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COMBINED LINEAR - GEOMETRICALLY NONLINEAR FEM MODELING FOR HIGHLY EFFICIENT DYNAMICAL SIMULATIONS *

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Abstract. *Software packages for multi-body system (MBS) dynamics are efficient tools for modeling interconnected rigid and/or flexible bodies. Consideration of flexible bodies in commercially available MBS software packages is limited to linear elastic behavior. In many cases though, structural behavior includes geometrical nonlinearities, which are, however, restricted to a relatively small structural sub-domain. The paper addresses the idea of combined linear - geometrically nonlinear FEM modeling that aims at high accuracy with optimal numerical effort. The approach can be of great importance in all areas where highly efficient MBS or FEM models are required, such as robotics, car industry, etc. The idea is demonstrated in the paper on an example involving a tower crane with a suspended load. The model reduction based on modal superposition technique is used for the linear part of the model, which further improves the numerical efficiency. Dynamics is resolved by means of an explicit time integration scheme. The results by the proposed approach are compared with those computed by rigorous geometrically nonlinear approach in commercially available software package ABAQUS.*

Key words: *geometrical nonlinearity, modal superposition, tower crane, explicit time-integration scheme*

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1. INTRODUCTION

Modeling, as one of the fundamental activities engineers are involved in, explores alternative solutions with the aim of achieving a satisfying compromise between the model complexity and the accuracy of the predicted behavior of physical system. The two aforementioned objectives are not always easy to conciliate and different techniques are used to achieve what is believed to be the best compromise.

While nonlinearity is intrinsic for a great number of mechanical systems, linear models are still used in many cases as a very good approximation of the actual system behavior within a certain, typically small domain in the vicinity of the original system configuration. In many areas of application, such as car industry or robotics, software packages for multibody system (MBS) dynamics are used to simulate dynamical behavior of complex mechanisms consisting of rigid or flexible bodies. The available algorithms for consideration of flexible body behavior in MBS dynamics are restricted to linear deformations and a model reduction is typically performed using modal space [1, 2]. If, however, the considered physical behavior cannot be described accurately enough by a linear model, then a nonlinear model becomes an imperative. Resolving such a problem often implies integration of MBS and FEM (finite element method) systems [3, 4], but such an integration is a rather demanding solution. There are solutions proposed to account for geometrical nonlinearities in modal space [5, 6], but they are restricted to moderate geometrically nonlinear structural behavior.

Certain cases, however, are characterized by local nonlinearities. This implies that a great part of the considered structure behaves linearly, but with certain structural sub-domains demonstrating nonlinear behavior. There might be various reasons for such a behavior, such as a relatively large overall motion of a sub-domain with respect to the rest of the structure. That would induce internal forces over the boundaries between the sub-domains that significantly change their line of action. Typical examples for such a behavior would be structures that involve several sub-domains with rather "weak" connections to each other. The effect becomes even more obvious if certain degrees of freedom are released, i.e. joints are involved in a structure so that actually a mechanism is considered. This would allow rigid-body motions of structural sub-domains.

For such cases, the authors propose the idea of combined linear – geometrically nonlinear modeling that accounts for nonlinearities locally, i.e. only in the part of the structure affected by nonlinear behavior, while the rest of the structure is described by a linear model. The approach offers a very good compromise between the model complexity and achieved accuracy of the predicted structural behavior. It would offer a highly efficient and sufficiently accurate model description in many various analyses such as the one given in [7] and which uses genetic algorithms to determine parameters of a 3D crane system described by a highly nonlinear model. The idea presented in this paper is demonstrated using a model of a tower crane with a suspended load. Transient analysis is conducted and the efficiency of the model is further improved by using the modal superposition technique for the linear part of the model. Dynamics is resolved by means of the central difference method, which brings further advantages in combination with the above mentioned approach. The results by the proposed approach are compared with those obtained by rigorous geometrically nonlinear results computed in commercially available software package ABAQUS.

