 + (477,745.83 × 0.07) = €511,118.04

4.3 Estimation by Payoff Matrix

The availability of the construction technology process is the most uncertain input in deciding the unit costs and total project costs. This feature of construction systems imposes the need to form more *alternatives* in the selection of the most likely scenario. Also, variations in average building norms have a significant impact on project cost estimates. These characteristics of construction production processes represent alternatives and criteria (*states of nature*) in the cost-risk analysis, using payoff matrices.

Three *alternatives*, for different availability values, are envisaged in the analyzed example. Also, three expected variations of building norms were selected as criteria (*states of nature*). The Table for the analyzed example should be created as is shown in Table 7:

Table 7 Alternative costs depending on variations of construction norms

Decision alternative	ACN × (1+0.05) SN1	Average Construction Norm ACN	ACN × (1- 0.05) SN3
System Availability - 0.99 (D1)	442,975.50	451,022.00	462,851.40
System Availability - 0.95 (D2)	466,290.00	474,760.00	487,212.00
System Availability - 0.90 (D3)	492,195.00	501,135.55	514,279.33

4.3.1. Determining Minimax

The first step of this technique is determining minimax. Minimax is the minimum cost among the maximum alternative values, as shown in Table 8. Minimax is €462,851.40 for decision alternative D1.

Table 8 Minimax alternative cost

Decision alternative	Minimum Payoff
System Availability – 0.99 (D1)	462,851.40
System Availability - 0.95 (D2)	487,212.00
System Availability - 0.90 (D3)	514,279.33

4.3.2. Determining Minimin

Minimin is the minimum cost among the minimum alternative values, as shown in Table 9. Minimin is €442,975.00 for decision alternative D1.

Table 9 Minimin alternative cost

Decision alternative	ACN × (1+0.05) SN1
System Availability - 0.99 (D1)	442,975.50
System Availability - 0.95 (D2)	466,290.00
System Availability - 0.90 (D3)	492,195.00

4.3.3 Determining Minimax Regret

Minimax regret represents the possible opportunity lost. The minimum of all maximum regrets is as shown in Table 9. Minimax regret is €0,000.00 for decision alternative D1.

Table 10 Minimax regret for cost risk occurrence

Decision alternative	ACN × (1+0.05) SN1	Average construction norm (ACN)	ACN × (1-0.05) SN3
System Availability - 0.99 (D1)	0.00	0.00	0.00
System Availability - 0.95 (D2)	23,314.50	23,738.00	24,360.60
System Availability - 0.90 (D3)	49,219.50	50,113.55	51,427.93

4.3.4. EMV

EMV could be the most appropriate approach because it takes into account the probabilities of event costs. This part of payoff matrices represents a very useful tool for well-experienced bidders. This means that the possibility of choosing probabilities could favorite the most likely scenario. The determined probabilities for SN1, SN2, and SN3 are 0.15, 0.75, and 0.1. EOL values are as shown in Table 11. The lowest cost assessment given the probabilities of all conditions is preferring the D1 alternative (*System Availability* = 0.99) for an EMV of €450,997.97.

Table 11 Expected monetary value for cost risk occurrence

EMV D_i	Total Cost (€)
EMV D1	450,997.97
EMV D2	474,734.70
EMV D3	501,108.85

4.3.5. EOL

Expected opportunity loss represents a variation of the expected monetary value. In this part, the probabilities of occurrence were multiplied by the minimax regret values from Table 9. This technique aims to minimize the expected opportunity loss, rather than maximizing the expected monetary value. The determined probabilities for SN1, SN2, and SN3 are 0.15, 0.75, and 0.1. EOL values are as indicated in Table 12. The minimum expected opportunity loss is €0.000.00 for alternative D1.

