Abstract. The brief bit error rate (BER) analysis of dual-hop amplify-and-forward relaying systems in the extended generalized K fading environment is presented in this paper. Assuming N nonidentical dual hops together with direct link between transmitter and receiver and maximal-ratio combining at the destination, the novel lower bound BER analytical expression is derived. The presented results illustrate that the BER lower bound is relatively tight to the obtained numerical integration and Monte Carlo simulation results. In some special cases, new derived expression is simplified to the ones previously reported in literature. This expression is utilized for determining the effects of fading and shadowing phenomena, as well as the number of relays on BER performance and can be used in designing contemporary mobile systems.

Key words: bit error rate, cooperative systems, diversity reception, fading channel, noise, shadowing

1. INTRODUCTION

The main advantage of cooperative diversity systems is to provide balanced network load and to diminish influence of different wireless impairments [1-4]. Separately, the utilization of relays is to redirect intensity traffic and to extend coverage area. The main role of implemented diversity receiver at the destination node of cooperative link is to reduce effects of multipath fading and to alleviate the shadowing effects to a lesser extent.

Implementation of amplify-and-forward (AF) relays improves system performance as simpler and inexpensive solution compared to decode-and-forward (DF) technique [5-7]. The assignment of AF relays is to amplify signals from the previous node and to forward it to the next node in specify wireless system. There are the channel-state-information

Received March 18, 2016
Corresponding author: Jelena Anastasov
University of Niš, Faculty of Electronic Engineering, Department of Telecommunications, Niš, Republic of Serbia
E-mail: jelena.anastasov@elfak.ni.ac.rs
(CSI)-based AF relays (or variable gain AF relays) and relays with fixed-gain, as two subcategories of AF relays.

As an interesting topic, the AF relaying systems have been very often analyzed in literature. The performance of dual-hop systems with AF relays over multipath Rayleigh, Nakagami-$m$ and asymmetric Rayleigh/Ricean fading environment were discussed in [5], [6] and [7], respectively. The performance evaluations of systems with CSI-based AF relays over channel influenced simultaneously by fading and shadowing effects, modeled by generalized-$K$ (GK) distribution, were given in [8].

In [9], maximal-ratio combining (MRC) scheme was employed at the AF relay and the bit error rate (BER) performance of proposed system over shadowed fading environment was analysed. The BER analysis of CSI-based relaying over Nakagami-$m$ fading channels when MRC scheme is applied at the destination was discussed in [10]. In that letter, geometric mean approximation of the signal-to-noise ratios (SNRs) of $N$ dual-hops was used for deriving moment-generating function (MGF) of upper bound overall SNR at the receiving end. Furthermore, using MGF expression, BER performance was numerically obtained. The minimum SNR approximation in deriving MGF for the similar system over Nakagami-$m$ channels was performed in [11]. The error performance of CSI-based systems over GK fading channels based on both, the minimum SNR approximation and geometric mean approximation, was presented in [12].

In [13], the authors have introduced a very general distribution, so-called the extended-generalized $K$ (EGK) distribution, easy to simplify into many other distributions, and adequate in modeling great variety of fading/shadowing channel conditions. This distribution is also applicable in modeling of radio wave propagation over wireless communication channels under very sharp shadowing condition [14]. The important performance evaluations of communication systems over EGK fading channels corrupted by noise are given in [13], and in [14] the brief performance analysis of mobile interference-limited systems over the same fading channels is proposed. Using an upper bound approximation for the end-to-end SNR and signal-to-interference ratio, the expressions for evaluating outage performance in the noise- and interference-limited AF dual-hops over EGK fading/shadowing links are derived in [15]. Transmission is only performed over relay because of the deeply shadowed direct link.

In this paper, the multi-user system of $N$ nonidentically distributed dual hops corrupted by EGK fading is analyzed. Considering geometric mean approximation of the end-to-end SNR, we derive a new formula of BER for different modulation formats. The obtained result is quite efficient lower bound for exact result which is unavailable in closed-form and can be only performed by simulations or numerical integration. In addition, presented solution in a form of special functions enables faster numerical computation compared to simulations. Based on analytics, BER performance is briefly discussed. Presented analytical result is quite general and can be simplified into other already published results [10], [12].

The paper is constructed as follows. In Section 2, system and channel models used in analysis are presented in brief. In Section 3, based on the geometric mean approximation method BER determination of proposed system is performed, and further appropriate numerical results are given in Section 4. Section 5 concludes the paper.
2. SYSTEM AND CHANNEL MODEL

We assume a system where \( N \) CSI-based AF dual-hop relays are incorporated in signal transmission originating from the source to the destination. As well as through \( N \) dual-hop links, the source terminal transmits signal through the direct line (Fig. 1). The MRC combining is implemented at the destination node.

