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RELIABILITY ANALYSIS OF STRUCTURES

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Abstract. *The process of building an object consists of planning, design, construction, use and eventual demolition [1]. In each of these components in the process of building uncertainties occurs, which can be caused by human or natural effect. Human cause of uncertainties includes intentional and unintentional deviation from the optimal realization, the use of inappropriate materials, unverified construction methods, poor quality of connections between elements, changes to the building design without the consent of all parties. Natural effects are often unpredictable.*
Reliability is the ability of the structure to meet the construction requirements set out under specific conditions during the service life, according which it is designed to [2]. It refers to the carrying capacity, serviceability and durability of construction and according to them different degrees of reliability can be defined.
One of the best ways of presenting the size of the uncertainty in the theory of reliability is the reliability index β.

Key words*: service life, structure, uncertainties, reliability, reliability index β*

1. INTRODUCTION

Throughout the service life of a structure, construction can be overloaded, inadequately maintained, sometimes even sabotaged. Any structure may collapse, and the term ,,absolute reliability'' cannot be applied to engineering structures [1]. Human uncertainties usually arrive from inadequate attention on building site and poor design management. With regards to natural uncertainties, they can be caused due to unforeseen effects of wind, seismic, snow, ice, water pressure, payload, as well as material properties, which can change from sample to sample [1].

1.1. Limit states

The limit state is the state of the structure at which the object performance transforms from acceptable to unacceptable. There are several types of limit states: design limit state,

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serviceability limit states, serviceability limit state of fatigue ... Any of these conditions can be defined by limit state function:

$$g = R - Q \tag{1}$$

where R is the resistance, i.e. the capacity of the construction and Q is a load, or the load on the construction [3]. If $g < 0$ it leads to unauthorized breakage of constructions and its performance, and if $g \geq 0$ structures performance are satisfied. In the case of design limit states or bending capacity, R represents structure bending capacity, while Q represents the load bending moment. In the case of serviceability limit states, R may represent a maximum allowable deflection of the structure, while Q can represent deflection under load. However, the limit state function may be more complex (eg. nonlinear) and its parameters can be variable in time.

2. Reliability Index β

Probability of achieving the ultimate limit state can be explained by the equation:

$$P_f = P((R - Q) < 0 = P(g < 0) \tag{2}$$

Probability P_f is equal to the cumulative distribution function of the random variable g. The reliability index is defined as a function of the probability P_f:

$$\beta = -\Phi^{-1}(P_f) \ \text{, respectively } P_f = \Phi(-\beta) \tag{3}$$

where Φ and Φ^{-1} are the functions of the standard normal cumulative distribution. The reliability index β is a measure of security that can be used in the comparison between the different elements or entire system [4]. Table 1 shows the values for P_f for selected values of β, given the limits that are applied in the engineering profession, while Table 2 shows the value of β for the selected value P_f.

Table 1 The probability of failure P_f. for selected values of reliability index β

β	1,0	1,5	2,0	2,5	3,0	3,5	4,0	4,5	5,0	5,5
P_f	$1,59 \cdot 10^{-1}$	$6,68 \cdot 10^{-2}$	$2,28 \cdot 10^{-2}$	$6,21 \cdot 10^{-3}$	$1,35 \cdot 10^{-3}$	$2,33 \cdot 10^{-4}$	$3,17 \cdot 10^{-5}$	$3,40 \cdot 10^{-6}$	$2,87 \cdot 10^{-7}$	$1,90 \cdot 10^{-8}$

Table 2 Reliability index β for selected values of probability of P_f

P_f	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}	10^{-8}	10^{-9}
β	1,28	2,33	3,09	3,71	4,26	4,75	5,19	5,62	5,99

2.1. Limit states

When R and Q are independent values of normally allocated random values, we calculate the reliability index as the shortest distance line $g(Z_R, Z_Q) = 0$ from the origin of reduced variables (equation Haushofer and Linda from 1974):

$$\beta = \frac{\mu_R - \mu_Q}{\sqrt{\sigma_R^2 + \sigma_Q^2}} = \frac{\bar{R} - \bar{Q}}{\sqrt{\sigma_R^2 + \sigma_Q^2}} \tag{4}$$

where μ_R and μ_Q or $\bar{R}$ and $\bar{Q}$ are mean values for R and Q, while the σ_R^2 and σ_Q^2 are variation of their values [3].

