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AVERAGE BER PERFORMANCE OF SIM-DPSK FSO SYSTEM WITH APD RECEIVER

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Abstract. *In this paper, average bit error rate (BER) analysis of the free-space optical (FSO) system employing subcarrier intensity modulation (SIM) with differential phase-shift keying (DPSK) and avalanche photodiode (APD) receiver is presented. The atmospheric turbulence is described by the Gamma-Gamma statistical model taking the pointing errors into account. Numerical results are presented and confirmed by Monte Carlo simulations. The effects of atmospheric turbulence, pointing errors and receiver parameters on the average BER performance are observed and discussed. Based on the presented results, it is concluded that the minimum of the average BER exists for an optimal value of APD gain, which is heavily dependent on receiver noise temperature, bit rate and atmospheric conditions.*

Key words: *Avalanche photodiode (APD), bit error rate (BER), differential phase-shift keying (DPSK), free-space optical (FSO) communications, pointing errors.*

1. INTRODUCTION

As an alternative technology to the radio frequency (RF), free-space optics (FSO) has gained an increased interest due to many benefits, such as: license-free operations, low-cost, high data rates capacity and wide bandwidth. FSO systems find the purpose primarily as a “last mile” solution [1]−[3]. Beside the mandatory existence of a line-of-sight between transmitter and receiver, the use of FSO systems is constrained due to the existence of atmospheric turbulence. As a consequence of the variation in the refractive index, the atmospheric turbulence can seriously cause the degradation of the system performance [2]−[4]. Furthermore, optical signal transmission is corrupted by optical beam divergence and vibration, leading to misalignment between FSO transmitter laser and receiver detector, which is called pointing errors or misalignment fading [1], [5]−[8]. The Gamma-Gamma distribution is adopted as most convenient model for describing the

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effect of atmospheric turbulence [4], while the pointing errors statistic is presented in [5], assuming the radial displacement at detector is modeled by Rayleigh distribution. The combined model, accounted for both Gamma-Gamma atmospheric turbulence and pointing errors, is given in [6], [7].

Due to simple implementation and design, the intensity modulation/direct detection (IM/DD) with on-off keying (OOK) is widely used scheme in commercial FSO systems. Still, the FSO systems with IM/DD and OOK are characterized by undesirable require for adaptive threshold settings. Borrowing the concept from RF networks, subcarrier intensity modulation (SIM) technique is proposed in order to improve the FSO system performance [2]. The performance of FSO applying SIM technique with various RF modulations has been widely analyzed in past literature [9]−[13]. Practical RF systems sometimes employ an alternative form of phase-shift keying (PSK) modulation, called differential PSK (DPSK). In DPSK systems, differential coding is used on the transmitting part, and differential detection on the receiver part of the system. The analysis of the FSO systems with SIM applying DPSK is presented in [14]−[17]. In previously mentioned papers, it is assumed that received optical signal is converted to electrical one by PIN photodiode. The FSO receivers with PIN photodetectors are usually used for short distance links. For the FSO signal transmission over long propagation path, it is more convenient to employ avalanche photodiode (APD) for optical-to-electrical signal conversion at the receiver. Due to the process of impact ionization, the APD gain is provided [18], [19]. The performance of APD based FSO system applying binary modulations, i.e. pulse-position modulation (PPM) and on-off keying (OOK), was analyzed in [20] and [21], respectively, while the performance analysis of the SIM based FSO systems with APD receiver was investigated in [22], [23]. More precisely, bit error rate (BER) performance of APD based FSO system with SIM applying rectangular quadrature amplitude modulation was studied in [22], and BER and channel capacity of FSO system with SIM−BPSK and APD receiver was analyzed in [23]. Furthermore, BER performance of FSO system with coherent DPSK and APD receiver was studied in [24].

Inspired by aforementioned works, in this paper we present average BER analysis of APD-based FSO system employing SIM−DPSK. It is assumed that the intensity fluctuations of received optical signal are described by combined model, which takes into account both Gamma-Gamma atmospheric turbulence and pointing errors. Numerical results are validated by Monte Carlo simulations. Optimization of the APD gain is performed in order to achieve minimal values of the average BER for different system and channel parameters.

The rest of the paper is organized as follows. The system and channel model is presented in Section II, while Section III describes the average BER analysis. Numerical results are given in Section IV. Concluding remarks are presented in Section V.