![System model](image)

For that scenario, we can define the end-to-end SNR in the following form [10]

\[
\gamma_{\text{end}} = \gamma_0 + \sum_{i=1}^{N} \frac{\gamma_h \gamma_g}{\gamma_h + \gamma_g + 1}
\]  

(1)

where, according to the Fig. 1, \( \gamma_0 \) denotes the instantaneous SNR of the direct link, \( \gamma_h \) presents the instantaneous SNR between the source and \( i \)th relay and \( \gamma_g \) is the instantaneous SNR between the \( i \)th relay and destination. In the analysis that follows, the fading/shadowing conditions of the direct path and over \( N \) relaying links are described by EGK distribution. So the probability density function (PDF) of SNRs, \( \gamma_0, \gamma_h, \gamma_g \) \( (i = 1, 2, ..., N) \) starting from [13, eq. (3)] and manipulating with transformation in [16, eq. (6.22)], can be presented in the following form

\[
f_\gamma(\gamma) = \frac{1}{\Gamma(m_s)\Gamma(m_s)} H_{1,0}^{2,0} \left( \begin{array}{c} \frac{\beta_s}{\bar{\gamma}} \beta_s \\ m_s, \frac{1}{\xi_s}, \frac{m_s}{1}\end{array} \right),
\]  

(2)

where the severity and shaping parameters of the multipath fading are denoted as \( m \) and \( \xi \) respectively; and the shadowing severity and shadowing shaping parameters are \( m_s \) and \( \xi_s \), respectively. The average value of SNR is denoted by \( \bar{\gamma} \), \( \beta = \Gamma(m+1/\xi)/\Gamma(m) \), \( \beta_s = \Gamma(m_s+1/\xi_s)/\Gamma(m_s) \), \( \Gamma(\cdot) \) is the Gamma function [17, eq. (8.31)] and special function...
is the Fox’s $H$ function [18, eq. (1.1.1)], [19, eq. (1.2)], where parameters $a_i, b_j \in \mathbb{C}$, with $C$ being the set of complex numbers, and parameters $A_i, B_j \in \mathbb{R}^+ = (0, \infty)$, $i = 1, \ldots, p; j = 1, \ldots, q$. Numbers $p, q, m$ and $n$ are integer such that $0 \leq m \leq q$, $0 \leq n \leq p$.

The fading/shadowing parameters of the first and the second hop of the $i$th link, and the direct path are assumed as nonidentical and in further analysis will be indexed as $h_i, g_i$ and $0$, respectively.

3. BER Evaluation Based on the Geometric Mean Approximation

In performance analysis of proposed system, an efficient upper bound for output SNR will be derived as more convenient compared to (1). If we start from the relation between harmonic and geometric mean of variables $\gamma_{h_i}, \gamma_{g_i}$ and $\gamma_{h_i}\gamma_{g_i}$, i.e. considering the following inequality

$$3 \left( \frac{1}{\gamma_{h_i}} + \frac{1}{\gamma_{g_i}} + \frac{1}{\gamma_{h_i}\gamma_{g_i}} \right)^{-1} \leq (\gamma_{h_i} \gamma_{g_i} \gamma_{h_i}\gamma_{g_i})^{1/3},$$

the end-to-end SNR in (1) can be bounded as [12, eq. (18)]

$$\gamma_{\text{end}} \leq \gamma_{\text{bound}} = \gamma_0 + \frac{1}{3} \sum_{i=1}^{N} (\gamma_{h_i}\gamma_{g_i})^{2/3}.$$

Adopted approximation of output SNR greatly facilitates derivation of MGF in a closed-form. Assuming the independence of variables in (3), the MGF of $\gamma_{\text{bound}}$ can be evaluated following [11, eq. (6)]

$$M_{\gamma_{\text{bound}}}(s) = M_{\gamma_0}(s) \times$$

$$\prod_{i=1}^{N} \int_{0}^{\infty} e^{-s(y_{h_i}\gamma_{g_i})^{2/3}} f_{\gamma_{h_i}}(y_{h_i}) f_{\gamma_{g_i}}(y_{g_i}) dy_{h_i} dy_{g_i},$$

where $M_{\gamma_0}(s)$ is the MGF of the SNR from the direct link, defined as

$$M_{\gamma_0}(s) = \int_{0}^{\infty} e^{-s\gamma} f_{\gamma_0}(\gamma) d\gamma.$$