Table 12 Expected opportunity loss for cost risk occurrence

EOL D_i	Cost (€)
EOL D1	0,000.00
EOL D2	23,736.74
EOL D3	50,110.88

The results obtained using the payoff matrices indicate the importance of decision alternative D1 in the decision–making. It is crucially significant to emphasize before discussing and comparing results. Namely, the costs of alternative D1 for altered states of building norms are based on the system availability of 0.99999, i.e., 1.0. This availability of construction production systems is not likely in practical examples, although it is envisaged as one of the alternatives. The reason for such an approach is to emphasize the importance of the expected failure states in project realization, with the associated risks. This means that alternative D1 is set up to warn inexperienced project managers of certain system failures and their impact to cost risk estimation. Therefore, the obtained results must be corrected without taking into account alternative D1. Table 13 shows the already stated (incorrect) and corrected values of cost risk strategies within the payoff matrices.

Table 13 Payoff matrix - cost risk results

Payoff Matrix	Cost Risk (€)	Payoff Matrix	Cost Risk (€)
Results	Correct	Results	Incorrect
Minimax - D3	487,212.00	Minimax – D1	462,851.40
Minimin - D2	466,290.00	Minimin – D1	442,975.50
Minimax Regret - D3	25,905.00	Minimax Regret - D1	0,000.00
EMV - D2	474,734.70	EMV - D1	450,997.97
EOL - D2	23,736.74	EOL - D1	0,000.00

5. RESULTS COMPARISON AND DISCUSSION

After comparing the results of the triangular distribution and the PERT method, the next step is comparing the outputs with the payoff matrix results.

5.1. Triangular Distribution vs. Beta Distribution (PERT Method)

The total bid costs (BC) according to Eq. 5 for the triangular distribution, and Eq. 3 are €476,087.00. Similarly, the total bid costs according to Eq. 6 for the PERT method and Eq. 3 are €475,423.67. The total costs for the confidence level of 75% according to the triangular distribution (TD) and PERT method (PM) are *€477,746.00* (TD), i.e., interval (*€473,101.51 ÷ €477,745.83*) for PM. According to Eq. 5 and Eq. 6., the total costs of TD are higher than the total costs of PM by 0.14%. The total costs of both techniques for a confidence level of 75% are equal in the case of the upper limit of the PM interval. Namely, according to the PM and the lower limit of the interval, there is a possibility to choose the costs of €473,101.51 ÷ €477,745.83. This means that those costs should be reduced by €4,644.32 compared to the stated costs. The bid costs, according to Fig. 5 are within the limited area of minimum and maximum value.

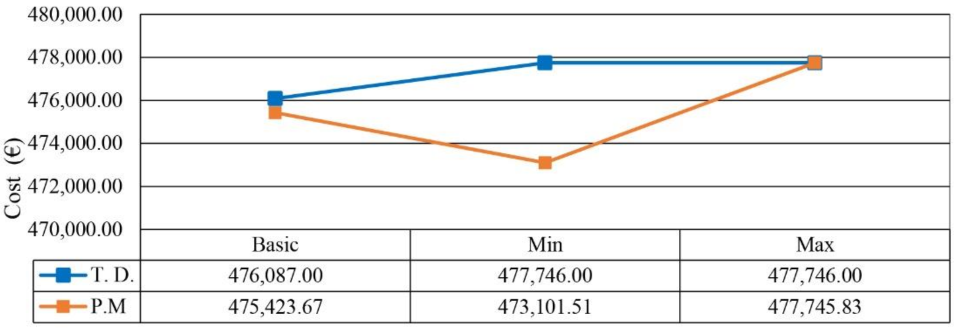


Fig. 5 Triangular distribution vs. PERT method

5.2. Minimax and Minimum vs. Expected Monetary Value – Payoff Matrix

The EMV value is based on the probabilities for each state of nature. Due to the comparison with the triangular and beta distribution (PERT), that follow in the next chapter, the preferred probability of occurrence of 75% was chosen. According to Fig. 5, there is a deviation of the minimax and minimin concerning EMV. Namely, the value of EMV cost is close to the middle of the interval determined by minimax and minimin. In this case, the EMV value of *€474,734.70* represents a reference value in the decision-making.