2. System and Channel Model

Fig. 1 presents a block diagram of the FSO system with SIM−DPSK and APD receiver. The electrical signal, bearing information, is sent to the electrical RF modulator, which in this case represents binary DPSK. The signal at the DPSK modulator output is further used to modulate the intensity of the optical source (laser), which directs optical signal to the transmitting telescope. The telescope determines direction and size of the optical beam, which is forwarded to the receiver via atmospheric channel.

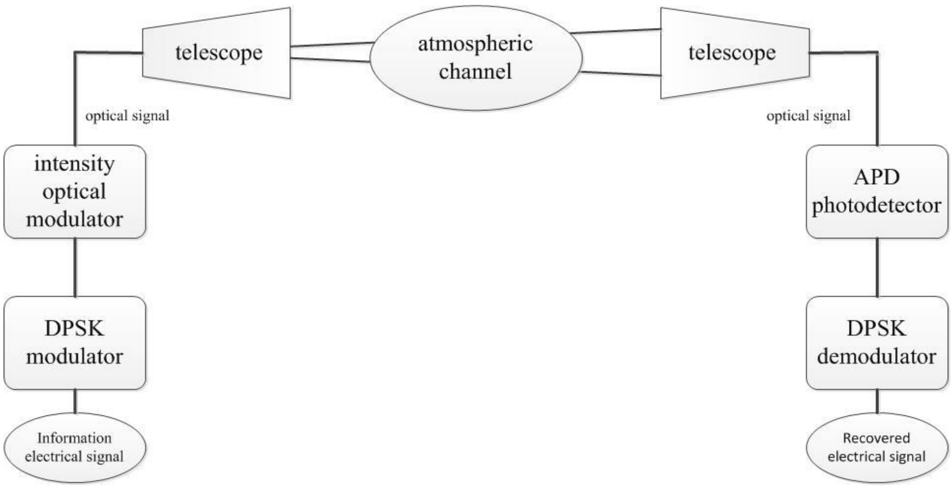

Fig. 1 Block diagram of FSO system employing SIM with DPSK

After transmission over atmospheric channel, the received optical signal is given by [13], [14]

$$r_{opt} = P_t I\left[1+ms\right], \tag{1}$$

where m denotes modulation index, s is signal at the DPSK modulator output, P_t is transmitted optical power, and I represents normalized irradiance accounted for the intensity fluctuations due to Gamma-Gamma atmospheric turbulence, pointing errors and path loss. After direct detection at the receiver, and removing dc bias, the optical signal conversion to electrical signal is done by APD photodetector. Further, electrical signal is demodulated by DPSK demodulator, whose output signal is expressed as

$$r = gRmP_t I + n, \tag{2}$$

where g and R represent APD gain and responsivity, respectively, and n is total APD noise.

2.1. Total APD noise

Total APD receiver noise is composed of shot noise, thermal noise and dark current contributions. It is assumed that the dark current is negligible, so n can be expressed as [21]–[23]

$$n = i_{Th} + i_{Sh}, \tag{3}$$

where i_{Th} represents thermal noise and i_{Sh} is APD shot noise.

Thermal noise happens as a result of the electrons thermal motion at any finite temperature, and it is not dependent on APD parts. It can be modeled as the stationary Gaussian random process with the zero-mean value and variance [21]–[23]

$$\sigma_{Th}^2 = 4k_B \frac{T}{R_L} F_n \Delta f, \tag{4}$$

where k_B and F_n denote the Boltzmann constant and amplifier noise figure, respectively, T is the receiver temperature in degree Kelvin, R_L represents APD load resistance, Δf is the symbol effective noise bandwidth, dependent on the bit rate, R_b, as $\Delta f = R_b/2$.

Different from thermal noise, shot noise is dependent on the APD parts, and also can be modeled as the stationary zero-mean Gaussian random process with variance [21]–[23]

$$\sigma_{Sh}^2 = 2qg^2 F_A R P_t m I \Delta f, \tag{5}$$

where q represents an electron charge and F_A denotes the excess noise factor of the APD given by

$$F_A = k_A g + (1-k_A)(2 - 1/g), \tag{6}$$

where k_A is the ionization factor. Since shot noise and thermal noise are independent Gaussian random processes, total APD noise is modeled as the stationary Gaussian random process with the zero-mean value and variance obtained as a sum of shot and thermal noise variances, expressed as

$$\sigma_n^2 = \sigma_{th}^2 + \sigma_{Sh}^2 = 4k_B \frac{T}{R_L} F_n \Delta f + 2qg^2 F_A R m P_t I \Delta f. \tag{7}$$