The general equation of limit state is:

$$g = R - Q = 0 \tag{5}$$

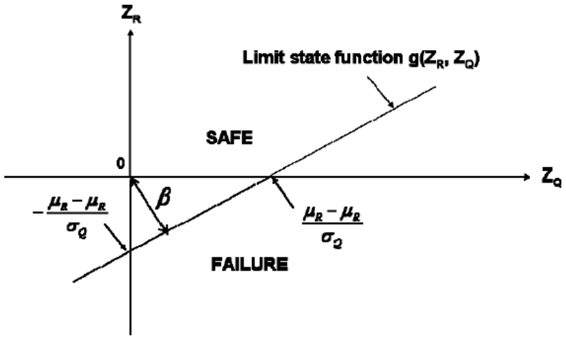

Fig. 1 Index of reliability as the shortest distance in the space of reduced variables

When the difference between R and Q is zero, i.e. when R=Q, the boundary between security and integrity of the structure is reached. Equation (5) changes shape when g is presented as a linear combination of random variables:

$$g(\{X\}) = a_0 + \sum_{i=1}^{n}(a_i X_i) = a_0 + \{a\}^T\{X\} \tag{6}$$

Reliability index is then equal to:

$$\beta = \bar{g}/[\sum_{i=1}^{n}(a_i \sigma X_i)^2]^{1/2} \tag{7}$$

where $\bar{g}$ is mean value of g:

$$\bar{g} = a_0 + \sum_{i=1}^{n}(a_i \bar{x}_i) = a_0 + \{a\}^T\{\bar{x}\} \tag{8}$$

Previous expressions (in which variation values occur) provide so-called reliability index of second mean values, as expressed by the mean values and the values of variation.

3. Numerical Calculation Examples of Reliability Index β

3.1. Calculation of reliability index for simple supported beam [5]

Calculate the reliability index β and the probability of fracture P_f for simple beam with span $l = 5\ m$, exposed to distributed load q, combined with the concentrated load W in the middle of the span. It is assumed that q, W and bending moment in the middle span M_r are unrelated normal random variables, which gives parameters of distribution: $\bar{q} = 20\frac{kN}{m}$, $V_q = 8\%$, $\bar{W} = 60\ kN$, $V_w = 12\%$, $\overline{M_r} = 290\ kNm$, $V_{M_r} = 10\%$.

The standard deviation is equal to the product of the mean value and coefficient of variation, so variations σ^2 of q, W and M_r are equal:

$\sigma_q^2 = [0{,}08 \cdot 20]^2 = 2{,}56 \quad \sigma_w^2 = [0{,}12 \cdot 60]^2 = 51{,}84 \quad \sigma_{M_r}^2 = [0{,}1 \cdot 290]^2 = 841$

There is a limit state equation:

$$R = M_r \qquad Q = \left(\frac{l^2}{8}\right) q + \left(\frac{l}{4}\right) W \qquad g = R - Q$$

The mean value and the variation of the structure capacity (in this case the bending capacity):

$$\bar{R} = \overline{M_r} = 290 \qquad \sigma_R^2 = \sigma_{M_r}^2 = 841$$

The mean value and load variations on the structure (in this case the bending moment):

$$\bar{Q} = \left(\frac{l^2}{8}\right) \bar{q} + \left(\frac{l}{4}\right) \bar{W} = \left(\frac{5^2}{8}\right) 20 + \left(\frac{5}{4}\right) 60 = 137{,}5$$

$$\sigma_Q^2 = \left(\frac{l^2}{8}\right)^2 \sigma_q^2 + \left(\frac{l}{4}\right)^2 \sigma_W^2 = \left(\frac{5^2}{8}\right)^2 2{,}56 + \left(\frac{5}{4}\right)^2 51{,}84 = 106$$

The reliability index is equal to:

$$\beta = \frac{\bar{R} - \bar{Q}}{\sqrt{\sigma_R^2 + \sigma_Q^2}} = \frac{290 - 137{,}5}{\sqrt{841 + 106}} = 4{,}95$$

The probability of failure is equal (see Table 1):

$$P_f = \Phi(-\beta) = \Phi(-4{,}95) = 3{,}12 \cdot 10^{-7}$$

3.2. Calculation of reliability index for steel cross-section [5]

Plastic moment capacity of steel-section can be represented as $M_p = f_y Z$, where f_y is the yielding strenght of steel, and Z is the plastic resistance moment of intersection. Let Q be the bending moment which loads the intersection. Reliability index and probability of failure needs to be found.

If it is assumed that f_y, Z and Q are independent random logarithmically normal variables, with mean values and coefficients of variation: $V_{f_y} = 8\%$, $\bar{Z} = 0{,}8 \cdot 10^{-3}\ m^3$, $\bar{f_y} = 280\ MPa$ $(280 \cdot 10^3\ kN/m^2)$, $V_Z = 7\%$, $\bar{Q} = 140\ kNm$, $V_Q = 10\%$.