2. EQUATIONS OF TRANSIENT STRUCTURAL DYNAMICS

The idea of combined linear – nonlinear modeling is generally applicable onto both static and dynamic structural behavior. The paper, however, puts focus onto transient dynamics, as the integration of dynamic equilibrium is more demanding and the advantages of the idea come more to the fore.

The FEM equation of transient structural dynamics can be generally given as:

$$[M]\ {}^{t}\{\ddot{u}\} + {}^{t}[C]\ {}^{t}\{\dot{u}\} = {}^{t}\{f_{ext}\} - {}^{t}\{f_{\mathrm{int}}\}, \tag{1}$$

where $[M]$ and $[C]$ are the mass and damping matrices, $\{f_{ext}\}$ and $\{f_{int}\}$ are the external (excitation) and internal (elastic) forces of the FEM assemblage, $\{u\}$are the structural displacements with dots above denoting the time derivatives (i.e. acceleration and velocity), while the left superscript denotes at which moment of time the quantity is taken.

If a linear model is considered, the change of the structural configuration is neglected and the material law is assumed to remain constant, which allows the computation of the internal forces based on the structural stiffness matrix $[K]$ determined for the initial structural configuration:

$${}^{t}\left\{f_{\mathrm{int}}^{lin}\right\} = [K]\ {}^{t}\{u\}, \tag{2}$$

with $\{u\}$ denoting the current nodal displacements.

A rigorous nonlinear analysis determines the internal forces based on the current stress state in the structure, $\{\sigma\}$, and current configuration:

$${}^{t}\left\{f_{\mathrm{int}}^{nl}\right\} = \int_{{}^{t}V} {}^{t}[B]\ {}^{t}\{\sigma\} d^{t}V, \tag{3}$$

where $[B]$ is the strain-displacement matrix of the FE assemblage. The nonlinear analysis requires the update of structural properties (geometrical and/or material) upon each time-step computed in the analysis. Depending on the analysis type, iterations within a time-step may also be a part of the solution procedure and are, in fact, required in most nonlinear computations. This additionally increases the required numerical effort.

The essence of the idea proposed in this paper is to conduct structural analysis by computing structural internal forces according to either (2) or (3), depending on the suitability of the expressions for a specific structural domain. For the part of the structure, the behavior of which can be well approximated by a linear model, (2) is the suitable expression. For the part of the structure with pronounced nonlinearities, (3) is used. The approach is reasonable for the structures, a great portion of which can be covered by using (2). This enables great computational savings as the computation of the linear stiffness matrix can be done in a pre-step prior to simulation and various model reduction techniques can also be applied.

3. MODAL DECOMPOSITION FOR THE LINEAR PART OF THE MODEL

FE models typically have a large number of degrees of freedom, i.e. displacements. This number could go up to several 100,000 and the solution of the resulting system of equations is numerically rather demanding, particularly if the model is developed for the purpose of control of structural dynamics. An effective control algorithm would require reduction of the number of degrees of freedom to be controlled. One of the very efficient strategies to achieve this goal consists in modal decomposition. The technique is especially convenient if a structure is excited by a band-limited excitation. The drawback of the idea is that it is limited to linear models only. Hence, within the proposed idea, it is applicable only to the linear part of the model. In the following, the modal decomposition technique is briefly described.

Equation (1) for time t and a linear system can be given in the following form:

$$[M]\{\ddot{u}\}+[C]\{\dot{u}\}+[K]\{u\}=\{f_{ext}\}\,. \tag{4}$$

It is transformed into a more efficient form for direct integration by performing the following transformation on the finite element nodal displacements:

$$\{u\}=[P]\{z\}\,, \tag{5}$$

where $[P]$ is a square matrix and $\{z\}$ is a time-dependent vector of order n (the number of degrees of freedom of the FE model), whose components are referred to as generalized displacements. The objective of the transformation matrix $[P]$ is to obtain new system mass, damping and stiffness matrices, which have smaller bandwidth. Pre-multiplying equation (4) with $[P]^{\mathrm{T}}$ one obtains:

$$[\tilde{M}]\{\ddot{z}\}+[\tilde{C}]\{\dot{z}\}+[\tilde{K}]\{z\}=\{\tilde{f}\}\,, \tag{6}$$

where:

$$[\tilde{M}]=[P]^{\mathrm{T}}[M][P], \qquad [\tilde{C}]=[P]^{\mathrm{T}}[C][P], \qquad [\tilde{K}]=[P]^{\mathrm{T}}[K][P], \qquad \{\tilde{f}\}=[P]^{\mathrm{T}}\{f\}\,. \tag{7}$$