2.2. Channel model

The intensity fluctuations of received optical signal are assumed to originate from Gamma-Gamma atmospheric turbulence, pointing errors and path loss, which can be written as [6]

$$I = I_l I_a I_p, \tag{8}$$

where I_a and I_p are the random attenuations caused by atmospheric turbulence and pointing errors, respectively, and I_l denotes the path loss. Assuming combined model presented in [6], [7], which takes into account both Gamma-Gamma atmospheric turbulence, pointing errors and path loss, the probability density function (PDF) of I is given by [7]

$$f_I(I) = \frac{\xi^2 \alpha\beta}{A_0 I_l \Gamma(\alpha)\Gamma(\beta)} G_{1,3}^{3,0}\left(\frac{\alpha\beta}{A_0 I_l} I \middle| \begin{matrix} \xi^2 \\ \xi^2 - 1, \quad \alpha - 1, \quad \beta - 1 \end{matrix} \right), \tag{9}$$

where $G(.)$ represents the Meijer's G function [25, eq. (9.301)], α and β denote the Gamma-Gamma atmospheric turbulence parameters, and ξ and A_0 represent the parameters determined by the pointing errors. When $\xi \to \infty$, the pointing errors can be neglected, and it can be assumed that the intensity fluctuations of the received optical signal originate only form Gamma-Gamma atmospheric turbulence. The path loss component, I_l, is described by the exponential Beers-Lambert law as [6]

$$I_l = \exp(-\sigma L), \tag{10}$$

where σ is the atmospheric attenuation coefficient, and L represents propagation distance.

The parameters α and β can be directly linked to atmospheric conditions as [2], [3]

$$\alpha = \left(\exp[0.49\sigma_R^2/(1+1.11\sigma_R^{12/5})^{7/6}]-1\right)^{-1}, \tag{11}$$

and

$$\beta = \left(\exp[0.51\sigma_R^2/(1+0.69\sigma_R^{12/5})^{5/6}]-1\right)^{-1}, \tag{12}$$

where the Rytov variance is defined as $\sigma_R^2 = 1.23 C_n^2 k^{7/6} L^{11/6}$, and k=2π/λ is the wave number with the wavelength λ, and the index of refraction structure parameter C_n^2.

The parameter ξ is defined as the ratio between the equivalent beam radius at the receiver and the pointing error (jitter) standard deviation at the receiver [6]

$$\xi = \frac{w_{L_{eq}}}{2\sigma_s}. \tag{13}$$

The radius of a circular detector aperture is denoted by a, and the equivalent beam radius at the receiver is dependent on the beam waist (radius calculated at e^{-2}) at distance L, w_L, as [6]

$$w_{L_{eq}}^2 = \frac{w_L^2\sqrt{\pi}\,\mathrm{erf}(v)}{2v\exp(-v^2)}, \quad v = \frac{\sqrt{\pi}a}{\sqrt{2}w_L}. \tag{14}$$

The parameter A_0 represents the fraction of the collected power at L = 0, and it is defined as $A_0 = [\mathrm{erf}(v)]^2$, where erf(.) is the error function [25, eq. (8.250.1)]. Further, the beam waist at the distance L, w_L, is related to optical beam waist at transmitter laser, denoted as w_0, and to the radius of curvature, denoted as F_0, as [8]

$$w_L = w_0\sqrt{\left((\Theta_o + \Lambda_o)(1+1.63\sigma_R^{12/5}\Lambda_1)\right)}, \tag{15}$$

where

$$\Theta_o = 1 - \frac{L}{F_0}, \quad \Lambda_o = \frac{2L}{kw_0^2}, \quad \Lambda_1 = \frac{\Lambda_o}{\Theta_o^2 + \Lambda_o^2}. \tag{16}$$

Based on (2) and (7), the instantaneous electrical SNR can be defined as

$$\gamma = \frac{\left(gRmP_tI\right)^2}{2\sigma_n^2} = \frac{\left(gRmP_tI\right)^2}{2\left(4k_B\dfrac{T}{R_L}F_n\Delta f + 2qg^2F_ARmP_tI\Delta f\right)}. \tag{17}$$