Substituting (2) with required parameters into previous definition formula (6) and transforming exp function into Meijer’s $G$ e.g. Fox’s $H$ function as $e^{-s} = H_{p,n}^{m,n} \left[ \begin{array}{c} \alpha_i \beta_i \cr \gamma_i \delta_i \end{array} \right] \left[ \begin{array}{c} \infty \\ 0,1 \end{array} \right]$, we get the resulting integral in the form

$$\int_{0}^{\infty} t^{\gamma}s^{-1} H_{p,n}^{m,n} \left[ \begin{array}{c} \alpha_i \beta_i \cr \gamma_i \delta_i \end{array} \right] H_{p,q}^{m,n} \left[ \begin{array}{c} \gamma_i \delta_i \cr \alpha_i \beta_i \end{array} \right] dt.$$
This integral can be solved with the help of [18, eq. (2.8.12)], in the following way

\[ M_{\gamma_s}(s) = \frac{1}{\Gamma(m_{\alpha})\Gamma(m_{\beta})} \times H_{2,2}^{1,1}\left[\frac{\beta_{\alpha}\beta_{\beta}}{\overline{\gamma}^0}, \left(\begin{array}{c} 1,1 \\ m_{\alpha}, \frac{1}{\overline{\gamma}_0} \end{array}\right) \right]. \] (8)

The first integral in (5), the one on \( \gamma_{h_2} \), can be solved tracking the same procedure as in finding \( M_{\gamma_s}(s) \), in a way

\[ I_1 = \frac{1}{\Gamma(m_{\alpha})\Gamma(m_{\beta})} \times H_{2,2}^{1,1}\left[\frac{2}{3} \left(\begin{array}{c} \beta_{\alpha}\beta_{\beta} \overline{\gamma}_h \\ \overline{\gamma}_h \end{array}\right) \right] \times H_{4,1}^{1,4}\left[\left(\begin{array}{c} 1 \\\n\end{array}\right) \right] \right]. \] (9)

Substituting (9) and (2) in integral \( I \) in (5), the remaining integration is solved using the integration procedure for product of two Fox’s \( H \) functions [18, eq. (2.8.12)], as

\[ I = \int_0^{\infty} e^{-(t/3)\gamma_h} f_{\gamma_h}(\gamma_h) I_1 d\gamma_h = \frac{1}{\Gamma(m_{\alpha})\Gamma(m_{\beta})\Gamma(m_{\gamma_3})\Gamma(m_{\gamma_1})} \times \]

\[ \times H_{4,1}^{1,4}\left[\left(\begin{array}{c} 1 \\\n\end{array}\right) \right] \right]. \] (10)

with \( \chi = \left(1 - m_{\gamma_3}, \overline{\gamma}_h \overline{\gamma}_h \right) \left(1 - m_{\gamma_3}, \overline{\gamma}_h \overline{\gamma}_h \right) \left(1 - m_{\gamma_3}, \overline{\gamma}_h \overline{\gamma}_h \right) \left(1 - m_{\gamma_3}, \overline{\gamma}_h \overline{\gamma}_h \right). \)

Finally, considering (8) and (10), novel expression for MGF of proposed system is derived as

\[ M_{\gamma_{\text{new}}}(s) = \frac{1}{\Gamma(m_{\alpha})\Gamma(m_{\beta})} \times H_{2,2}^{1,1}\left[\frac{\beta_{\alpha}\beta_{\beta}}{\overline{\gamma}^0}, \left(\begin{array}{c} 1,1 \\ m_{\alpha}, \frac{1}{\overline{\gamma}_0} \end{array}\right) \right] \times \]

\[ \times \prod_{i=1}^{N} \frac{1}{\Gamma(m_{\alpha})\Gamma(m_{\beta})\Gamma(m_{\gamma_3})\Gamma(m_{\gamma_1})} \times H_{4,1}^{1,4}\left[\left(\begin{array}{c} 1 \\\n\end{array}\right) \right] \right]. \] (11)

The previously derived expression (11) in a form of special Fox’s \( H \) functions can be simplified transforming Fox’s \( H \) functions into more familiar Meijer’s \( G \) functions using [21, eq. (8.3.2.22)] and further into hypergeometric functions on the basis of relation given in [21, eq. (8.2.2.3)].
This result can be used directly for the error analysis of binary noncoherent frequency-shift keying (NFSK) and differential phase-shift keying (BDPSK) signaling, and also using integration for other modulation formats. Due to generality, new derived solution for EGK fading environment unifies the results for GK [12] and Nakagami-\(m\) [10] fading scenarios.