Although theoretically it is possible to define a number of different transformation matrices that would reduce the bandwidth of the system matrices, in practice an effective transformation matrix is obtained using the solution of the free vibration equilibrium equation with damping neglected [8]:

$$[M]\{\ddot{u}\}+[K]\{u\}=\{0\}\,. \tag{8}$$

The solution is assumed in the form:

$$\{u\}=\{\phi_i\}e^{j\omega_i t}\,, \tag{9}$$

where $\{\phi_i\}$ and ω_i must satisfy the eigenvalue problem:

$$\left([K_{uu}]-\omega_i^2[M_u]\right)\ \{\phi_i\}=0\,, \tag{10}$$

and, hence, $\{\phi_i\}$ is the i^{th} mode shape vector, and ω_i is the corresponding eigenfrequency, where the number of modes is equal to the number of degrees of freedom, n.

It can be demonstrated that the mode shapes corresponding to distinct natural frequencies are orthogonal with respect to both the mass and stiffness matrices. The orthogonality conditions are often given as:

$$\{\phi_i\}^T[M]\{\phi_j\} = \mu_i\delta_{ij}\,, \tag{11}$$

$$\{\phi_i\}^T[K]\{\phi_j\} = \mu_i\omega_i^2\delta_{ij}\,, \tag{12}$$

where δ_{ij} is the Kronecker delta (δ_{ij}=1 if i=j, and δ_{ij}=0 otherwise), μ_i is the modal mass of mode i. Since mode shapes can be scaled arbitrarily, they are usually normalized so that μ_i=1. Now, defining the matrix of the mode shapes [Φ] so that $[\Phi]=[\{\phi_1\}\ \{\phi_2\}\ \dots\ \{\phi_n\}]$, the orthogonality conditions come down to:

$$[\Phi]^T[M][\Phi] = \mathrm{diag}\,(\mu_i)\,, \tag{13}$$

$$[\Phi]^T[K][\Phi] = \mathrm{diag}\,(\mu_i\omega_i^2)\,. \tag{14}$$

If the damping matrix is defined in the form of the Rayleigh damping, then similarly to (13) and (14):

$$[\Phi]^\mathrm{T}[C][\Phi] = \mathrm{diag}\,(2\xi_i\mu_i\omega_i^2)\,, \tag{15}$$

$$\text{with} \qquad \xi_i = \frac{1}{2}\left(\frac{\alpha}{\omega_i} + \beta\omega_i\right), \tag{16}$$

where α is the coefficient of the mass proportional damping and β the coefficient of the stiffness proportional damping.

Adopting [P]=[Φ], (6) can be rewritten as:

$$\{\ddot{z}\} + 2\ \ [\xi][\Omega]\{\dot{z}\} + [\Omega]^2\{z\} = [\mu]^{-1}[\Phi]^\mathrm{T}\{f\}\,, \tag{17}$$

$$\text{with } [\xi] = \mathrm{diag}\,(\xi_i)\,,\ \ [\Omega] = \mathrm{diag}\,(\omega_i)\,, [\mu] = \mathrm{diag}\,(\mu_i)\,. \tag{18}$$

Hence, the finite element equations are decoupled and the response of the structural subdomain described by the linear model is then obtained by a superposition of the response in each mode. In modal coordinates, the number of degrees of freedom is significantly reduced.