3. Average BER Analysis

Using well-known expression for the average BER of binary DPSK, the conditional average BER of the FSO system with SIM-DPSK is expressed as [26, eq. (5.2-69)]

$$P_{b/\gamma(I)} = \frac{1}{2}\exp(-\gamma), \quad (18)$$

where γ is the previously defined instantaneous electrical SNR. Substituting (17) into (18), and utilization of σ_n^2 definition given by (7), the conditional average BER is now

$$P_{b/\gamma(I)} = \frac{1}{2}\exp\left(-\frac{\left(gRmP_t I\right)^2}{2\sigma_n^2}\right) = \frac{1}{2}\exp\left(-\frac{\left(gRmP_t I\right)^2}{2\left(4k_B \frac{T}{R_L} F_n \Delta f + 2qg^2 F_A RmP_t I \Delta f\right)}\right). \quad (19)$$

The average BER of investigated FSO system is obtained by averaging (19) over received optical irradiance I as

$$P_b = \frac{1}{2}\int_0^\infty \exp\left(-\frac{(gRmP_t I)^2}{2\left(4k_B \frac{T}{R_L} F_n \Delta f + 2qg^2 F_A RmP_t I \Delta f\right)}\right) f_I(I)dI, \quad (20)$$

where $f_I(I)$ is the PDF of I previously defined by (9). Substituting (9) into (20), the average BER of the FSO system under the investigation is derived as

$$P_b = \frac{\xi^2 \alpha\beta}{2A_0 I_l \Gamma(\alpha)\Gamma(\beta)} \int_0^\infty \exp\left(-\frac{\left(gRmP_t I\right)^2}{2\left(4k_B \frac{T}{R_L} F_n \Delta f + 2qg^2 F_A RmP_t I \Delta f\right)}\right) \times G_{1,3}^{3,0}\left(\frac{\alpha\beta}{A_0 I_l} I \middle| \begin{matrix} \xi^2 \\ \xi^2 - 1, \quad \alpha - 1, \quad \beta - 1 \end{matrix}\right) dI. \quad (21)$$

The integral in (21) has no closed form, so the final BER expression is evaluated numerically.

4. Numerical results

The numerical results are obtained based on average BER expressions given by (21). All results are validated by Monte Carlo simulations. Turbulence parameters α and β are calculated based on (11) and (12), considering different values of turbulence strength determined by the refractive index parameter as: $C_n^2 = 6\times10^{-15}$ m$^{-2/3}$ for weak, $C_n^2 = 2\times10^{-14}$ m$^{-2/3}$ for moderate and $C_n^2 = 5\times10^{-14}$ m$^{-2/3}$ for strong turbulence conditions [23]. The values of wavelength $\lambda = 1550$ nm, modulation index is $m = 1$, responsivity R=1 A/W and atmospheric attenuation coefficient $\sigma = 0.1$ dB/km are assumed, while the radius of the circular detector aperture is $a = 5$ cm, and radius of curvature is $F = -10$ m [8]. The electron charge takes a value of

$q = 1.6\times10^{-19}$ C, amplifier noise figure $F_n = 2$, the Boltzmann constant is $k_B = 1.38\times 10^{-23}$ W/kHz, APD load resistance is $R_L = 1000\ \Omega$ and ionization factor $k_A = 0.7$ for InGaAs APD [23].

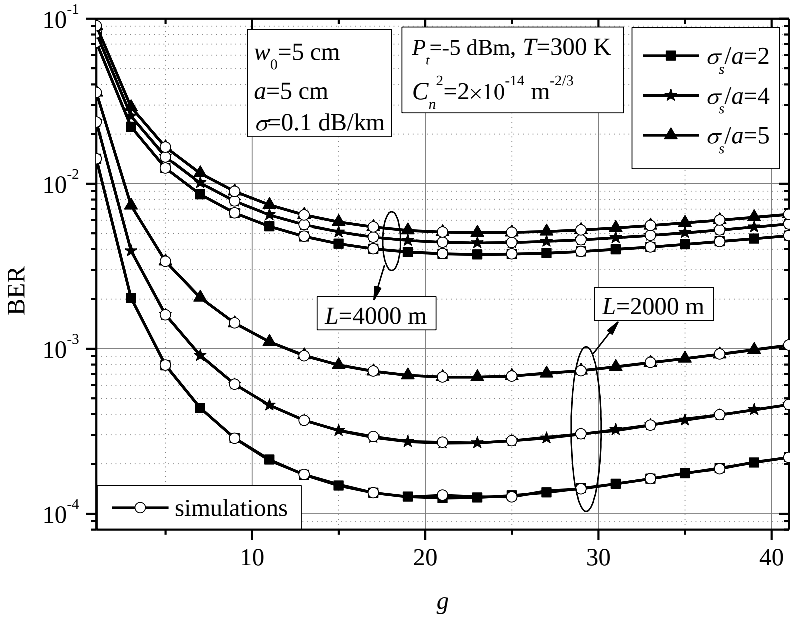

Fig. 2 Average BER dependence on APD gain for different values of normalized jitter standard deviation and propagation distance