By initializing parameters \(s_0 = s_s = 1\) in (8), one can simplified it into [12, eq. (4)] following the relation [18, eq. (2.1.3)] and then transforming the Fox’s \(H\) function into Meijer’s \(G\) function using [16, eq. (6.108)] in a way:

\[
H^{m,n}_{p,q}\left(\frac{a_1, c, \ldots, (a_p, c)}{b_1, c, \ldots, (b_q, c)}\right) = \frac{1}{c} G^{m,n}_{p,q}\left(\frac{a_1, \ldots, a_p}{b_1, \ldots, b_q}\right).
\]  

(12)

By setting \(s_{sh} = s_{hi} = s_{sg} = s_{gi} = 1\) in (10) and using relation between Fox’s \(H\) and Meijer’s \(G\) function [21, eq. (8.3.2.22)], after some algebra, the Fox’s \(H\) is reduced into Meijer’s \(G\) function and (10) becomes [12, eq. (24)]. Finally, (11) gives counterbalance to [12, eq (26)] which simplifies our analysis into analysis for proposed system over GK fading channels.

Furthermore, considering limiting operation [20, eq. (07.34.0005.01)], where \(m_{ob}, m_{sh}, m_{eqi}\) tend to infinity, derived result (11) reduces to the result presented in [10, eq. (11)] for Nakagami-\(m\) fading environment.

4. NUMERICAL RESULTS

In this section, numerical results of BDPSK and BPSK modulation scheme for proposed system are presented. The BER in detecting BDPSK signals transmitted over EGK fading channels can be obtained directly based on new expression (11) as \(M_{\text{bound}}(1)/2\). Numerical results of BER for BPSK modulation scheme for proposed system scenario are also performed using

\[
\frac{1}{\pi} \int_{0}^{\pi/2} M_{\text{bound}}\left(-1, \frac{1}{\sin^2 \theta}\right) d\theta.
\]

The BER for BDPSK signal detection when multiple dual hops as well as the direct link are employed in transmission of AF relaying system is presented in Fig. 2. In order to justify the accuracy of proposed approximation method, the simulation results are also obtained. One can noticed that adopted BER approximation method is improved with shadowing shaping parameters decreasing, namely for worst channel conditions. Also, the tightness of numerical results with the simulations is more evident for smaller values of MRC diversity order \(N\).

Fig. 3 presents the BER performance for BPSK modulation format when unbalanced average SNRs scenario is introduced. The geometric mean performance bound at this figure gives results that are a little closer to the required exact ones for the balanced power case compared with unbalanced one.
Fig. 2 BER for BDPSK modulation format and different number of dual-hop links

Fig. 3 BER for BPSK modulation format of AF system with $N=1$ and unbalanced average SNRs at hops
In order to confirm our derived formula (11) for generalized fading environment, Fig. 4 and Fig. 5 present the BER evaluation of proposed multiuser system for BPSK modulation schemes over GK and Nakagami-$m$ fading environment, respectively. One can noticed BER performance improvement when fading and shadowing conditions over relaying channels become better, as expected. In Fig. 4 the case with one dual-hop link
and the direct path is presented. In Fig. 5 more complex scenarios with multiple dual hop relays over Nakagami-\(m\) fading channels are given. In this figure the importance of cooperative systems can be observed. The best BER performance is noticeable for the case with \(N=4\) dual hop links, and the worst one when there is no relaying links and the transmission is only performed over the direct path.

Numerical results at all figures are performed in Mathematica 7, using the relation between the Fox's \(H\) and Meijer's \(G\) function [21, eq. (8.3.2.22)]. Actually, all presented numerical results are evaluated on the basis of original program given in [22, Appendix]. Also, the efficient computation of the Fox’s \(H\) function can be performed using program published in [23, Appendix A]. Simulation results are obtained by simulating the EGK fading envelope using [14, eq. (9)]. One can notice better coincidence of proposed bounds to corresponding simulations (numerical integration results) for lower values of the average SNR that is significant for practical implementation because great majority of users in contemporary mobile systems operate just in the regime of low SNR values.

5. CONCLUSION

In this paper, new BER evaluation for the system with \(N\) dual-hop links, the direct path and MRC at the destination over composite EGK fading channels was presented. Novel analytical expression based on the geometric mean approximation require significantly less time (<10^{-2} seconds) for evaluating a specific value of BER compared to exact results. Additionally, this bound is confirmed to be accurate for smaller average SNR values. Presented numerical results match better simulation results for higher values of fading/shadowing parameters and/or for less number of relays. This result can be used as a good solution in the design of multiple dual-hop relay links for the previously mentioned scenarios.

Acknowledgement: This paper was realized as a part of the projects under Grants III-44006 and TR-32052 financed by the Ministry of science and technology development of the Republic of Serbia within the framework of integrated and interdisciplinary research for the period 2011-2014.

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