According to Fig. 1, the final step in decision-making is comparing the results of all researched techniques.

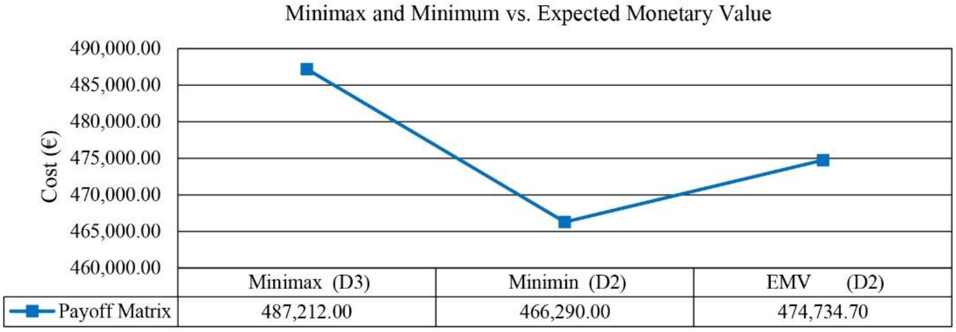


Fig. 6 Comparison of the Payoff strategies

5.3. Triangular Distribution and PERT Method vs. Payoff Matrix Strategies

Fig. 7 and Fig. 8 show the cost risk values of the applied techniques. Results are presented by two figures due to the interval limits of the PERT method. Fig. 7indicates all the obtained results with the minimum value of the PERT method (strategy 1).

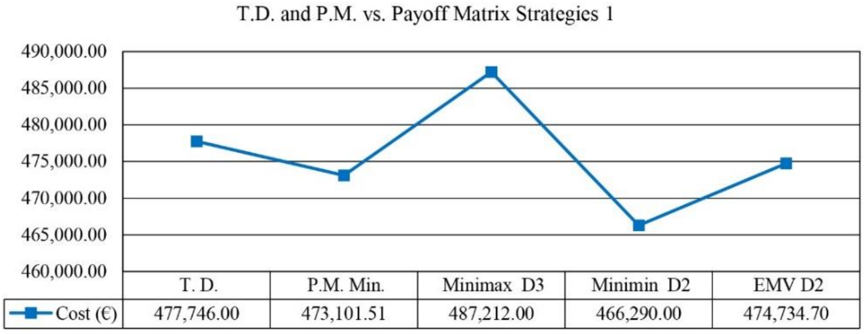


Fig. 7 Graph of all results obtained with the minimum value of the PERT method

Also, Fig. 8 shows all results with the maximum value of the PERT method (strategy 2).

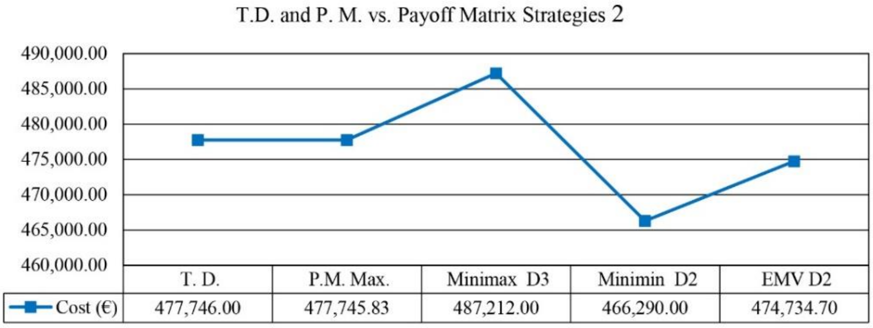


Fig. 8 Graph of all results obtained with the maximum value of the PERT method

The value of minimax refers to alternative D3, with the availability of 0.9. The stated value should not be taken into account due to a significant deviation from the others. Besides, the average availability of optimally maintained building systems in practice is around the value of 0.95. Also, the minimin D2 is the minimum cost that can jeopardize the competitiveness of the bid and potential losses in the event of a contract award. Besides, considering the obtained values is based on theoretical and practical experience in bidding. Hence, decision-making can be presented as a choice between the two alternatives, as indicated in Fig. 9 and Fig. 10.