The probability of failure:

$$P_f = P\left(\frac{M_P}{Q} < 1\right) = P\left[\left(\frac{f_y Z}{Q} < 1\right)\right]$$

Variances of sizes f_y, Z and Q are:

$$\sigma_{f_y}^2 = [0{,}08(280 \cdot 10^3)]^2 = 501{,}76 \cdot 10^6 \quad \sigma_Z^2 = [0{,}07(0{,}8 \cdot 10^{-3})]^2 = 3{,}136 \cdot 10^{-9}$$

$$\sigma_Q^2 = [0{,}10(140)]^2 = 196$$

Let $Y_1 = ln f_y$, $Y_2 = ln Q$, $Y_3 = ln Z$. Variances and mean values for Y_1 , Y_2 and Y_3 are:

$$\sigma_{Y_1}^2 = \ln\left(V_{Y_1}^2 + 1\right) = \ln(0{,}08^2 + 1) = 6{,}38 \cdot 10^{-3}$$

$$\overline{y_1} = ln\overline{f_y} - \frac{1}{2}\sigma_{Y_1}^2 = \ln(280 \cdot 10^3) - \frac{1}{2} 6{,}38 \cdot 10^{-3} = 12{,}54$$

$$\sigma_{Y_2}^2 = \ln\left(V_{Y_2}^2 + 1\right) = \ln(0{,}10^2 + 1) = 9{,}95 \cdot 10^{-3}$$

$\overline{y_2} = ln\bar{Q} - \frac{1}{2}\sigma_{Y_2}^2 = \ln 140 - \frac{1}{2}9{,}95 \cdot 10^{-3} = 4{,}94$
$\sigma_{Y_3}^2 = \ln\left(V_{Y_3}^2 + 1\right) = \ln(0{,}07^2 + 1) = 4{,}89 \cdot 10^{-3}$
$\overline{y_3} = ln\bar{Z} - \frac{1}{2}\sigma_{Y_3}^2 = \ln(0{,}8 \cdot 10^{-3}) - \frac{1}{2}4{,}89 \cdot 10^{-3} = -7{,}13$

Let $X_0 = \ln(\frac{f_y Z}{Q})$ and let $Y = \ln X_0 = \ln f_y + \ln Z - \ln Q = Y_1 + Y_3 - Y_2$. Mean values and variations of Y are equal:

$\bar{y} = \overline{y_1} + \overline{y_3} - \overline{y_2} = 12{,}54 - 7{,}13 - 4{,}94 = 0{,}47$
$\sigma_Y^2 = \sigma_{Y_1}^2 + \sigma_{Y_3}^2 + \sigma_{Y_2}^2 = 6{,}38 \cdot 10^{-3} + 4{,}89 \cdot 10^{-3} + 9{,}95 \cdot 10^{-3} = 21{,}22 \cdot 10^{-3}$

The probability of failure is equal to:

$$P_f = P\left(\frac{f_y Z}{Q} < 1\right) = \Phi\left[\left(\frac{\ln(1) - \bar{y}}{\sigma_Y}\right)\right] = \Phi\left[\left(\frac{0 - 0{,}47}{\sqrt{21{,}22 \cdot 10^{-3}}}\right)\right] = \Phi[-3{,}08] = 1{,}24 \cdot 10^{-3}$$

Thus, the reliability index $\beta = 3{,}08$.

4. Application of Reliability Analysis of Structures in Eurocodes

Recent regulations in the construction industry that are used in almost all countries of the European Union include a detailed analysis of the loads that can be expected for engineering objects. Eurocode 0 (EN 1990): Principles of design, provides information of materials and safety issues in structural design included in Eurocodes. EN 1990 sets out the principles and requirements for security, usability and structural durability [2].

EN 1990 establishes principles and requirements in terms of safety and serviceability of structures for use in all other Eurocodes, provides the basis and general principles for the design of structures and their assessment of the reliability and durability and it is based on the principle of limit states that are used in combination with the partial safety coefficients for effects and materials, applied together with the standard EN 1991 which defines the effects on structures. At the same time it describes the basis for the design of structures and provides guidelines for related aspects of structural reliability. It is designed for boards of standardization and regulations, investors, designers, contractors and government agencies responsible for public safety and security in civil engineering.

4.1. Structural requirements

A structure must be designed and constructed so that: it maintains the required service properties; it takes into account the anticipated service life and cost performance; with increased reliability it opposes actions that may occur during construction period and service life; it has an exemplary durability in relation to maintenance costs [2]. Potential damage can be limited by a variety of measures:

- Choose the form of construction that is less sensitive to certain risks,
- Reduce or avoid the risk that structure must withstand,
- Avoid systems that might collapse without a warning,
- Connect the entire construction in the whole.

In the case of the residential building when it is not possible to connect elements of the building in horizontal and vertical direction, instances must be computed when most important supporting elements are removed from building. In case of failure of single element more than 15% of the building or 70m^2 should not collapse (relevant is greater value). In the case of industrial buildings with main span of more than 9m, we examine the situation when any of the elements are demolished, and in this case only elements that are directly adjacent to the destroyed main girder may yet be pulled down, and the rest of the structure must stay functional.