4. Explicit Time Integration

Within an FE transient solution, the integration of dynamic equilibrium equations is the most time-consuming part. The direct integration methods are divided into the group of explicit methods and group of implicit methods. The main differences between them are the expense of calculating one time step and the time step size due to stability criteria [8]. The implicit methods are unconditionally stable, which accounts for their advantage. This means that the time step size is dependent only on the accuracy requirements of the user. However, the choice of the time step size should also take into account a relatively large computational effort in each time step due to the necessary iteration, especially in a geometrically nonlinear analysis. Contrary to the implicit methods, the time-marching-forward scheme of explicit methods does not require a factorization of the stiffness matrix. Though savings are made on avoiding the use of a matrix inverter, the price is paid by being restricted in the size of the time step, which has to be smaller than a certain critical value for the solution to be stable. The critical time step directly depends on the largest eigenfrequency of the finite element assemblage influenced by the discretization of the structure (smallest element). Hence, a short time step has to be used in the simulation, which has a negative effect on the overall computational time, but on the other hand, the iteration errors due to nonlinearities are negligible and hence, no iterations are performed.

The authors use the central difference method, which assumes the acceleration by the following finite difference expression:

$$ {}^{t}\{\ddot{u}\} = \frac{1}{\Delta t^2}\left({}^{t-\Delta t}\{u\} - 2\,{}^{t}\{u\} + {}^{t+\Delta t}\{u\} \right). \tag{19}$$

The error in (19) is of order $(\Delta t)^2$ and to have the same order of error in the velocity expansion, it is defined as:

$$ {}^{t}\{\dot{u}\} = \frac{1}{2\Delta t}\left({}^{t+\Delta t}\{u\} - {}^{t-\Delta t}\{u\} \right). \tag{20}$$

The displacements at time $t+\Delta t$ are given as:

$$ {}^{t+\Delta t}\{u\} = {}^{t}\{u\} + \Delta t\ {}^{t+\frac{1}{2}\Delta t}\{\dot{u}\}, \tag{21}$$

and starting from this one, (19) and (20) can be rearranged so that it can be written:

$$ {}^{t}\{\dot{u}\} = \frac{1}{\Delta t}\left({}^{t+\frac{1}{2}\Delta t}\{u\} - {}^{t-\frac{1}{2}\Delta t}\{u\} \right), \tag{22}$$

$$ {}^{t}\{\ddot{u}\} = \frac{1}{\Delta t}\left({}^{t+\frac{1}{2}\Delta t}\{\dot{u}\} - {}^{t-\frac{1}{2}\Delta t}\{\dot{u}\} \right). \tag{23}$$

A stability consideration of the central difference scheme gives the limit of the time step length as:

$$ \Delta t \le \Delta t_{cr} = \frac{2}{f_{n\max}}, \tag{24}$$

with $f_{n\ max}$ denoting the highest eigenfrequency of the finite element assemblage with n degrees of freedom.

Introducing (22) and (23) into (1) one obtains:

$$ \left(\frac{1}{\Delta t}\,{}^{t}[M] + \frac{1}{2}\,{}^{t}[C] \right) {}^{t+\frac{1}{2}\Delta t}\{\dot{u}_e\} - \left(\frac{1}{\Delta t}\,{}^{t}[M] + \frac{1}{2}\,{}^{t}[C] \right) {}^{t-\frac{1}{2}\Delta t}\{\dot{u}_e\} = {}^{t}\{f_{ext\,e}\} - {}^{t}\{f_{\mathrm{int}\,e}\}. \tag{25}$$

If the matrices $[M]$ and $[C]$ are diagonal, the resulting system of equations is decoupled, so that it can be rewritten for each degree of freedom i as:

$$ {}^{t+\frac{1}{2}\Delta t}\dot{u}_i = \frac{2 - c\Delta t}{2 + c\Delta t} \cdot {}^{t+\frac{1}{2}\Delta t}\dot{u}_i + \frac{2\Delta t}{m_{ii}(2 + c\Delta t)}\left({}^{t}f_{ext_i} - {}^{t}f_{\mathrm{int}_i} \right), \tag{26}$$

and (21) is used to perform the time integration.