Fig. 2 shows the average BER dependence on APD gain assuming different values of normalized jitter standard deviation and propagation distance. The system performance is better when the value of σ_s/a is lower, corresponding to the weaker pointing errors and better positioning of FSO apertures. The longer FSO link leads to the deterioration of the system performance. Furthermore, the influence of the normalized jitter standard deviation, i.e. pointing errors, is more pronounced when the distance between transmitter and receiver is shorter. In addition, it is noticed that minimum of the average BER exists for a certain optimal value of APD gain, denoted as g_{opt}. Hence, the system performance can be significantly improved by the proper choice of receiving aperture during system design. From the obtained results we can conclude that the normalized jitter standard deviation has no influence on the value of g_{opt}, while the different FSO link lengths lead to the varying in the value of g_{opt}. For example, for L=2000 m (σ_s/a=2,4 and 5), the optimal value of APD gain is g_{opt} =21, and for L=4000 m (σ_s/a=2,4 and 5), it holds g_{opt} =23.

The average BER dependence on APD gain for different values of receiver temperature in various atmospheric turbulence conditions is presented in Fig. 3. As it was expected, system has better performance in weak turbulence conditions, as well as in lower temperature environment. The BER minimum, determined by optimal value of the APD gain, is highly dependent on receiver temperature T. With higher temperature, the value of

g_{opt} is greater. Based on the presented results, it is noticed that worse turbulence conditions reflects in higher optimal APD gain. Also, influence of the temperature has stronger effect on average BER performance when the optical signal transmission via free space suffers from weak atmospheric turbulence.

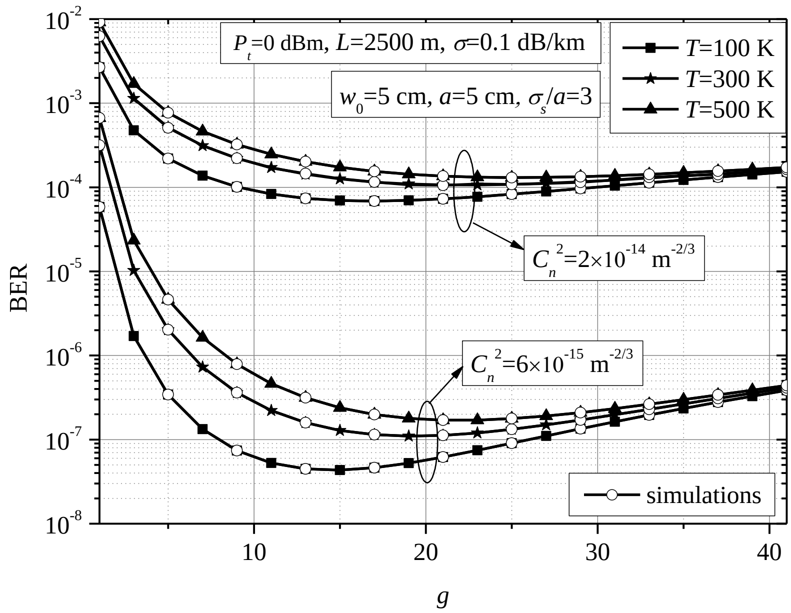

Fig. 3 Average BER dependence on APD gain for different values of receiver temperature in various atmospheric turbulence conditions

Fig. 4 presents the average BER dependence on APD gain for different values of bit rate and transmitted optical power. Increase in the optical power P_t leads to the improvement of the BER performance. Also, lower value of R_b reflects in better performance. Based on the presented results: $g_{opt} = 25$ for R_b=1 Gb/s (P_t = 0 dBm and P_t=10 dBm); $g_{opt} = 17$ for R_b = 5 Gb/s (P_t = 0 dBm and P_t = 10 dBm); $g_{opt} = 15$ for R_b = 10 Gb/s (P_t = 0 dBm and P_t = 10 dBm), it is concluded that the optimal value of APD gain is not dependent on transmitted optical power, and the lower values of R_b correspond to the higher g_{opt}.