There are several types of structural requirements, which must be respected in the design calculations:

- Security, usability, robustness (strength) and flammability of construction,
- Reliability of structures,
- Service life of the structure,
- Sustainability of construction.

The structure should be designed and implemented in such a manner that during its life cycle, with a certain degree of reliability and under certain conditions of economic viability, it should withstand all forces that can happen and be deformed within the design limits, so that it will not collapse in the event of an explosion, shock (accidental) loads or human error.

In the selection of the degree of reliability for a particular design, next terms should be taken into account: possible cause for reaching the limit states, the possible consequences of fracture risk in terms of loss of life, injury and possible economic losses, public opinion about the possible fracture of the structure and the costs and procedures necessary to reduce the risk of fracture [2].

Table 3 Defining consequences classes according to EN 1990

Consequences class	Description	Examples of buildings and industrial objects
CC3	Serious consequences for loss of human life, or for economic, social or environmental concerns	Grandstands, public buildings where consequences of failure are high (e.g. a concert hall)
CC2	Moderate consequence for loss of human life; economic, social or environmental consequences considerable	Residential and office buildings, public buildings where consequences of failure are medium (e.g. an office building)
CC1	Low consequence for loss of human life; economic, social or environmental consequences small or negligible	Agricultural buildings where people do not normally enter (storage buildings, greenhouses)

In Table 4 are given reliability indexes of structures for different consequence classes, depending on the return period of 1 year or 50 years [2]. The consequences affect the acceptable probability, and depending on the type of construction individual elements of the structure can be designed into a lower, the same or a higher consequence class in relation to the global structure.

Table 4 Differentiation of reliability to EN 1990

Consequences class	Reliability class	Ultimate bearing capacity		Fatigue		Serviceability	
		β for return period		β for return period		β for return period	
		1 year	50 years	1 year	50 years	1 year	50 years
CC3	RC3	5,2	4,3				
CC2	RC2	4,7	3,8	/	1,5-3,8	2,9	1,8
CC1	RC1	4,2	3,3				

The service life of the structure is assumed period in which the structure is used for a particular purpose with expected maintenance but no major work on the rehabilitation or reconstruction. Description is given in Table 5.

Table 5 Designed service life of different constructions

Class of designed service life	Designed service life (years)	Examples
1	10	Temporary structures, other than those that can be re-installed
2	10 to 25	Substituting structural parts, such as moving bearings
3	15 to 30	Agricultural and similar structures
4	50	Residential and other buildings
5	100	The monumental buildings, bridges, highways and railways, other civil engineering structures

5. CONCLUSIONS

Reliability is the ability of the structure to meet the construction requirements set out under specific conditions during the service life, according to which it is designed [2]. It refers to the carrying capacity, serviceability and durability of construction and according to them different degrees of reliability can be defined. One of the best ways of presenting the size of the uncertainty in the theory of reliability is the reliability index β, because it is a measure of security that can be used in the comparison between the different elements or entire system.

The most engineering constructions consist of a system with connected components and members. When thinking about the reliability of the system, it is important to recognize whether failure of the individual elements may or may not cause a failure of the entire structure.

EN 1990 establishes principles and requirements in terms of safety and serviceability of structures for use in all other Eurocodes, provides the basis and general principles for the design of structures and their assessment of the reliability and durability, it is based on the principle of limit states that are used in combination with the partial safety coefficients for effects and materials, applied together with the standard EN 1991 which defines the effects on structures.

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ANALIZA POZDANOSTI KONSTRUKCIJA

Proces izgradnje nekog objekta se sastoji iz planiranja, projektovanja, izgradnje, upotrebe i eventualnog rušenja [1]. U svakoj od navedenih komponenata procesa izgradnje objekta se javljaju nesigurnosti, koje mogu biti uzrokovane ljudskim ili prirodnim dejstvom. Ljudski uzroci nesigurnosti podrazumevaju namerno i nenamerno odstupanje od optimalne realizacije, korišćenje neadekvatnih materijala, neproverenih metoda gradnje, lošeg kvaliteta veza između elemenata, promene na objektu bez saglasnosti svih potrebnih strana. Prirodnu uzroci su često nepredvidljivi.

Pouzdanost konstrukcije predstavlja sposobnost konstrukcije da zadovolji postavljene zahteve pod specifičnim uslovima tokom upotrebnog veka, prema kojem je projektovana [2]. Ona se odnosi na kapacitet nosivosti, upotrebljivosti i trajnosti konstrukcije, i prema njima se mogu definisati različiti stepeni pouzdanosti.

Jedan od najboljih načina predstavljanja veličine nesigurnosti u Teoriji pouzdanosti jeste indeks pouzdanosti β.

Ključne reči: *upotrebni vek, konstrukcija, nesigurnosti, pouzdanost, indeks pouzdanosti* β