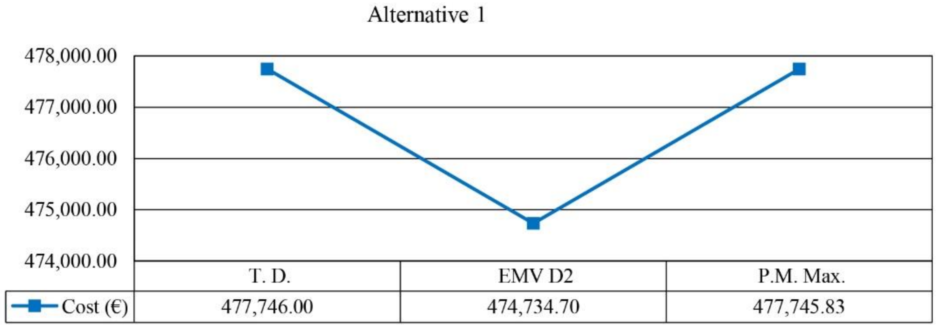


Fig. 9 Alternative 1 with P. M. Maximum

Analogous to alternative 1 is alternative 2, with the maximum value of the used PERT method (Fig. 10).

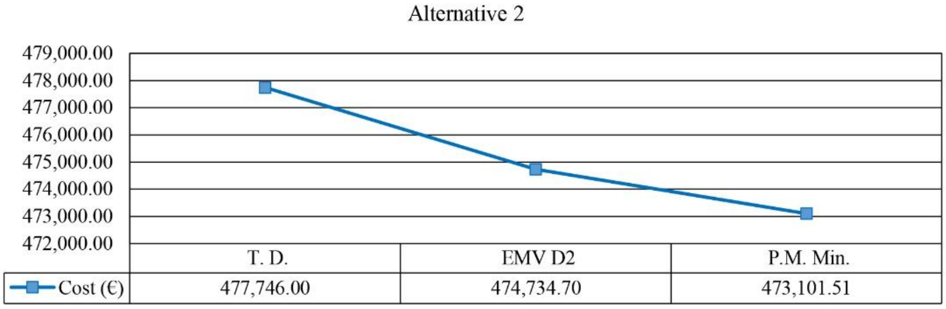
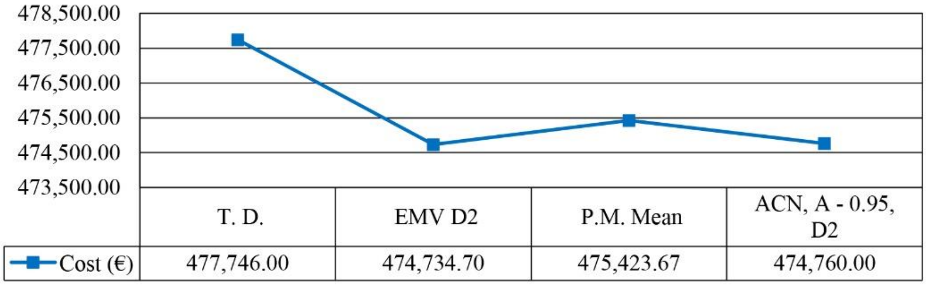


Fig. 10 Alternative 2 with P. M. Minimum

In such situations, the decision-maker should decide on the narrowest possible interval of the final bid costs. In a specific example, the cost variation interval can be further reduced by taking into account the following values:

- mean value obtained by the PERT method (without variation)
- most likely value for the availability of 0.95, and average building norms (Table 6, Payoff Matrix)