5. APPLICATION TO DYNAMIC ANALYSIS OF A TOWER CRANE

A tower crane is a large span structure and certain points of the structure, such as the working jib tip, undergo large displacements during the operation of the tower crane. However, compared to the structural dimensions, those displacements are, for typical operational conditions and loads of a tower crane, still in the realm of the physical behavior that can be accurately enough described by a linear model.

The tower crane performs a combination of motions to reach any point within its working radius. Attached to the very top of the mast is the slewing unit comprised of a gear and motor that gives the crane the ability to rotate. A trolley is fastened to the long working jib and carries the load along the jib. Steel ropes and a hook are used to suspend a load. Due to inertial forces during the transport of the load, which involves rotational motion and translational motion along the working jib, longitudinal and side sways of the load are easily initiated. The forces induced in the ropes are the actual excitation of the tower crane's structure. The load sways are the cause of changing line of action of the steel rope internal forces. Omitting this aspect, erroneous simulation results are obtained for the dynamical behavior of both the hanging load and the structure of the tower crane. A linear model "freezes" the initial configuration and the internal forces of the rope would act along the line of the original rope direction, which is typically vertical. Pulling the load along the direction of the jib would result in rather large displacements as there is no stiffness that would resist this motion in the original configuration.

Hence, the authors apply the idea outlined in this paper to adequately model a tower crane with a suspended load. The tower crane structure is considered by a linear model. As proposed in the 3rd section, the modal superposition technique is used to further increase the efficiency of the model. The first 10 eigenmodes of the structure are taken for the demonstration purpose. The steel ropes with suspended load are considered separately by a geometrically nonlinear model coupled to the linear model of the tower crane's structure (Fig. 1).

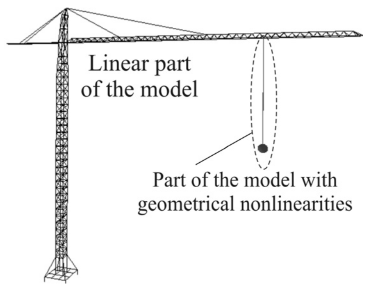


Fig. 1 Combined linear – geometrically nonlinear model of a tower crane with suspended load

If the model depicted in Fig. 1 is entirely considered as linear, any excitation that acts horizontally onto the suspended mass (perpendicular to the rope) would result in unrealistic large displacements (Fig. 2b), as no stiffness is associated to such a motion of the suspended mass and an artificial enlargement of the rope can be noticed. But taking the local rotations of the steel rope into account resolves the problem successfully (Fig. 2c).

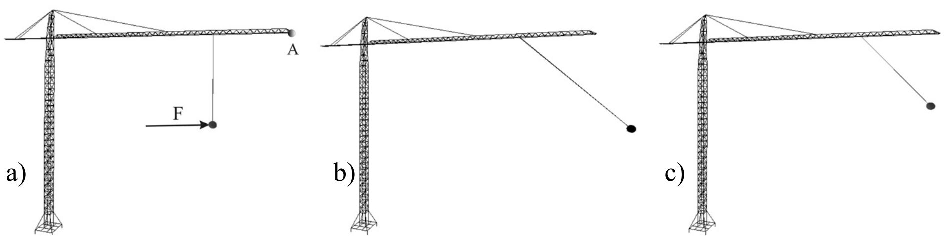
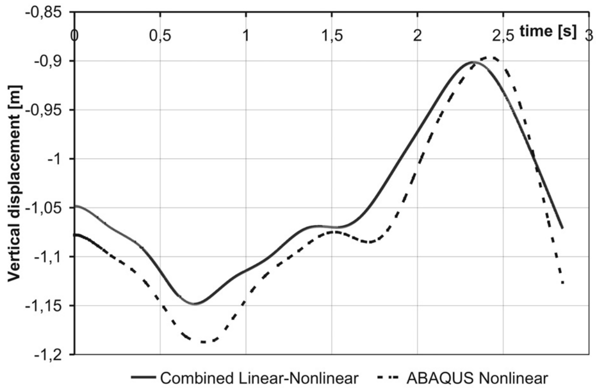