The average BER dependence on transmitted optical power for different values of normalized jitter standard deviation in various turbulence conditions is presented in Fig. 5. As it has been already concluded, the best system performance is achieved in weak atmospheric turbulence, and when the normalized jitter standard deviation is lower. Furthermore, the pointing errors, determined by σ_s, have more dominant effect on BER performance when the optical signal transmission is impaired by weak atmospheric turbulence.

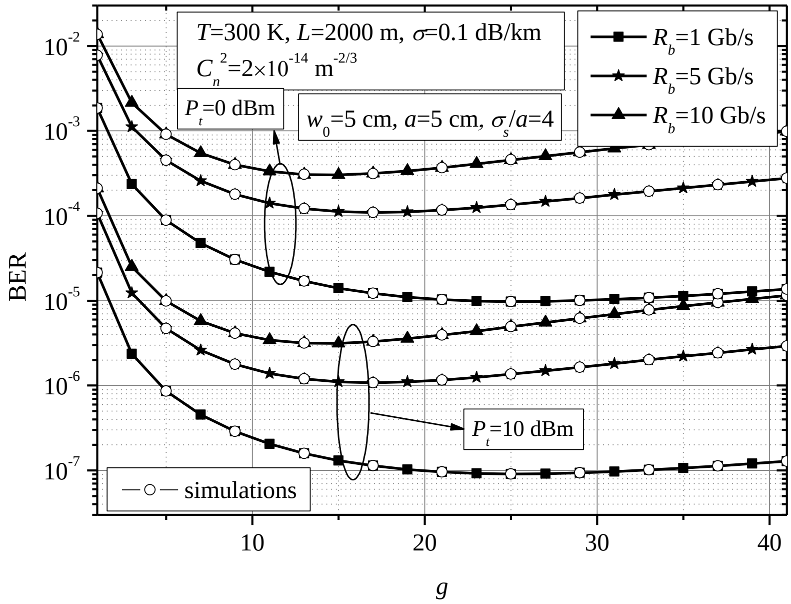

Fig. 4 Average BER dependence on APD gain for different values of bit rate and transmitted optical power

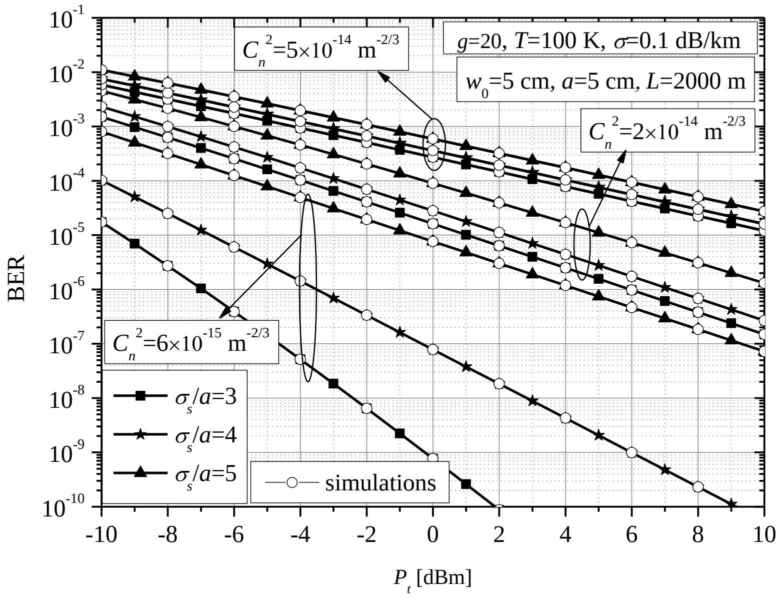

Fig. 5 Average BER dependence on transmitted optical power for different values of normalized jitter standard deviation in various turbulence conditions

5. Conclusion

This paper presents the average BER analysis of the APD-based FSO system employing SIM-DPSK. The FSO channel is under the influence of combined effect of the Gamma-Gamma atmospheric turbulence and the pointing errors. Numerical results are presented and validated by Monte Carlo simulations. The presented results show the effect of atmospheric turbulence, pointing errors and APD receiver parameters on the average BER performance. The minimum of the average BER is noticed for an optimal value of APD gain. Unlike transmitted optical power and normalized jitter standard deviation, the atmospheric turbulence, receiver noise temperature, bit rate and propagation distance have a significant effect on the optimal values of APD gain, so the proper selection of FSO apertures can lead to the FSO performance improvement.

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