So, decision-making is simplified, as shown in Fig. 11:

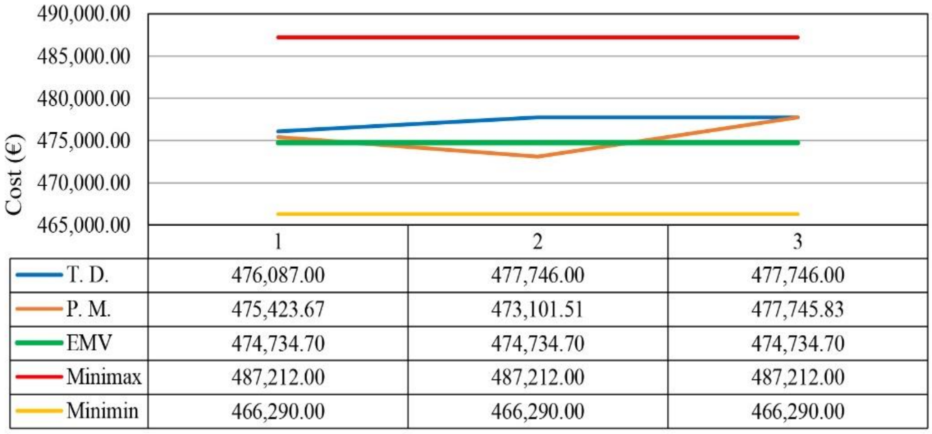


	T. D.	EMV D2	P.M. Mean	ACN, A - 0.95, D2
Cost (€)	477,746.00	474,734.70	475,423.67	474,760.00

Fig. 11 Decision-Making Chart

The results of the cost risks, according to Fig. 11, emphasize the importance of the EMV technique with multiple aspects. Primarily, EMV represents the minimum costs that are within the confidence interval of the PERT method. Also, EMV refers to alternative D2 with a system availability of 0.95. Besides, the differences between the EMV value and other techniques are 0.1%. The triangular distribution was not taken into account in the decision-making due to a greater deviation from the others.

These cost ratios and the reasons for the final decision-making are clearly illustrated by Fig. 12.



	1	2	3
T. D.	476,087.00	477,746.00	477,746.00
P. M.	475,423.67	473,101.51	477,745.83
EMV	474,734.70	474,734.70	474,734.70
Minimax	487,212.00	487,212.00	487,212.00
Minimin	466,290.00	466,290.00	466,290.00

Fig. 12 The ratio of adopted costs (EMV) to the results of the analyzed techniques

Due to the total bid cost of *€474,734.70* and a profit rate of 7%, a pre-bid layout is as shown in Table 13:

Table 13 Pre-bid layout

Item	Text	Quantity	Unit	Unit Cost (€/m2)	Total Cost (€)
3	Asphalt Constructions				
3.1	Apply asphalt base layer AC 32 TS 50/70, thickness 12 cm	11000.00	m2	18.83	207,076.85
3.2	Apply asphalt binder layer AC 16 BS 25/55-55, thickness 6 cm	11000.00	m2	12.82	140,988.50
3.3	Apply asphalt surface AC 11 DS 25/55-55 A, thickness 4 cm	11000.00	m2	11.52	126,669.35
Cost bid sum					*474,734.70*
Profit 7%					33,231.43
VAT 20%					*101,593.23*
Gross bid sum					*609,559.35*

Bids contain unit prices without division into unit costs and profit in practical examples. With this approach, contractors protect the company policy. Therefore, the final bid form is as shown in Table 14:

Table 14 Final bid

Item	Text	Quantity	Unit	Unit Price (€/m2)	Total Price (€)
3	Asphalt Constructions				
3.1	Apply asphalt base layer AC 32 TS 50/70, thickness 12 cm	11000.00	m2	20.14	221,572.23
3.2	Apply asphalt binder layer AC 16 BS 25/55-55, thickness 6 cm	11000.00	m2	13.71	150,857.69
3.3	Apply asphalt surface AC 11 DS 25/55-55 A, thickness 4 cm	11000.00	m2	12.32	135,536.21
Net bid sum					507,966.13
VAT 20%					*101,593.23*
Gross bid sum					*609,559.35*