Fig. 2 Tower crane model: a) initial configuration with excitation force; b) simulation by linear model 2; c) simulation by linear – geometrically nonlinear model

To demonstrate this effect, a mass of 300 kg suspended on the rope has been exposed to a rather short impulse force of 5 kN in duration of 10^{-3} s (Fig. 2a) and the displacements of the working jib tip (point A in Fig. 2a) have been observed. The linear model yields no displacements of the working jib tip and in Fig. 3 those displacements would have been given as coinciding with the x-axis. With the linear model, the steel ropes can transfer the force only in vertical direction, while the motion of the mass is strictly horizontal. As already noted, the combined model can resolve this successfully and Fig. 3a gives the displacement of the working jib tip in the vertical direction, i.e. parallel to the tower crane mast, while Fig. 3b shows the displacement of the same point in the horizontal direction, i.e. parallel to the working jib, predicted by the combined model (solid line) and by rigorous geometrically nonlinear FEM computed in ABAQUS (dashed line) for the simulation period of 3 s. In the conducted analysis, the static computation is performed first in order to determine the statically deformed initial configuration.

a)



b)

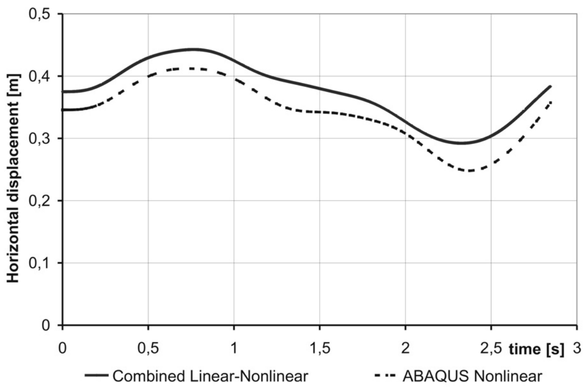


Fig. 3 Displacements of the working jib tip by presented formulation and by rigorous geometrically nonlinear computation in ABAQUS: a) in vertical direction; b) in horizontal direction

The displacements at time t = 0 s correspond to this configuration. One may notice that there are differences between the initial displacements computed in ABAQUS and by the presented formulation. This is clearly the consequence of the fact that the computation in ABAQUS is geometrically nonlinear and done with the full FE model, whereas the computation by the presented formulation is strictly linear for the static case (the rope remains vertical) and performed in the modal space with only the first 10 eigenmodes as degrees of freedom. The dynamic analysis shows a relatively good agreement between the two formulations. It should be emphasized at this point that the rigorous geometrically nonlinear computation in ABAQUS is performed with the model containing 1773 degrees of freedom, while the model used with the presented formulation has only 13 degrees of freedom (10 eigenmodes and the 3 displacements of the suspended mass). Consequently, performing the both computations with the same time-step, the computational time in ABAQUS was two orders of magnitude greater (measured in seconds) than the one needed for the presented formulation.

6. Conclusions

The paper elaborates the idea of combined linear - geometrically nonlinear FEM modeling with the aim of improving simulation accuracy whereby the required numerical effort is kept moderate. If the approach is used for structural dynamics, then a possibility for further improvement of model efficiency has been proposed in the form of model reduction for the linear part of the model. The principles of modal superposition as one of the simplest solutions for model reduction are briefly given. In practical applications, however, different possibilities for the choice of modes should be considered, such as application of Craig-Bamption method of component mode synthesis [9]. This would offer higher modeling flexibility as variable boundary conditions can be considered during the simulation.

A relatively simple example of a tower crane with suspended load is chosen to demonstrate the idea. The cause of geometrical nonlinearity resides in local rotations of the steel ropes as a consequence of load sways. The dynamics is resolved by means of central difference method. An entirely linear model would yield practically useless results in this case as they would be rather unrealistic. The comparison of the results obtained by the proposed approach with those of the rigorous geometrically nonlinear results obtained in ABAQUS yields a relatively high level of agreement, whereby the numerical effort with the presented approach is enormously reduced.

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