5. CONCLUSION

The most important part of construction bidding relates to determining real cost inputs. Assessing building norms for each project is the most complex task in the costing process. Due to variations in building norms, the potential contractor anticipates several scenarios before decision-making on the final bid costs and bid price. The paper used known methods for estimating bid cost probabilities of occurrence.

The results of the analyzed techniques confirmed the assumptions of the decision-making complexity in the cost risk bidding. Namely, relations between optimistic costs, pessimistic costs, and most likely costs influenced the choice of research techniques.

By testing the techniques on a specific example, the results of the PERT method and Expected Monetary Value were matched. This means that EMV was in the cost range obtained by the PERT method.

The minimax cost significantly deviated from the most likely costs and those listed above. Also, minimax costs have a high probability of a non-competitive bid. Besides, minimin costs have a high probability of contract awards and potential losses during the project realization. The results got by the triangular distribution method have small deviations from the mean of the PERT method for the same confidence level. Figure 12 indicates the grouping of cost risk values got by the PERT method, the triangular distribution method, and the EMV method.

The uncertainty of the obtained results is confirmed by the difference of 4.80%, between the minimax and minimin technique. Besides, the differences among other technique results are in the range of 0.1 to 1%. Also, the mentioned techniques influenced compromise solutions due to the same probability of cost risk occurrence. In this case, it is EMV.

It is important to emphasize that the chosen technique provides the highest probability of minimum costs and the highest probability of occurrence of the given profit in case of winning the tender. Also, finding results represent the complexity and uncertainty within cost risk in construction bidding and decision-making.

For further research of cost risk with the same methods, it is necessary to vary the probabilities of occurrence, i.e., compare results for different confidence limits of 50% to 95%.

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POREDJENJE METODE TRIANGULARNE DISTRIBUCIJE I PERT METODE SA MATRICAMA RANGIRANJA ZA PODRŠKU ODLUČIVANJU U ANALIZI RIZIKA GRADJEVINSKIH PONUDA: ANALIZA SLUČAJA

Donošenje odluka u gradjevinskom nadmetanju je složen proces zbog prisutnog rizika. Rizik ili neizvesnost ne može se zanemariti i treba biti tretirati kao sastavni deo donošenja odluka. Cilj rada je da naglasi važnost teorije verovatnoće upoređivanjem nedovoljno primenljivih metoda u praktičnom nadmetanju. Metoda trougaone raspodele i PERT metoda pripadaju troparametarskim tehnikama procene, dok matrice rangiranja predstavljaju višekriterijumski pristup. Takođe, odabrane metode spadaju u kvantitativne tehnike za analizu troškova rizika. Određivanje jediničnih troškova i ukupnih troškova ponuda, još uvek se vrši na osnovu intuitivnog pristupa. Prema tome, upoređeni rezultati trougaone raspodele, PERT metode i tehnike matrice rangiranja (minimin, *minimax, očekivana novčana vrednost i očekivana mogućnost gubitka) ukazuju na značaj procene troškova rizika na tenderima. Analiza rezultata pokazala je preklapanje vrednosti troškova rizika dobijenih PERT metodom i tehnikom očekivane novčane vrednosti. Navedeni rezultati su posledica specifičnosti izabranog praktičnog primera i ne mogu se usvojiti kao pravilo. To znači da su odabrane metode i tehnike veoma korisne za sve procene ponuda. Rad je dokazao složenost odlučivanja, gde je primarni cilj dodela ugovora.*

Ključne reči: *nadmetanje, rizik, trošak, jedinični trošak, ukupni trošak, cena ponude*